KEY STAGE 3
GRADES 6-8

Mathematical Treasure Hunt

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Instructions for Teachers

Introduction

The mathematical treasure hunt is a great activity for fun and engaging mathematics lessons: the pupils follow a trail of clues and mathematical problems around the school site; each clue contains a hint to where the next clue is hidden.

This document includes clues and questions intended for Key Stage 3 (UK) or grades 6–8 (US).

The treasure hunt works best when the class is divided into groups of about 5 children of different abilities. Working in a team, and in a competition, supports team working skills, and even children with difficulties in mathematics can participate.

The questions are taken from a wide range of different topics, and often not directly related to the mathematics curriculum. Some of the problems lend themselves to further discussion afterwards; often there is an article on that topic in the Mathigon World of Mathematics.

The answer to each problem is an integer, and all the answers – once decoded into letters – spell the location of the treasure.: the library.

The Questions

	Name	Locations	Solution	Order of Teams				
A	Cryptography		18	1	9	7	5	3
B	Combinatorics		18	2	10	8	6	4
C	Graph Theory		2	3	1	9	7	5
D	Number Pyramid		9	4	2	10	8	6
E	Pascal's Triangle		8	5	3	1	9	7
F	Prime Numbers		25	6	4	2	10	8
G	Probability		5	7	5	3	1	9
H	Platonic Solids		12	8	6	4	2	10
I	Geometry		1	9	7	5	3	1
J	Sequences		20	10	8	6	4	2

Preparation

First choose 10 locations in your school where to hide the the different questions (see previous table). Either use the prepared clues (pages 9–10) or come up with your own clues (pages 11–12) to lead to these questions. Print the clues once for each team.

Make sure that the class is able to solve all the problems. Print the introductory sheets and questions (pages 3–8) once for every team and cut them in the middle. Print and cut the additional materials for various problems (pages 13–14).

Put the questions, materials as well as the clues leading to the *next* question into an envelope, and hide the 10 envelopes around the school site. Keep the two introductory sheets for each team, as well as a different clue for each team – the ones leading to their first problem.

At the beginning of the lesson, divide the class into a couple of teams and give each team the two introductory sheets, as well as their first clue. The treasure is hidden in the library – usually chocolate works well…

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Mathematical Treasure Hunt

INSTRUCTIONS

Professor Integer was one of the world's most famous mathematicians, who made discoveries that changed the world forever: from algorithms for computers and internet to statistical calculations and quantum mechanical predictions.

When he died, he had no relatives or close friends but a very large fortune. He believed that only the best mathematicians deserved to find his treasure and created a trail of puzzles and problems.

Many of his diary pages, notes and letters are archived at the University of Cantortown, and they all include clues and hints regarding the location of the treasure.

> *This treasure hunt will require you to move around your school, find the hidden clues and solve mathematical problems. Each question will contain a clue about where the next problem will be hidden, but every team solves the problems in a different order.*
>
> *When you find an envelope, take one problem page and one clue. Try to solve the problem, sometimes using additional materials in the envelope; then look for the next problem. You may not find the problems in the correct order!*
>
> *There are many other children in the school, so avoid any unnecessary noise. Don't leave your solutions behind for the next team to see, and don't take more than one copy of each problem – otherwise following teams might not be able to solve the problem.*

You are now ready to receive the first clue and a copy of the last letter written by Professor Integer.

Good luck!

The Integer Files	
Archive of the University of Cantortown	Item 0
Item 0: Last letter of Prof. Integer	Catalogue Nr. 0010

Dear Mathematicians,

When you read this letter, I will be dead, and my treasure will be hidden in a very safe location. Only the best mathematicians deserve to find it.

In my notes and diaries, I have left 10 problems which you need to solve. The answer to every problem is a single number, which you can write down here:

A	B	C	D	E	F	G	H	I	J

Once you have solved all problems, turn the numbers into letters (1-a, 2-b, 3-c and so on) and bring the letters into the correct order to spell the location of the treasure:

_ _ _   _ _ _ _ _ _ _

Hurry, though, because other teams may be onto it as well...

Regards – and good Luck!
Prof. Integer

Problem A: Cryptography

I think somebody has broken into my study and stolen important documents and calculations. It is a disaster that I have lost my notes, but it is even worse that the thief can read my discoveries and ideas.

