

# **Composite Estimators in Statistical Analysis**

## **1. Introduction**

Statistical estimation forms the bedrock of quantitative research, providing the means to infer characteristics of a population based on a subset of observed data. The fundamental aim is to obtain precise and accurate approximations of population parameters, enabling informed decision-making and a deeper understanding of underlying phenomena. However, achieving this goal is often fraught with challenges, particularly when dealing with intricate survey designs, limited sample sizes for specific subgroups, or the need to track changes over time. In such scenarios, traditional estimation methods may fall short, necessitating the adoption of more advanced techniques. Composite estimation emerges as a powerful statistical tool designed to address these complexities. By strategically combining multiple sources of information or different types of estimators, it seeks to enhance the overall quality of the final estimate, often resulting in improved precision and accuracy. This report aims to provide a comprehensive exploration of composite estimators, encompassing their definition, the underlying rationale for their use, a detailed examination of various types, their mathematical formulations, inherent advantages and disadvantages, and their diverse applications within the field of statistics.

## **2. Definition of a Composite Estimator**

A composite estimator, at its core, represents a statistical approach that synthesizes two or more distinct estimators to yield a single, hopefully superior, estimate of a particular population parameter.<sup>1</sup> This combination is typically achieved through a process of assigning weights to each of the individual estimators. The determination of these weights is not arbitrary but is based on specific criteria, often related to the estimated statistical properties of the component estimators, such as their precision (variance) or potential for bias. The primary objective behind employing composite estimation is to harness the strengths of the individual estimators while simultaneously mitigating their respective weaknesses. This strategic amalgamation often leads to a final estimator that exhibits a lower mean squared error (MSE) than any of its constituent parts.<sup>2</sup> In essence, it's a method of creating a more robust and reliable estimate by leveraging the complementary information provided by different statistical approaches.<sup>2</sup> As noted, composite estimators can be viewed as weighted averages of current and previous estimates, particularly in the context of longitudinal surveys.<sup>2</sup>

### 3. Rationale for Using Composite Estimators

The adoption of composite estimators in statistical practice is driven by several key motivations, primarily centered around enhancing the quality and reliability of estimates. One of the foremost reasons is the potential for **reducing variance**. This is particularly crucial when dealing with individual estimators that suffer from high variability, such as direct estimators applied to small subpopulations where sample sizes are inherently limited.<sup>2</sup> For instance, in the Current

Population Survey (CPS), composite estimators are employed to improve the efficiency of over-the-month change estimates.<sup>2</sup> Moreover, composite estimation can be effectively utilized in rotation designs to decrease the variance of level estimates.<sup>2</sup> In repeated surveys with overlapping samples, these techniques exploit correlations in the data arising from the sample overlap between different survey times, thereby enhancing the precision of current estimates.<sup>3</sup> Even in contexts like environmental monitoring, where spatially overlapping surveys are conducted, composite estimation offers a statistical means to obtain a precision-weighted combination of survey estimates, resulting in improved population estimates with lower variances.<sup>5</sup>

Another significant rationale lies in the ability of composite estimators to **leverage correlations in repeated surveys**. Surveys that interview the same units over multiple time periods, often employing rotating panel designs, generate correlated data. Composite estimators are specifically designed to capitalize on these temporal correlations between measurements from the same individuals or households across different survey waves.<sup>2</sup> By incorporating information from previous survey rounds, these estimators can refine the estimates for the current period, leading to more accurate assessments of both current levels and changes over time.

Composite estimators also play an indispensable role in **improving estimates for small domains** or subpopulations where the sample size is insufficient to yield reliable direct estimates. In such situations, composite estimators bridge the gap by combining direct estimates, which might be unstable due to small within-domain sample sizes, with synthetic estimates that "borrow strength" from

other, larger domains or related populations.<sup>6</sup> Direct estimators, which rely solely on data from the area of interest, can exhibit high variance when sample sizes are small.<sup>7</sup> Synthetic estimators, while often having lower variance, may introduce bias by assuming homogeneity across different areas.<sup>7</sup> Composite estimators, by combining these two types, aim to strike a balance, leveraging the unbiasedness of direct estimators (when sample size permits) and the stability of synthetic estimators to produce more accurate estimates for small areas. This approach recognizes that in many practical scenarios, particularly when granular data is required, a hybrid approach can be more effective than relying on either direct or synthetic estimation alone.

Furthermore, composite estimation offers a mechanism for **incorporating prior information** or knowledge derived from past experience into the estimation process.<sup>1</sup> This is particularly relevant in situations where sample data might be sparse or of limited quality. By allowing for the combination of sample-based estimates with information stemming from other sources, composite estimation provides a more comprehensive and potentially more accurate assessment of the parameter of interest.

