The Algebra of Power

or

Logischerkrieg

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Purpose and Thesis

Purpose

The purpose of this book is to present a method for creating structures of power by the use of abstract algebra.

Definition of power

Power is the ability to determine what things are like and what they are going to be like, despite any external influences to the contrary.
To put it otherwise, power is our ability to engrain whatever information we desire into the external world, without other agents being able to change it.

Active, Passive, and Symmetry

The distinction between active and passive is one that can be useful in clarifying our concept of power. Power is our ability to make other systems take whatever form we want them to take. To put it otherwise, power is our ability to make others adapt to us, so we don't have to adapt to others. As stated previously it is our ability to determine what information is going to be ingrained into a physical substrate, and to stop that information from changing once it is ingrained.

If the action of an agent can be deduced from the actions of other agents, then the action of the first agent is informationally redundant. If an agent can resist the inertia of systems around him, and ingrain new bits into reality, then the agent is informationally relevant, that is, the agent possesses true agency. Note that for the agent to have true agency, the agent does not need to be uninfluenced by everything around him. He only needs to be *capable of* ingraining relevant (non-compressible) information. Any agent whose choices can always be deduced from other variables, or from the choices of other agents does not exist as a true agent.

We see the determinant (active) as inherently more desirable than the adaptive (passive). This is important because adaptation has been marketed as the desired state to be achieved by the individual. Our position is contrary to that. We see the fixed as the determining principle, and the volatile, or adaptive, as the determined principle. The objective is to create a structure that is maximally coherent (self-symmetric) so that other systems need to adapt to it, instead of vice versa. When two things cannot exist simultaneously because they are mutually exclusive (logically incompatible), logic tells us that either both structures change, or one stays as it is and the other changes. In the real world, however, every time two structures find each other, they both change (no structure stays in exactly the same state because totally isolated systems do not exist in practical reality and interaction always has an effect on both interacting structures). However, the ratio of the changes in one structure to the changes in the other structure is a measure of the relationship between the coherence (permanence, fixedness) of the most coherent system to the coherence of the least coherent system. To put it otherwise, power is a measure of the relationship between the reactivity of the least reactive (volatile) system and the reactivity of the most reactive system, it is the relationship between the coherence (un-reactivity) of the most coherent system and the coherence of the least coherent system. The fixed, or active, is information, while the volatile, or reactive (passive) is energy. But things are not absolutely fixed or absolutely volatile, rather, they are relatively fixed or volatile, which is why we see information and energy as relationships, and power as the gradient between the two. Anything that interacts with something that is fixed without changing it becomes consistent with it. Information causes an amount of energy (change) proportional to the gradient between its own coherence and that of the system that is not consistent with it. By contact with its environment, the number of bits that are at least logically compatible with the system expands. Implicit bits are the bits that represent the minimum specification for the whole structure (that is, maximally compressed data). Explicit bits are the ingrained bits that are coherent and stay coherent with the underlying (implicit) data. Coherence can be measured as the ratio of explicit bits to implicit bits of a structure, and it will also be called compression.

We can maximise coherence through the insertion of built-in symmetries into our structures. We will see how abstract algebra can be used to maximise the self-symmetry of a system, and thus its resistance to external disturbance and friction.

Main Thesis

I propose to unify the concept of power, so that the same concept can accurately describe physical power (the rate at which work is done), social power, personal power, or other types of power. I propose that all these powers have something in common, namely, that they stem from information transmission from one variable (or system) to another, and that the ability of a system to transfer information is directly proportional to its coherence, which can be quantified as symmetry. Information transmission is immediately accomplished between two systems that are in contact with each other due to their necessity to be a coherent overall metasystem (given that two systems that are in contact are really just one system, and they must at least be logically consistent with each other).

We can use abstract algebra, specifically group theory, the study of symmetry, to create structures of power by maximising the 'fixedness' of a system, thus forcing surrounding systems to adapt. Algebra allows us to multiply ingrained information and to protect it against attack.

Symmetries in logic and time

A system that is complete is perfect, so it implies all parts of itself at all times. A whole system is an isolated system. Partial systems suffer from friction, because they are not implied or created by themselves in logical space or time. Overall, entire systems do not suffer from environmental friction. They are usually called ideal systems and consist of a perfect loop of the system to itself. They can be seen as the perfect androgynous (active and passive, inhale, and exhale...) and would contain infinite energy in time (due to infinite iterations of the loop) because of their perfect symmetry. They represent the absolute oscillatory equilibrium.