In the future, I need to decipher my notes, so that only I can read them. A very easy method was invented by Julius Caesar: you just shift ever letter along the alphabet, for example

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s

The word 'mathematician' for example would be shifted to 'ftmaxftmbvbtg'.

To decipher this code, one would have to try all 24 possibilities to shift the letter, which could take a very long time. This should keep my notes safe in the future!

Note: Cryptography is the area of mathematics about finding and breaking codes. It was especially important in wars: during the second world war, the Cambridge Mathematician Alan Turing successfully built one of the first computers to decode the German Enigma coding machine. This could have well been the single most important achievement that led to the allied victory.

There are many much more complicated methods to decode sentences today, some of which (we think) are unbreakable and without which internet banking would be impossible. They use prime numbers and many important mathematical results.

MAX TGLPXK BL XBZAMXXG

Problem B: Combinatorics

Yesterday I was invited to a Birthday party. There were 36 guests and everybody shook hands with everybody else exactly once.

Afterwards, I wondered how many hand shakes there were in total. It clearly is impractical to count them all one by one; we need a clever mathematical idea to find a simple equation...

To get the key number for this problem, divide the total number of handshakes by 35.

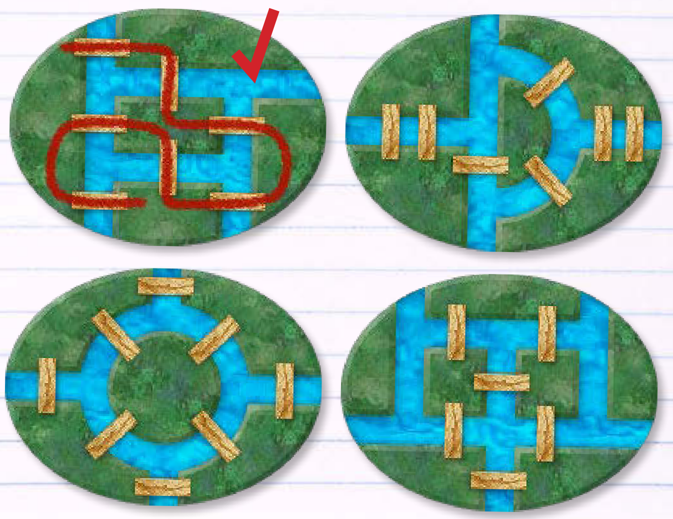
I wonder whether you can use similar ideas to calculate the probability to win in lotto: How many ways are there to choose 6 numbers out of 49. This is related to an area of maths called Combinatorics.

Problem C: Graph Theory

Last week I visited Königsberg, a city in Russia. Königsberg is divided into several parts by a river, and the islands are connected by bridges.

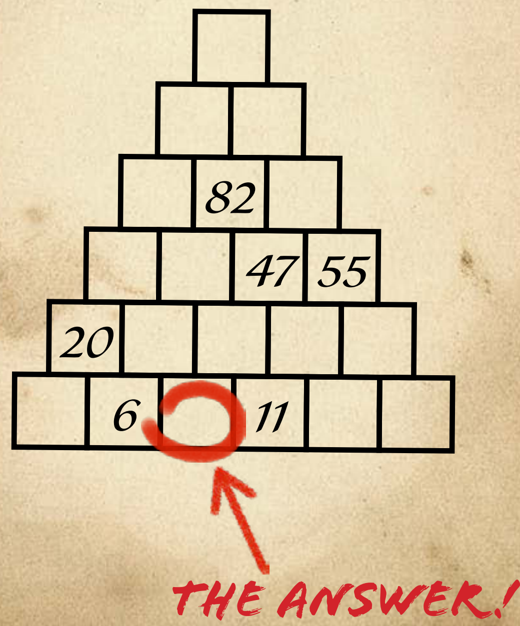
Many years ago, the mathematician Leonard Euler asked whether it would be possible to tour Königsberg, so that you cross every bridge once, but not more than once.

Here are a couple of other city maps. In how many maps is it IMPOSSIBLE to find a tour that crosses every bridge exactly once? You can start and finish wherever you want.



Problem D: Number Pyramid

Last night I was thinking about a large number pyramid. Unfortunately I spilled my coffee, and I lost many of the numbers – only 6 remained legible. I was thinking about it for some time, and I think it is possible to reconstruct the whole pyramid using only those 6 numbers!