The effectiveness of composite estimators in reducing variance is often directly related to the strength of the correlation being exploited in the data. When there is a high degree of correlation, such as in repeated measurements of relatively stable characteristics, the gains in efficiency achieved by using a composite estimator tend to be more substantial. This is because previous data provides a strong indication of current values, allowing the composite estimator to smooth out random fluctuations in the current sample more effectively. Conversely, if the correlation is

weak, the benefits of using a composite estimator for variance reduction might be less pronounced.

The design and application of composite estimators also involve a critical consideration of the trade-off between bias and variance. While the primary aim is often to reduce variance and improve precision, the act of combining different estimators, especially those with differing bias properties, can potentially introduce or exacerbate bias. This is particularly true if the weights assigned to the component estimators are not chosen judiciously. The goal in composite estimation is to find a balance that minimizes the overall error, often quantified by the mean squared error (MSE), which takes into account both the variance and the squared bias of the estimator. Therefore, a thorough understanding of the bias and variance characteristics of the individual estimators being combined is essential for the successful application of composite estimation.

## **4. Types of Composite Estimators**

A variety of composite estimators have been developed to address different statistical needs and survey designs. These estimators differ in their underlying principles, mathematical formulations, and the specific types of component estimators they combine.

### **4.1 Simple Linear Combination of Direct and Synthetic Estimators:**

This represents a fundamental type of composite estimation, particularly prevalent in the context of small area estimation.<sup>8</sup> It involves creating a weighted average of two primary types of estimators: direct and synthetic. Direct estimators utilize data solely from the specific geographic area or subpopulation of interest to

produce an estimate.<sup>7</sup> These estimators are generally unbiased, meaning that on average, they will accurately reflect the true population parameter for that area. However, when the sample size within the area is small, direct estimators can suffer from high variance, making them less reliable.<sup>7</sup> Synthetic estimators, in contrast, employ data from outside the specific area of interest, relying on the assumption that the small area is similar to other areas with more abundant data.<sup>7</sup> While synthetic estimators often exhibit lower variance, they are generally biased, as the assumption of similarity might not perfectly hold.<sup>7</sup>

The mathematical formula for a simple linear composite estimator is typically expressed as:

$$Y_{\sim hC} = (1 - \phi_h) y_{\sim h} + \phi_h y^- \quad ^8$$

where  $Y_{\sim hC}$  is the composite estimate for area  $h$ ,  $y_{\sim h}$  is the direct estimator for area  $h$ ,  $y^-$  is the synthetic estimator for area  $h$ , and  $\phi_h$  is a weight ranging between 0 and 1 that determines the relative contribution of each estimator.<sup>8</sup> The choice of the weight  $\phi_h$  is crucial and is often determined by attempting to minimize the mean squared error (MSE) of the composite estimator. This involves finding a balance between the unbiasedness of the direct estimator and the lower variance of the synthetic estimator.<sup>8</sup> A weighting scheme that considers the inverse of the mean square error of each component estimator has also been discussed.<sup>18</sup>

The optimal weight in this simple linear combination is not a fixed value but rather depends on the specific characteristics of the direct and synthetic estimators for each individual domain, particularly their relative precision (variance) and accuracy (bias). For domains where the direct estimator is based on a very small sample size and

thus has a high variance, a larger weight would likely be assigned to the synthetic estimator to enhance stability. Conversely, for domains with more substantial direct data, a greater weight might be given to the direct estimator to minimize the potential for bias from the synthetic component. The simplicity of this linear combination approach offers computational ease and interpretability. However, it might not fully capture the complex relationships between the area of interest and the areas from which synthetic information is derived, potentially limiting the extent of efficiency gains or bias reduction compared to more sophisticated methods.

## 4.2 The AK Estimator:

The AK estimator stands as a specific and widely used composite estimator within the context of official labor force statistics in the United States. It has been employed by the U.S. Current Population Survey (CPS) since 1998 for the estimation of monthly labor force figures.<sup>2</sup> The primary purpose of the AK estimator is to reduce the variance associated with estimates of over-the-month change in key labor market indicators, such as employment and unemployment levels.<sup>2</sup> This variance reduction is achieved by strategically exploiting the correlations inherent in the CPS's rotating panel survey design.<sup>2</sup>

The mathematical formula for the AK composite estimator is given by:

$$\hat{Y}_{t|AK} = (1 - K)\hat{Y}_{t|SS} + K(\hat{Y}_{t-1|AK} + \Delta_t) + A\hat{\beta}_t$$

Here,  $\hat{Y}_{t|SS}$  represents the second-stage estimate for the current month  $t$ .<sup>2</sup> Second-stage estimation in the