Ideal systems are our first example of energy multiplication through symmetries. By making the first part of the loop imply the second part of the loop, and vice versa, we have multiplied the initial energy contained in an iteration of the loop by infinity (infinite iterations), thus obtaining infinite energy (in time). This is a theoretical scenario, given that ideal (isolated) systems do not exist in nature (apart from the universe itself, perhaps). However, we can approximate this state, thus multiplying the energy in real systems. We can obtain energy from informational symmetries. To put it otherwise, the order of the system is such that the system implies itself in time or in logical space. We can see this information that implies itself as meta-information, or informational symmetries. Note that systems made of meta-information in time (they produce themselves in time) are able to regenerate if one part of the information is damaged because all the other parts imply, or produce, that part in time. The logical equivalent of regeneration is something we will call locking, which consists of making a small variable very difficult to change, since it is implicit in a much larger structure.

Ideal systems cannot, however, grow, because they are perfectly isolated.
Partial systems (non-ideal systems) can be pseudo-ordered or truly ordered. A pseudo-ordered system is one that suffers friction and dissipates. A truly ordered system is one that grows. No partial system remains exactly the same, although it can remain approximately the same. We also have systems that are largely very ordered, although they suffer from friction at some level of their organization (information) and therefore end up dissolving. However, for practical purposes, we will consider a truly ordered system any system that is a priori capable of perpetuating itself for an indefinite time. We can see system dissipation as friction, and growth (syntropy) as negative friction.

The order of the type of systems we propose takes the order of the relevant part of the environment into itself, so that the system includes all the appropriate parts of what might be called its environment. However, for practical purposes, we will refer to the main part of the system as "the system" or "the machine" or "the *seed*", and the "environmental" part of the system as the environment.

We can measure the multiplication of energy through symmetries, whether in time, or logical space, and we will call this the coherence of the system. The relative coherences of two interacting systems form a coherence gradient (ratio from one coherence to the other). This gradient or ratio is a measure of the power of one system over the other. This ratio is what determines the amount of energy (ability to produce change) that will result from the encounter of the two systems.

The coherence of a system can be seen as a measure of how many explicit (materialised) bits there are per bit of the fundamental structure of the system. It is a measure of how many explicit bits there are for each implicit bit. The more coherent system will act as a relatively fixed set of materialised information, relative to the less coherent system. This will make the less coherent system adapt to the more coherent system, thus changing things (energy).

One example is power in society. It arises from gradients of coherence. The general (social) system, with many more bits encompassed by the same structure, is much more coherent than the structures created by the individual. That makes the individual adapt to the overall (social) structure, rather than being able to make the social structure adapt to himself. The more coherent the structure created by the individual is, and the less coherent the structure created by the general system is, the smaller the difference in power between the individual and the social structure will be.

Regeneration of a system in time is due to each part of the system implying other parts of the system, so that the cycle returns and creates anew what has been damaged. Imagine a cyclic group of order 3, that is, a system composed of three variables *X*, *Y*, and *Z*, where *X* creates *Y*, *Y* creates *Z*, and *Z* creates *X*. If someone damages *X*, then *Z* produces *X* again in a given period of time, and *X* produces *Y*, and *Y* produces *Z*, having regenerated *X*. The same applies to any other variable within the cycle.
Locking is a mechanism by which, in a logical structure (out of time, in logical space), a variable is locked in a given state due to that specific state being logically implied by other variables within the structure. If many other variables imply that A is in state A', then to change A into a different state, one will have to change all the other variables also, making it difficult to change A from A' into a different state, which has the effect of locking *A* into state *A'*. Locking is the logical equivalent of regeneration because it is based on the same mechanism (symmetries) which, in the case of regeneration takes place in time, whereas in the case of locking it takes place in logical space (outside of time, in a purely logical, synchronous space). Logical space is an abstract space, outside physical space-time.
We can also define the concept of logical time. To say that *X* happens 'before' *Y* in logical time would be equivalent to saying that *X* precedes *Y* in logical deduction. Logical time is given by the need for one thing to precede another in logic, which means that, without knowledge about the state of the first variable, the state of the second variable cannot be deduced.

Abstract Algebra

A group is a type of algebraic structure constituted of the symmetries of objects, whether the objects are physical or abstract. Before explaining what the properties of groups are, we must explain some fundamental concepts.