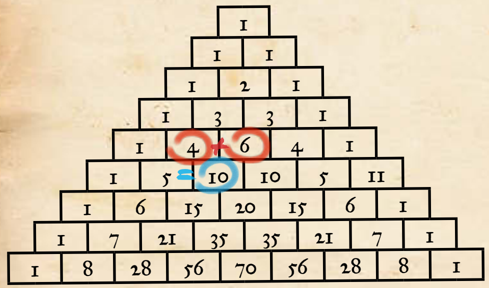


Two lines or curves are orthogonal if they are perpendicular at their point of intersection. Two vectors are orthogonal if and only if their dot product is zero.

Pascal's Triangle

In mathematics, Pascal's triangle is a triangular array of binomial coefficients. It is named after the French mathematician Blaise Pascal, but other mathematicians studied it centuries before him in India and China.

A simple construction of the triangle proceeds in the following manner. In the first row, write only the number 1. Then, to construct the elements of following rows, add the two numbers above a cell to make the number in the new cell. For example, the first number in the first row is 0 & 1 = 1, whereas the numbers 1 and 3 in the third row are added to produce the number 4 in the fourth row.



Problem E: Pascal's Triangle

I tried colouring in all cells divisible by 3 in Pascal's triangle with 16 rows. Guess how long the base of the largest coloured triangle was ...

Pascal's triangle has many interesting properties. It is symmetric, the diagonals are all 1s, the second diagonals are the integers 1, 2, 3, ... and the third diagonal are the triangle numbers 1, 2, 6, 10, ... Many other interesting number sequences and patterns can be found if you look more closely.

A particularly interesting thing happens when you colour in all cells that are divisible by 2 or 3. The result will be a pattern of many more triangles of various sizes. As you try this with bigger and bigger versions of Pascal's triangle, it starts looking like a fractal, a shape which repeats itself on

Problem F: Prime Numbers

We say that a number y is a factor of a number x if you can make x by multiplying y with another number. For example, 7 is a factor of 21 since, 21 = 7 × 3.

A number which has no factors apart from 1 and itself is called a prime number. Note however that 1 itself is not a prime number! Prime numbers play a very important role in mathematics, since they can't be divided any further. They are like the "atoms" of numbers.

Eratosthenes, a Greek mathematician, found an easy way to calculate all the prime numbers less than 100. It is called the Sieve of Eratosthenes. You will need one of the 100-tables in the envelope.

We start by circling the smallest prime number, 2. Then we cross out all multiples of 2 less than 100 – these numbers can't be prime numbers, since they are divisible by 2.

Now we circle the next number which isn't crossed out, in this case 3, and cross out all multiples of 3; again these numbers can't be prime.

Since 4 is crossed out, the next number we circle is 5 and we cross out the remaining multiples of 5. We continue until all numbers are either circled or crossed out (some of them may be crossed out several times!). Then all remaining circled numbers are prime numbers.

How many prime Numbers are there less than 100?

Problem G: Probability

I love playing cards - poker, blackjack and many other games. And card games are very much related to probability!

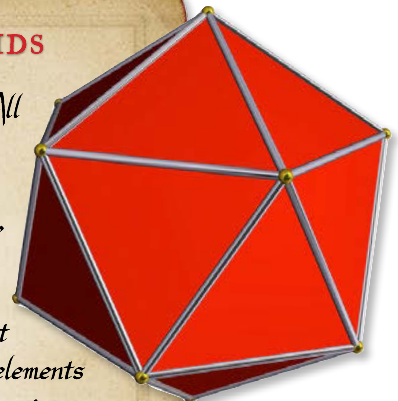
Here is one question I was thinking about: A normal deck of cards contains 52 cards in four different suits (clubs and spades are black, hearts and diamonds are red.

Suppose we choose one card at random and put it back, then choose a second card at random and put it back, and then choose a third card at random.

What is the probability (in percent) that all three cards have the same colour (red or black)? If the probability is X%, i am looking for the square root of the number X.

Playing Cards by Wikimedia Users Asimzb and Jfitch

Problem H: Platonic Solids



This shape is called an Icosahedron. All faces are equilateral triangles, and it looks the same from every direction. Therefore it is called a Platonic Solid, named after the Greek mathematician Plato. Plato showed that there are only five solids of this kind. He though that they corresponded to the four classical elements fire, air, earth and fire, as well as the universe.