CPS involves a weighting process that benchmarks sample estimates to independent population controls, thereby reducing variance.<sup>2</sup>  $\hat{Y}_{t-1}^{AK}$  denotes the AK composite estimate from the previous month ( $t - 1$ ).<sup>2</sup> The term  $K$  is a recursion weight coefficient, with a value of 0.7 for employed persons and 0.4 for unemployed persons.<sup>2</sup> This difference in  $K$  values suggests that the temporal correlation structure differs between these two labor market states.  $\Delta_t$  is an estimate of the over-the-month change specifically within the continuing rotation groups of the CPS sample.<sup>2</sup> The CPS employs a rotating panel design where households are in the sample for four months, out for eight, and then back in for four more. The continuing rotation groups are those households that were also in the sample in the previous month. The coefficient  $A$  has a value of 0.4 for employed and 0.3 for unemployed.<sup>2</sup> Finally,  $\hat{\beta}_t$  represents an adjustment for potential rotation group effects, calculated as the average difference between the second-stage estimates of the new rotation groups entering the survey and the continuing rotation groups.<sup>2</sup> This component aims to account for any systematic differences that might arise due to the different time-in-sample profiles of the respondents.

The AK estimator is inherently recursive, as the estimate for the current month depends on the composite estimate from the previous month.<sup>2</sup> While the AK estimator has proven effective in reducing the variance of over-the-month change estimates, it does come with a trade-off. The use of unequal compositing coefficients and the adjustments for rotation group effects can introduce a persistent bias into the estimates.<sup>2</sup>

Parameter	Description	Value (Employed)	Value (Unemployed)



$\hat{Y}_{t SS}$	Current month's second-stage estimate	-	-
$\hat{Y}_{t-1 AK}$	Previous month's AK composite estimate	-	-
K	Recursion weight coefficient	0.7	0.4
$\Delta_t$	Estimate of change in continuing rotation groups	-	-
A	Coefficient for rotation group effects adjustment	0.4	0.3
$\hat{\beta}_t$	Average difference between new and continuing rotation groups	-	-

The different values of the K and A coefficients for employed and unemployed persons likely reflect the underlying statistical differences in the stability and temporal correlation of these two labor market indicators. Employment tends to be a more persistent state than unemployment, justifying a higher weight on the previous

month's estimate for employed individuals. The recursive nature of the AK estimator implies that any bias introduced in one month's estimate can potentially propagate to subsequent months, making it crucial to understand the sources and magnitude of this persistent bias for accurate interpretation of CPS data.

### **4.3 Composite Regression Estimators:**

Composite regression estimators represent a more sophisticated class of techniques that utilize regression methodology to combine information from various sources or time points. These are particularly valuable in the context of repeated surveys that employ rotating panel designs.<sup>3</sup> A key aspect of some composite regression approaches is the concept of simultaneous calibration of sampling weights from the current and previous survey times.<sup>3</sup> This involves adjusting the weights to align with known population totals for auxiliary variables while also ensuring consistency across different time periods. These methods can lead to the development of multivariate composite regression estimators that efficiently integrate information from multiple survey waves.<sup>3</sup> Notably, certain advanced composite regression techniques avoid the need for micro-matching of data in the common sample, simplifying the process and mitigating potential issues associated with linking records across time.<sup>3</sup> The Fuller & Rao (2001) regression composite method, and its adaptation by Statistics Canada, stands as a prominent example of this type of estimator.<sup>19</sup> Modified regression estimators (MR1 and MR2), initially developed for the Canadian Labour Force Survey (LFS), also fall under this category.<sup>3</sup> Furthermore, the Fuller-Rao method has been recognized for its ability to address potential "drift problems" that can arise in regression composite estimation, where estimates might deviate

from underlying trends over time.<sup>19</sup> Empirical studies have indicated that the Fuller-Rao estimator can offer advantages over the AK estimator in terms of weighting procedures, consistency, and efficiency gains.<sup>19</sup>

The evolution of composite regression estimators reflects a continuous effort to refine these methods, addressing challenges like drift and enhancing their efficiency and applicability in dynamic survey environments. Early modified regression approaches laid the groundwork, but the identification of limitations such as the drift problem spurred the development of more advanced techniques like the Fuller-Rao method. The use of simultaneous calibration in some of these estimators signifies a move towards more integrated approaches that consider multiple constraints and relationships within the survey data, potentially leading to more robust and coherent estimates compared to sequential or independent calibration methods.

#### **4.4 Modified Regression (MR) Estimators:**

Modified Regression (MR) estimators were specifically developed for use in the Canadian Labour Force Survey (LFS) to improve the efficiency of estimates.<sup>3</sup> Two primary versions, MR1 and MR2, were introduced. MR1 has been found to perform better for estimating the level of a characteristic at a specific point in time, while MR2 is generally more effective for estimating the change in a characteristic between two time periods.<sup>19</sup> Recognizing the need for an estimator that performs well across both types of estimates, Fuller and Rao (2001) proposed a composite modified regression estimator as a compromise between MR1 and MR2.<sup>19</sup> This compromise aims to provide a more balanced performance for both point-in-time and

movement estimates.