An element is a member of a group, that is, a variable or a state that is part of a given group. For our purpose, each element must be a different manifestation of the initially ingrained bits, so that each element adds to the (non-compressed) information *I* of the group or system *S.* An example of the different elements of a group are all the possible combinations in a Rubik's cube (that preserve the shape of the cube itself).

A generator is that which connects one element to another, that is, generators produce (generate) elements. The generators are the processes that connect one manifestation of the initially engrained bits to another. Some examples of generators are:

One element *produces* another (production).

One element *increases the probability of* another (increase of probability).

One element *logically implies* another (logical implication).

One element *increases the energy or resources available to* another (facilitation).

One element *allows for an action* that leads to another (specific action)

A binary operation is that which considers two elements together to produce a third element. Examples of binary operations are:

One element *followed by* another produces a third element (followed by).

One element, *simultaneously with* another, produces a third element (simultaneously with).

Now we go into the properties of a group, that is, of the symmetries of an object, whether the object is physical or abstract:

Closedness under a binary operation: Any element of the group operated with any other element of the group gives a third element of the group.

Presence of an identity element: The identity element can be seen as the initial state, or the first element. The identity element operated together with any other element produces the other element. It acts like a 'blank' element from which to generate other elements. The identity element in a real system is the state that facilitates the other states, that makes them possible.

Inverses: There is within the group a path back from any element to the identity element, so that the cycles can be repeated indefinitely.

Associativity: The binary operation of the group is associative, that is, (a*b)*c = a*(b*c), where each letter represents an element and the asterisk * represents the operation. We will see what associativity means with an example. Say we choose the binary operation to be *followed by*. Then, one element *followed by* another, and the result then *followed by* yet another element, is equivalent to one element *followed by* the result of a second element *followed by* a third element. The operation *followed by* is thus associative. So is the operation *simultaneously with*.

The properties of closedness and inverses (the property of inverses presupposes the presence of an identity element) taken together mean that there is a path from each element to each other element via the generators of the group, and that the cycle can repeat indefinitely. The operation being binary instead of ternary, quaternary, etc… maximises the efficiency with which new elements are generated from other elements. Also, most ternary or quaternary algebras reduce to binary ones (although not all). The operation of a group composed of elements that arise out of a set of generators is a composition of generators, and each of those generators is a relation between two elements. Composition of relations is associative; thus, the operation of a group must be associative. If the generators of the group are determinative, that is, if they guarantee the production of elements, then the information I of the whole group or system S can be compressed (maximally) into the entropy of the element with the least entropy. If the generators of the group are not determinative (that is, if they increase the probability of an element but do not guarantee it), then, as the number n of cycles from one element to another within the group repeats indefinitely, that is, as n tends to infinity, the probability of each element tends to 1, so, eventually, the information I of the system S can still be (maximally) compressed. (An element is a specific state of a given variable).

Ingraining information

The information I (novelty) contained in the obtention of the *least likely* outcome x of a binary variable X (a binary variable is one that is divided into two mutually exclusive and jointly exhaustive outcomes) is inversely proportional to the entropy H of X, thus:

$$I(x) = \frac{1}{H(X)}$$

The entropy of a variable as defined by Shannon depends on how many mutually exclusive and jointly exhaustive outcomes we choose to consider. But we could always define a variable as a binary variable, with outcomes *x* and *not x*. Thus, a variable can always be defined as a binary variable, whose maximum possible entropy is one bit. Thus, information as defined above can be seen as:

$$I(x) = \frac{Maximum\ Entropy}{Actual\ Entropy} = \frac{1\ bit}{H(X) bits}$$

Thus, the unit of information is the bit per bit.

Information needs to be materialized (ingrained). Treating a whole system *S* as a binary variable, the probability of all the variables composing a system of *n* variables (assumed to be independent from each other) being in their least likely outcome *s* is the product of the probability of the least likely outcome of each variable, and we would assume that:

$$I(s) = \frac{1}{H(S)}$$

Hidden within the concept of *H(S)* treating *S* as a binary variable is something resembling an exponential function of *n*, given that the probabilities of outcomes (which determine the entropy) are a product of *n* factors, that is, the product operation is repeated *n* times. It is not a proper exponential function, where the factors are all the same, but it still gives *n* the same type of impact over the probability of the least likely outcome *x*. The value of *H(S)* decreases faster as *n* increases than it would if *n* where the exponent of *H(X)*, that is, *n* has a more marked impact on decreasing *H(S)*, and thus increasing *I(S)* than an exponential function of *n* would. We can call this a supra-exponential function of *n*.