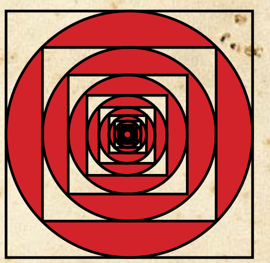
Here is a table showing all 5 platonic solids. Can you find a pattern and fill in the gaps?

Name	Model	Faces	Vertices	Edges
Tetrahedron		4		6
Cube		6	8	12
Octahedron		8	6	
Dodecahedron			20	30
Icosahedron		20		30

MAYBE THINK ABOUT FACES + VERTICES!

My dear friend,

Here's a fun problem: Can you work out which proportion of this square is red?

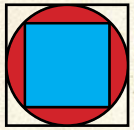


I received these two letters from Prof. Interger just a couple of days before he died.

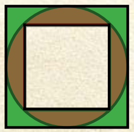
Problem I: Geometry

My dear friend,

I realised that my previous problem was rather hard, so here are some hints: Let us assume that the big square has length 1. First, we need to calculate the area of the biggest circle and the area of the second biggest square.



Now notice that the original shape consists of a single frame, which is repeated again and again – just smaller. Thus the proportion red in the final shape is exactly the same as the proportion red in the frame.



Can you work out the proportion red in this frame? Here are three possibilities: $p = 0.57$ $p = 0.49$ $p = 0.68$

KEY: 1 2 3

Problem J: Sequences

A sequences is a list of numbers which follow a certain pattern. For example, the square numbers, the powers of two or the prime numbers are all sequences.

A very famous sequence are the Fibonacci numbers. Starting with 1, 1, every following number is the sum of the previous two numbers. The third number is 1+1=2, the fourth number is 1+2=3 and so on. We get

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Discovered by the Italian mathematician Leonardo Fibonacci, these numbers appear in many places in nature: from rabbit populations to sunflower seeds.

I love playing around with sequences. Here are a few examples, you need to find the pattern and fill in the missing numbers.

1, 3, 6, 10, __, __, __

1, 3, 9, 27, __, __, __

3, 6, 5, 10, 9, __, __, __, __

The answer to this problem is the sum of the individual digits of the numbers in the circles.

Full of paper,
books and files,
Pay the school office
some smiles!

School Office

Bonjour, Hola,
Goddag, Ni Hao,
And more if
languages allow.

Languages Room

Find the riddle
that is given,
Where the bits and
bytes are livin'.

Computer Room

With watercolour,
crayons, pen,
The next puzzle is
waiting then.

Art and Crafts Room

IN BREAKTIME ITS BRAWLING,
IN LESSONS IS STILL,
ON THE PLAYGROUND THE NEXT
RIDDLE FINDING YOU WILL.

Playground

Where smoke and where fire
are common event,
The following mystery
I will present.

Chemistry Lab

Where 10 divided 5 is 2,
The next questions,
it waits for you.

Mathematics Room

No pupil may enter,
no child may come in,
Where the next clue is hidden,
so you can begin!

Staff Common Room

Trumpet fanfares –
no delay!
And music sounds
will lead your way.

Music Room

Hurry, less than
80 days,
For you to reach
the problem's place.