The development of two distinct MR estimators, each tailored for a specific estimation goal (level vs. change), underscores that the optimal composite estimator can vary depending on the parameters of interest in a survey. This suggests that a one-size-fits-all approach might not always be best, and estimators should be designed with specific objectives in mind. The compromise offered by the Fuller-Rao estimator reflects a practical need for methods that can provide good overall performance across different analytical objectives, particularly in routine statistical production where a single estimator is often preferred for simplicity and consistency.

#### **4.5 Fuller-Rao Regression Composite Estimator:**

The Fuller-Rao regression composite estimator was specifically developed to address a problem known as "drift" that was observed in earlier regression composite estimators.<sup>19</sup> This drift refers to a tendency for the estimates to move away from the true underlying values over time. The Fuller-Rao method tackles this issue by employing a weighted combination of the proxy variables that were used in the MR1 and MR2 estimators.<sup>20</sup> Empirical studies comparing the Fuller-Rao estimator with the AK estimator have indicated that the Fuller-Rao method offers several advantages, including a more robust weighting procedure, greater consistency, and improved efficiency.<sup>19</sup> These findings suggest that the Fuller-Rao estimator has the potential to improve the estimation of both levels and month-to-month changes in important statistics like unemployment rates.<sup>20</sup>

The identification and subsequent rectification of the drift problem

highlight the importance of rigorous evaluation and continuous refinement of statistical methods, especially when dealing with time-series data. The superior performance of the Fuller-Rao estimator over the AK estimator in some studies suggests that more advanced regression-based methods might offer further improvements in the accuracy and efficiency of labor force statistics estimation, potentially warranting consideration for future methodological updates.

#### **4.6 Composite Calibration Estimators:**

Composite calibration estimators represent another class of methods that leverage calibration techniques to develop composite estimates. Calibration involves adjusting the sampling weights of a survey so that the weighted sums of certain auxiliary variables match their known population totals. In the context of composite estimation, particularly in successive sampling designs where surveys are conducted repeatedly over time, calibration can be used to incorporate information from previous occasions.<sup>3</sup> These estimators have the capability to simultaneously incorporate constraints related to both auxiliary totals and variances across successive time periods.<sup>24</sup> A composite calibration estimator can be constructed by combining calibrated estimators that are based on both the matched (units present in both current and previous samples) and unmatched samples across the time periods of interest.<sup>24</sup>

The application of calibration techniques in composite estimation reflects a broader trend towards making survey estimates more aligned with known population characteristics, potentially reducing bias and improving representativeness, especially when combining

data across multiple time points. The ability of composite calibration estimators to handle different sampling designs in each phase of successive occasions provides a valuable flexibility for real-world survey practice where methodologies might evolve over time.

## 5. Mathematical Formulation of Composite Estimators

The precise mathematical formulation of a composite estimator is contingent upon the specific type of estimator being considered and the underlying statistical model it is based on.

For the simple linear combination of direct ( $y^h$ ) and synthetic ( $y^-$ ) estimators, the formula is:

$$Y^h_C = (1 - \phi_h)y^h + \phi_h y^- \quad ^8$$

The AK composite estimator used by the U.S. Current Population Survey (CPS) has the formula:

$$\hat{Y}_{t,AK} = (1 - K)\hat{Y}_{t,SS} + K(\hat{Y}_{t-1,AK} + \Delta_t) + A\hat{\beta}_t \quad ^2$$

Composite regression estimators often involve more complex mathematical formulations, typically expressed using matrix notation. These formulations often include terms for simultaneous calibration of sampling weights and adjustments based on the differences observed between full and matched samples across different survey times. For instance, as discussed, an augmented regression matrix and associated calibration totals and weights are used to derive calibrated weights for the combined sample from two

consecutive time periods.<sup>3</sup> The resulting estimator can be expressed in terms of regression coefficients and the difference between the current-time regression estimator and the previous-time composite regression estimator updated with the change estimator from the matched sample.<sup>3</sup>

Composite calibration estimators, particularly in the context of successive sampling, often involve combining estimators derived from matched and unmatched samples. For example, a composite estimator for the population total ( $Y^t$ ) can be expressed as a linear combination of the matched sample estimator ( $Y^m$ ) and the unmatched sample estimator ( $Y^u$ ) with weights  $\phi$  and  $(1-\phi)$ :

$$Y^t = \phi Y^m + (1-\phi) Y^u \quad ^{24}$$

Variations of this formula exist, such as  $Y^d = \phi Y^{m*} + (1-\phi) Y^u$ , where  $Y^{m*}$  represents another estimator based on the matched sample.<sup>24</sup> These formulations often incorporate auxiliary information through calibration techniques to improve the precision of the estimates.