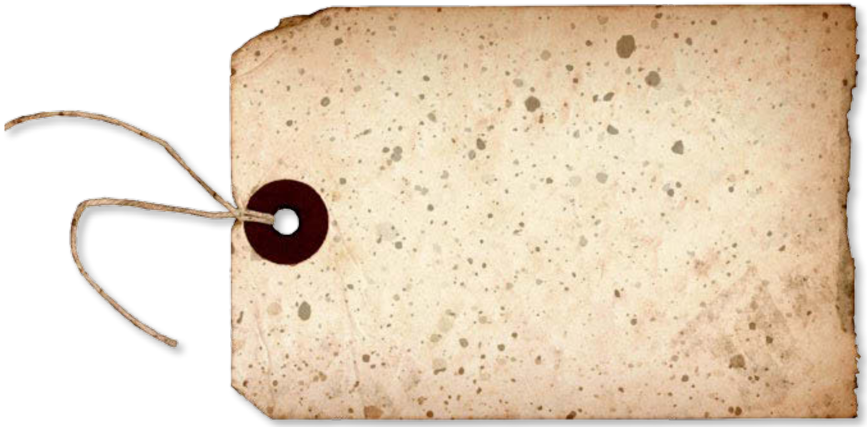
Geography Room

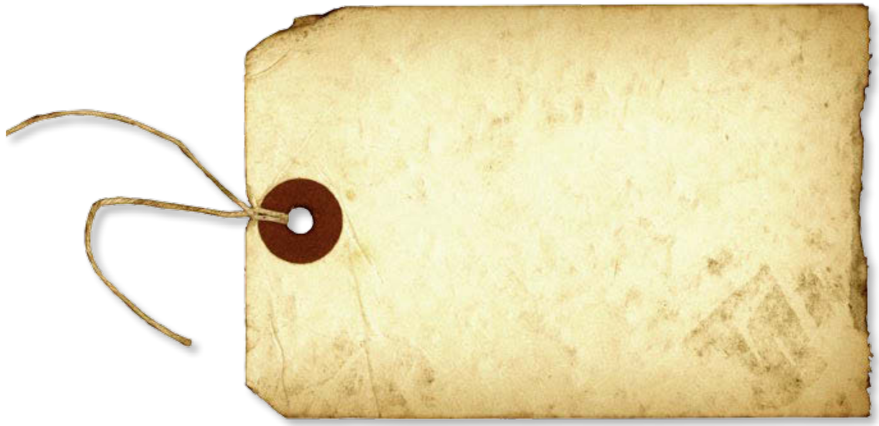
The biggest room
that is in sight,
But try to knock –
it is polite.

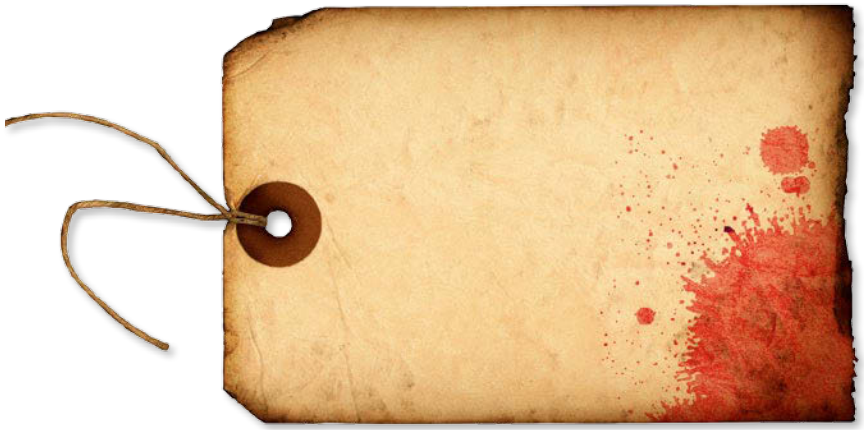
Hall / Auditorium

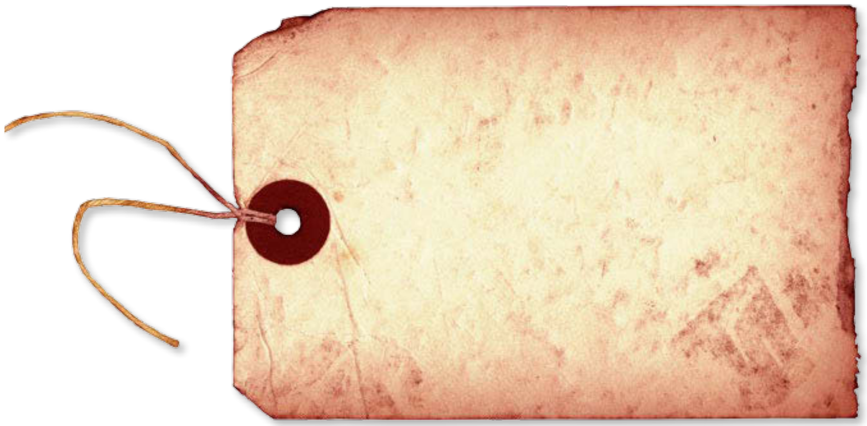
Up and down
and left and right,
The staircases
they do excite.

Starircases

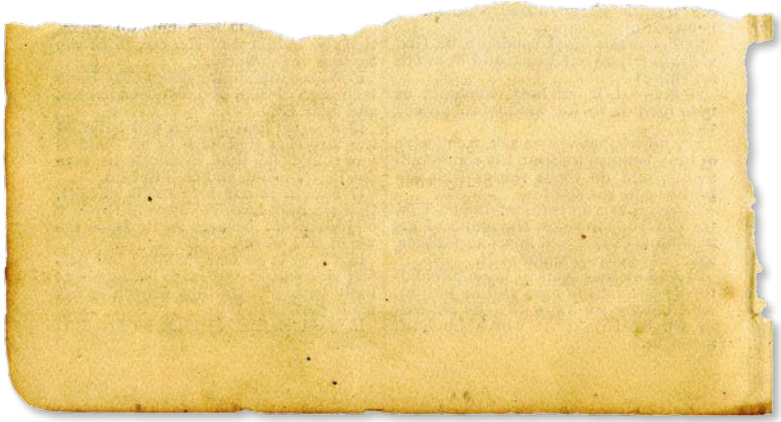


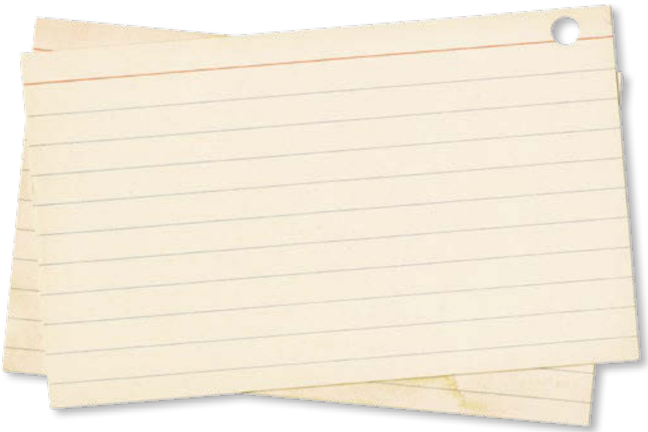


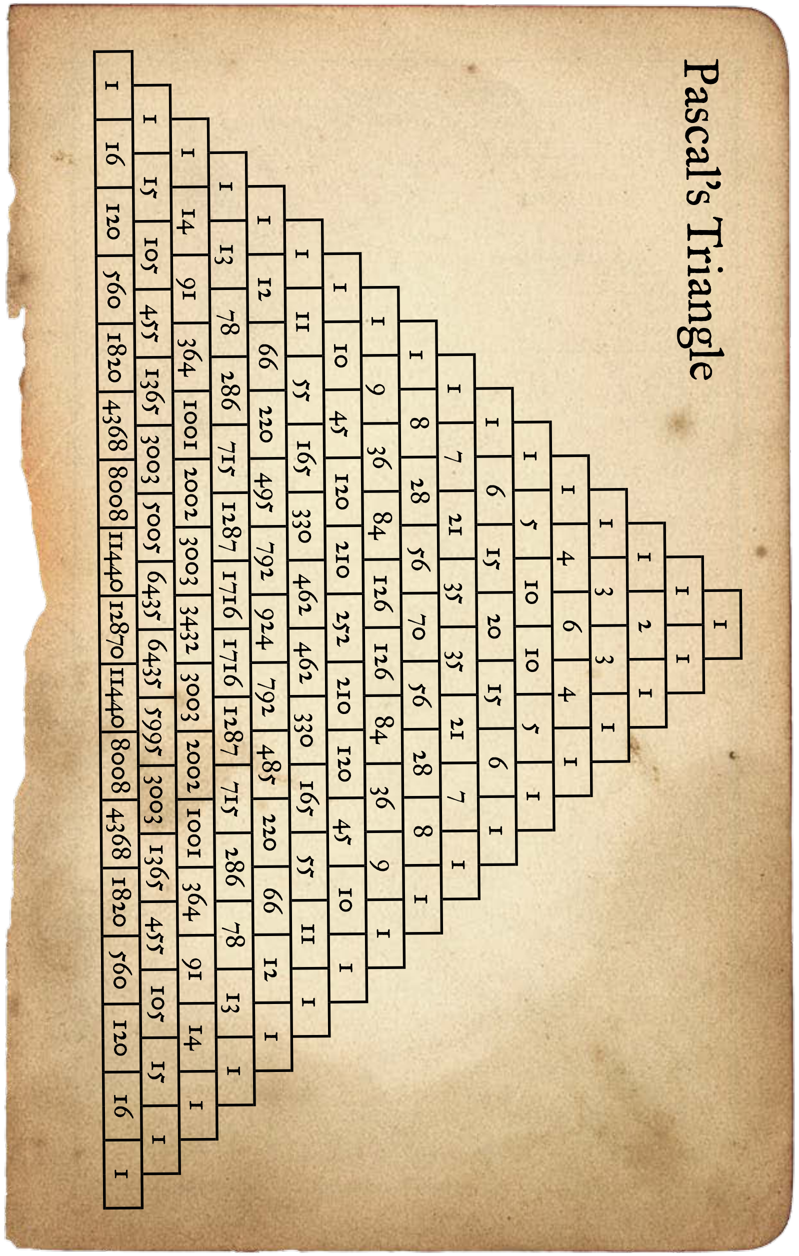


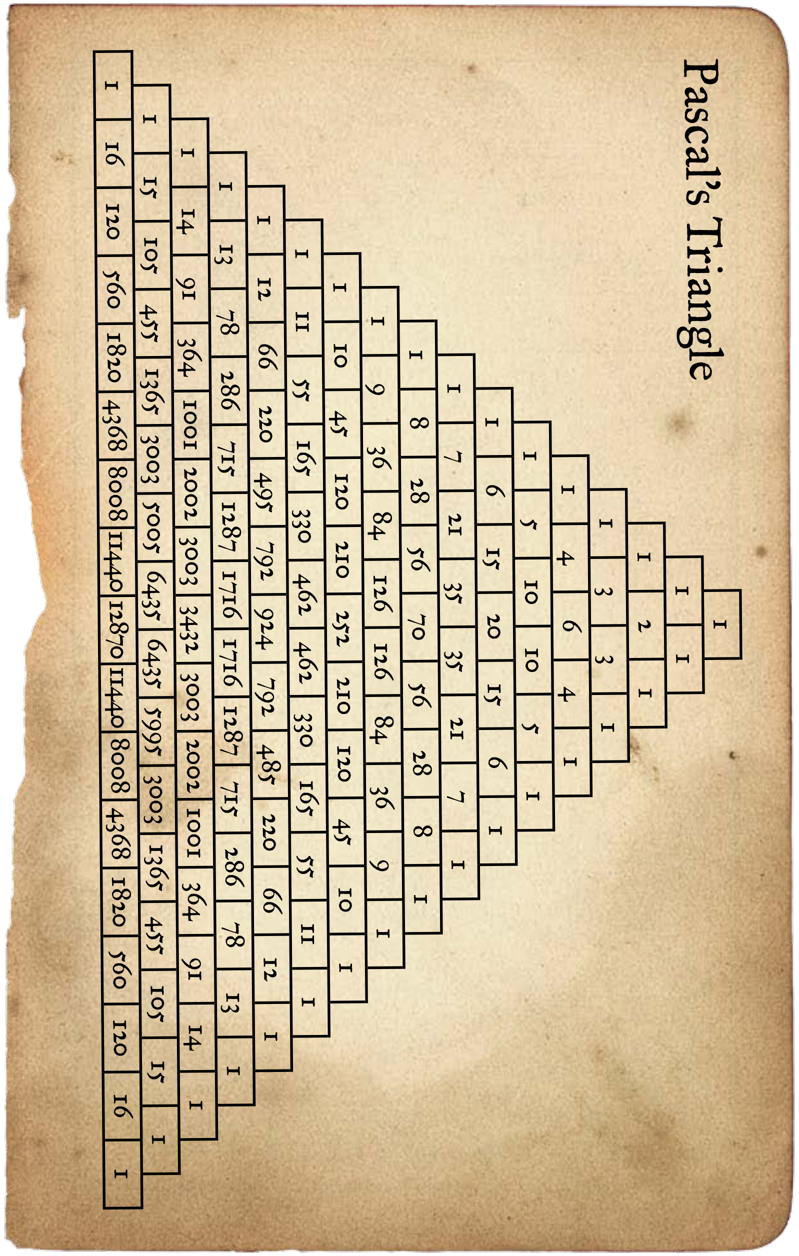










Pascal's Triangle

Print several times for each group, cut out and add to problem E

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

100 Number Table

Print several times for each group, cut out and add to problem F