In the realm of domain estimation, composite estimators might take forms that combine different existing estimators for the domain, such as simple direct estimators and direct ratio estimators, using a weighted approach.<sup>9</sup> Similarly, composite ratio estimators have been developed for specific sampling designs like two-phase sampling, often involving multiple supplementary variables.<sup>28</sup>

The mathematical formulations of these diverse composite estimators underscore a fundamental principle: the strategic weighting of different sources of information or different initial estimates to arrive at an improved final estimate. The complexity of the formula often reflects the sophistication of the method in accounting for various factors relevant to the data and the survey



design.

## 6. Advantages of Composite Estimators

Composite estimators offer several compelling advantages that contribute to their widespread use in statistical analysis. A primary benefit is **improved precision**, often manifested as a reduction in variance compared to some of the individual estimators that are combined.<sup>2</sup> This is particularly true for estimates of change over time and for estimates pertaining to small domains where sample sizes are limited.

Composite estimators are also known for providing **better accuracy for estimates of change**, especially in surveys that utilize rotating panel designs. By leveraging the correlation between repeated measurements on the same units, these estimators can produce more reliable assessments of how population characteristics evolve over time.<sup>2</sup>

A significant advantage lies in their ability to **leverage correlations** present in the data. This can include temporal correlations in longitudinal surveys, spatial correlations in geographically overlapping surveys, or correlations between different variables.<sup>2</sup> By effectively utilizing these relationships, composite estimators can extract more information from the available data.

Furthermore, composite estimators offer the **flexibility to incorporate information from multiple sources**. This can include data from current and past surveys, external auxiliary data, and even different types of estimators that might each provide a unique perspective on the parameter of interest.<sup>1</sup>

In the context of **small area estimation**, composite estimators are invaluable for generating more reliable estimates for subpopulations or geographic regions where direct survey data is sparse and might lead to unstable estimates.<sup>3</sup> By combining direct estimates with synthetic estimates that borrow strength from related areas, composite estimators can significantly improve the precision and accuracy of small area statistics.

The **flexibility and adaptability** of composite estimation techniques to various survey designs and diverse estimation needs are also noteworthy.<sup>1</sup> This allows statisticians to tailor the estimation approach to the specific challenges and characteristics of the data at hand.

Finally, the improved precision offered by composite estimators can potentially lead to the ability to achieve desired levels of accuracy with **reduced sample sizes** in some survey designs, offering cost and efficiency benefits.<sup>14</sup>

The advantages of composite estimators are most evident when the individual estimators being combined possess complementary strengths and weaknesses. For example, combining a direct estimator with low bias but high variance with a synthetic estimator that has lower variance but potential bias allows the weighting scheme to strategically balance these properties. The widespread adoption of composite estimators by government agencies for producing crucial official statistics underscores their practical value and the confidence placed in their ability to provide more accurate and reliable estimates compared to traditional methods.

## **7. Disadvantages and Limitations of**

# Composite Estimators

Despite their numerous benefits, composite estimators also come with certain disadvantages and limitations that need to be carefully considered. One potential drawback is the **introduction of bias**. While the primary goal is often variance reduction, the process of combining estimators, particularly if a biased estimator is given significant weight or if the weights are not optimally determined, can introduce or exacerbate bias in the final estimate.<sup>2</sup> For example, the AK estimator used by the CPS is known to have a persistent bias.<sup>2</sup>

Another significant challenge lies in the **complexity of determining optimal weights** for combining the component estimators. The ideal weights often depend on unknown population parameters, such as variances and covariances, which must be estimated from the sample data. This estimation process can be intricate and might rely on assumptions that do not always hold true.<sup>1</sup>

Assessing the overall performance of a composite estimator, typically measured by its **mean squared error (MSE)**, can also be difficult. The MSE encompasses both variance and bias, and estimating the bias component, in particular, can be challenging.<sup>10</sup> Without accurate estimates of the MSE, it can be hard to definitively determine if the composite estimator is indeed an improvement over its individual components.

In some specific applications, such as composite sampling used in environmental monitoring, the very act of combining physical samples can lead to a **loss of information about individual sampling units**, which might be important for other aspects of the analysis.<sup>31</sup>

The accuracy of composite estimators is also highly **sensitive to the quality and relevance of the data** used to derive the component estimators. If the underlying data is flawed or not representative, the composite estimate will likely be affected as well.<sup>32</sup>

Traditional design-based composite estimators can also face **difficulties with statistical inferences**, making it harder to draw conclusions or test hypotheses based on these estimates.<sup>7</sup>

Finally, when dealing with small domains, estimating the optimal weights or the MSE for each domain individually can be **unstable**, potentially leading to unreliable weights and thus suboptimal composite estimates.<sup>10</sup>

The difficulty in determining optimal weights often necessitates the use of sophisticated statistical techniques or reliance on assumptions that might not always be valid, introducing uncertainty into the estimation process. The trade-off between variance reduction and potential bias highlights the importance of a thorough evaluation of any composite estimator before its widespread use to ensure that the benefits outweigh the drawbacks in the specific context.

## 8. Applications of Composite Estimators

Composite estimators have found widespread applications across various fields of statistical analysis, particularly in situations where traditional estimation methods face limitations. One of the most prominent areas is in the production of **official labor force statistics**. Agencies such as the U.S. Bureau of Labor Statistics, through the Current Population Survey (CPS) and its AK estimator, and Statistics Canada, with the Canadian LFS employing modified

regression estimators, extensively use composite estimation to generate national and regional employment and unemployment figures.<sup>2</sup>

Another critical application domain is **small area estimation**. Composite estimators are fundamental in producing reliable estimates for subpopulations or geographic areas that have small sample sizes in surveys. Their ability to combine direct and synthetic estimates makes them invaluable for generating granular statistics across various domains.<sup>3</sup>

In **environmental monitoring**, composite estimation is used to combine data from spatially overlapping surveys, leading to improved population estimates and enhanced precision in assessing environmental conditions.<sup>5</sup>

The concept of composite estimation also extends to other fields like **cost estimation**. In the construction industry, for example, composite rates are used to quickly estimate costs based on historical data and average rates for various construction activities.<sup>32</sup>

Beyond labor force statistics, composite estimators are also employed in other national surveys with rotation sampling designs, such as the Australian monthly Labour Force Survey.<sup>20</sup>

Furthermore, composite estimators can be utilized in **variance estimation**, such as in combining estimates from different survey systems, as seen in the context of crash statistics analysis by the National Highway Traffic Safety Administration (NHTSA).<sup>34</sup>

The diverse range of applications underscores the broad utility and adaptability of composite estimators in addressing various estimation challenges across different fields. The specific methods

and formulas used are often tailored to the unique characteristics of the application domain, highlighting the need for domain-specific knowledge and statistical expertise in their effective development and application.

## **9. Conclusion**

Composite estimators represent a sophisticated and versatile set of statistical techniques designed to enhance the precision and accuracy of population parameter estimates by combining multiple sources of information or different types of estimators. The underlying rationale for their use is multifaceted, encompassing the reduction of variance, the leveraging of correlations in data (particularly in repeated surveys), the improvement of estimates for small domains, and the incorporation of prior knowledge. Various types of composite estimators exist, each with its own mathematical formulation and specific applications, ranging from simple linear combinations of direct and synthetic estimates to more complex methods like the AK estimator, composite regression estimators, and composite calibration estimators.

The advantages of using composite estimators are significant, including improved precision, better accuracy for estimates of change, the ability to leverage correlations and incorporate diverse information sources, and the potential for more reliable estimates in small area estimation. However, it is crucial to acknowledge the disadvantages and limitations, such as the potential for introducing bias, the complexity of determining optimal weights, and the challenges in accurately estimating the mean squared error.

The widespread application of composite estimators across diverse fields, including official labor force statistics, small area estimation,

environmental monitoring, and cost estimation, underscores their practical value in addressing real-world estimation challenges. As statistical analysis continues to evolve to meet the demands of increasingly complex data and the need for more granular insights, composite estimation will likely remain a vital tool in the statistician's arsenal. Future research could focus on developing more robust methods for determining optimal weights, addressing the challenges of bias estimation in complex composite estimators, and exploring novel applications of these techniques in emerging areas of data science and statistical inference.

### Works cited

1. ENCLOSURE - A-I-71 U.s. Department of Agriculture Statistical Reporting Service COMPOSITE ESTIMATION Earl E. Houseman Some Intro - USDA NASS, accessed May 9, 2025,  
[https://data.nass.usda.gov/Education\\_and\\_Outreach/Reports,\\_Presentations\\_and\\_Conferences/Survey\\_Reports/Composite%20Estimation.pdf](https://data.nass.usda.gov/Education_and_Outreach/Reports,_Presentations_and_Conferences/Survey_Reports/Composite%20Estimation.pdf)
2. Optimizing the Current Population Survey Composite Estimator - Bureau of Labor Statistics, accessed May 9, 2025,  
<https://www.bls.gov/osmr/research-papers/2022/pdf/st220090.pdf>
3. A New Approach to Composite Estimation for Repeated Surveys with Rotating Panels - American Statistical Association, accessed May 9, 2025,  
<https://www2.amstat.org/meetings/proceedings/2021/data/assets/pdf/1913757.pdf>
4. Variance Formulae for the Generalized Composite Estimator Under ..., accessed May 9, 2025,  
<https://www.census.gov/library/working-papers/1988/adrm/rr88-26.html>
5. Composite estimation to combine spatially overlapping environmental monitoring surveys, accessed May 9, 2025,  
<https://pmc.ncbi.nlm.nih.gov/articles/PMC10959383/>
6. A composite estimator for small area statistics - PubMed, accessed May 9, 2025,  
<https://pubmed.ncbi.nlm.nih.gov/114852/>
7. Direct, synthetic and composite estimation of parameters in population domains, accessed May 9, 2025,  
[https://www.dst.dk/ext/40769109402/0/jordan2022/Activity-2-1-1-Introduction-to-direct-synthetic-and-composite-estimation-SL-\(ENG\)--pdf](https://www.dst.dk/ext/40769109402/0/jordan2022/Activity-2-1-1-Introduction-to-direct-synthetic-and-composite-estimation-SL-(ENG)--pdf)
8. 2. Composite estimation - Statistique Canada, accessed May 9, 2025,  
<https://www150.statcan.gc.ca/n1/pub/12-001-x/2015002/article/14230/02-eng.htm>
9. Composite estimators for domain estimation and sensitivity performance interval



- of their weights - Statistics in Transition new series, accessed May 9, 2025,  
[https://sit.stat.gov.pl/SiT/2024/1/gus\\_2024\\_01\\_piyush\\_kant\\_rai\\_sweta\\_singh\\_composite\\_estimators\\_for\\_domain\\_estimation.pdf](https://sit.stat.gov.pl/SiT/2024/1/gus_2024_01_piyush_kant_rai_sweta_singh_composite_estimators_for_domain_estimation.pdf)
10. isi-web.org, accessed May 9, 2025,  
<https://isi-web.org/sites/default/files/import/pdf/239-day5-cps022-design-based-composite-estimat.pdf>
  11. Design-based composite estimation rediscovered - ResearchGate, accessed May 9, 2025,  
[https://www.researchgate.net/publication/353838192\\_Design-based\\_composite\\_estimation\\_rediscovered/fulltext/61149a2c169a1a0103f536c5/Design-based-composite-estimation-rediscovered.pdf](https://www.researchgate.net/publication/353838192_Design-based_composite_estimation_rediscovered/fulltext/61149a2c169a1a0103f536c5/Design-based-composite-estimation-rediscovered.pdf)
  12. Introduction to Small Area Estimation Techniques: A Practical Guide for National Statistics Offices - Asian Development Bank, accessed May 9, 2025,  
<https://www.adb.org/sites/default/files/publication/609476/small-area-estimation-guide-nsos.pdf>
  13. (PDF) Design-based composite estimation rediscovered - ResearchGate, accessed May 9, 2025,  
[https://www.researchgate.net/publication/353838192\\_Design-based\\_composite\\_estimation\\_rediscovered](https://www.researchgate.net/publication/353838192_Design-based_composite_estimation_rediscovered)
  14. Using Composite Estimators to Improve both Domain and Total Area Estimation | Request PDF - ResearchGate, accessed May 9, 2025,  
[https://www.researchgate.net/publication/23695503\\_Using\\_Composite\\_Estimators\\_to\\_Improve\\_both\\_Domain\\_and\\_Total\\_Area\\_Estimation](https://www.researchgate.net/publication/23695503_Using_Composite_Estimators_to_Improve_both_Domain_and_Total_Area_Estimation)
  15. From Start to Finish: A Framework for the Production of Small Area Official Statistics - Oxford Academic, accessed May 9, 2025,  
<https://academic.oup.com/jrsssa/article/181/4/927/7072057>
  16. Review and Synthesis of Estimation Strategies to Meet Small Area Needs in Forest Inventory - Frontiers, accessed May 9, 2025,  
<https://www.frontiersin.org/journals/forests-and-global-change/articles/10.3389/ffgc.2022.813569/pdf>
  17. Synthetic and composite estimators for small area estimation under Lahiri – Midzuno sampling scheme - Munich Personal RePEc Archive, accessed May 9, 2025,  
[https://mpira.ub.uni-muenchen.de/22783/1/ON\\_SYNTHETIC\\_AND\\_COMPOSITE\\_ESTIMATORS\\_FOR\\_SMALL\\_AREA\\_ESTIMATION\\_UNDER\\_LAHIRI\\_a\\_MIDZUNO\\_SAMPLING\\_SCHEME.pdf](https://mpira.ub.uni-muenchen.de/22783/1/ON_SYNTHETIC_AND_COMPOSITE_ESTIMATORS_FOR_SMALL_AREA_ESTIMATION_UNDER_LAHIRI_a_MIDZUNO_SAMPLING_SCHEME.pdf)
  18. 1978: CHOOSING WEIGHTS FOR COMPOSITE ESTIMATORS FOR SMALL AREA STATISTICS - Proceedings of the Survey Research Methods Section, accessed May 9, 2025, [http://www.asasrms.org/Proceedings/papers/1978\\_159.pdf](http://www.asasrms.org/Proceedings/papers/1978_159.pdf)
  19. Regression Composite Estimation: An Alternative Approach for the Current Population Survey - Federal Committee on Statistical Methodology, accessed May 9, 2025,  
[https://www.fcsfm.gov/assets/files/docs/G2\\_Bonnery\\_2013FCSM\\_AC.pdf](https://www.fcsfm.gov/assets/files/docs/G2_Bonnery_2013FCSM_AC.pdf)
  20. An evaluation of design-based properties of different composite estimators - EconStor, accessed May 9, 2025,

- [https://www.econstor.eu/bitstream/10419/236787/1/10.21307\\_stattrans-2020-037.pdf](https://www.econstor.eu/bitstream/10419/236787/1/10.21307_stattrans-2020-037.pdf)
21. Full article: Optimal AK composite estimators in current population survey, accessed May 9, 2025, <https://www.tandfonline.com/doi/full/10.1080/24754269.2017.1359437>
  22. 1. Introduction - Statistique Canada, accessed May 9, 2025, <https://www150.statcan.gc.ca/n1/pub/12-001-x/2015001/article/14160/01-eng.htm>
  23. A Regression Composite Estimator with Application to the Canadian Labour Force Survey, accessed May 9, 2025, <https://www150.statcan.gc.ca/n1/pub/12-001-x/2001001/article/5853-eng.pdf>
  24. Full article: Class of composite estimators of population total using calibration estimation based on matched and unmatched sample under successive sampling, accessed May 9, 2025, <https://www.tandfonline.com/doi/full/10.1080/00949655.2024.2441469?src=>
  25. Full article: Class of composite estimators of population total using calibration estimation based on matched and unmatched sample under successive sampling - Taylor & Francis Online, accessed May 9, 2025, <https://www.tandfonline.com/doi/full/10.1080/00949655.2024.2441469?ai=1il&mi=i&cwyn&af=R>
  26. Composite estimators for domain estimation and sensitivity performance interval of their weights - Biblioteka Nauki, accessed May 9, 2025, <https://bibliotekanauki.pl/articles/56307671.pdf>
  27. Composite estimators for domain estimation and sensitivity performance interval of their weights - Biblioteka Nauki, accessed May 9, 2025, <https://bibliotekanauki.pl/articles/56307671>
  28. COMPOSITE RATIO ESTIMATORS IN A TWO-PHASE SAMPLING USING MULTIPLE ADDITIONAL SUPPLEMENTARY VARIABLES | Far East Journal of Applied Mathematics - Pushpa Publishing House, accessed May 9, 2025, <https://pphmjopenaccess.com/index.php/fejam/article/view/1606>
  29. Optimal Weighting and Overlap for Composite Estimation in Repeated Surveys, accessed May 9, 2025, <https://2011.isiproceedings.org/papers/950144.pdf>
  30. 1983: Alternative Estimators to the Current Composite Estimator - Proceedings of the Survey Research Methods Section, accessed May 9, 2025, [http://www.asasrms.org/Proceedings/papers/1983\\_076.pdf](http://www.asasrms.org/Proceedings/papers/1983_076.pdf)
  31. Decommissioning: Composite Sampling Application and Regulatory Guidance., accessed May 9, 2025, <https://www.nrc.gov/docs/ML1114/ML111400153.pdf>
  32. What does Composite Rate mean in Construction? - Vergo, accessed May 9, 2025, <https://www.getvergo.com/define/composite-rate>
  33. Cost Estimating: How to use Composites - Cost Engineering Consultancy, accessed May 9, 2025, <https://www.costengineering.eu/blog-article/cost-estimating-how-to-use-composites>
  34. Crash Report Sampling System: Composite Estimator Variance Estimation - CrashStats - NHTSA, accessed May 9, 2025, <https://crashstats.nhtsa.dot.gov/Api/Public/ViewPublication/813225>

35. Crash Report Sampling System: Composite Estimator Variance Estimation -  
CrashStats - NHTSA, accessed May 9, 2025,  
<https://crashstats.nhtsa.dot.gov/Api/Public/Publication/813225>