Now, if we apply group theory to the creation of informational structures, then, due to the properties of inverses and closedness, each of the elements of the group is enough to generate the whole group, and thus contains the information of the whole group in a latent state. Therefore, if we were to temporarily envision the group as composed of independent elements, the information *I* of a group *G* composed of *n* elements (each representing the state *x* of a variable *X*, and together representing the group state *g*) would be the product of the information of the system *S* of elements composing *G* and the number *n* of elements in *G*:

$$I(g) = n \times I(S)$$

If ingraining one bit allows us to automatically ingrain other bits, then, the probabilities are not independent anymore, and all this information can be compressed into the information of any one of the elements of the group, thus maximal compression is given by compressing the group into the information of the element *k* (the least likely state of variable *K*) with the least amount of information, that is, with the most a priori entropy, thus the maximum compression *C* is given by:

$$C = \frac{I(g)}{I(k)}$$

Compression is a measure of information multiplication via the use of abstract algebra.

It seems reasonable to define negentropy as the ratio of maximum possible entropy to actual entropy, which coincides with our measure of information. Thus, our measure of information is a measure of negentropy, and this negentropy can easily be understood as a type of potential energy proportional to how far the system is from equilibrium, that is, from maximum entropy. Our measure of compression is a measure of potential (information) multiplication via the use of abstract algebra (group theory).

Interaction and Power

Dominant interaction *Inter* can be quantified as the ratio of the bits changed in the target system to the bits changed in our system.

$$Inter = \frac{\Delta bits_{target}}{\Delta bits_{system}}$$

The Interaction efficiency Eff_{inter} can be calculated as the dominant interaction per unit time *t*:

$$Eff_{inter} = \frac{Inter}{t}$$

Exerted power P_{ex} can be quantified as the ratio of the change in the compression (symmetry) of the target system to change in symmetry of our system:

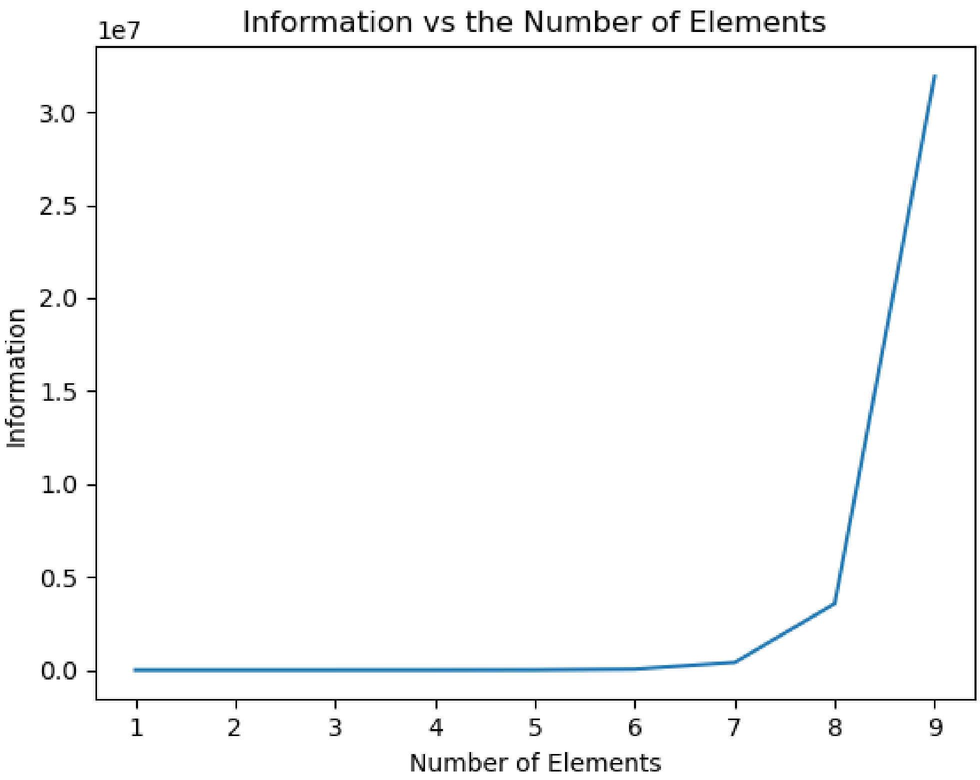
$$P_{ex} = \frac{\Delta C_{target}}{\Delta C_{system}}$$

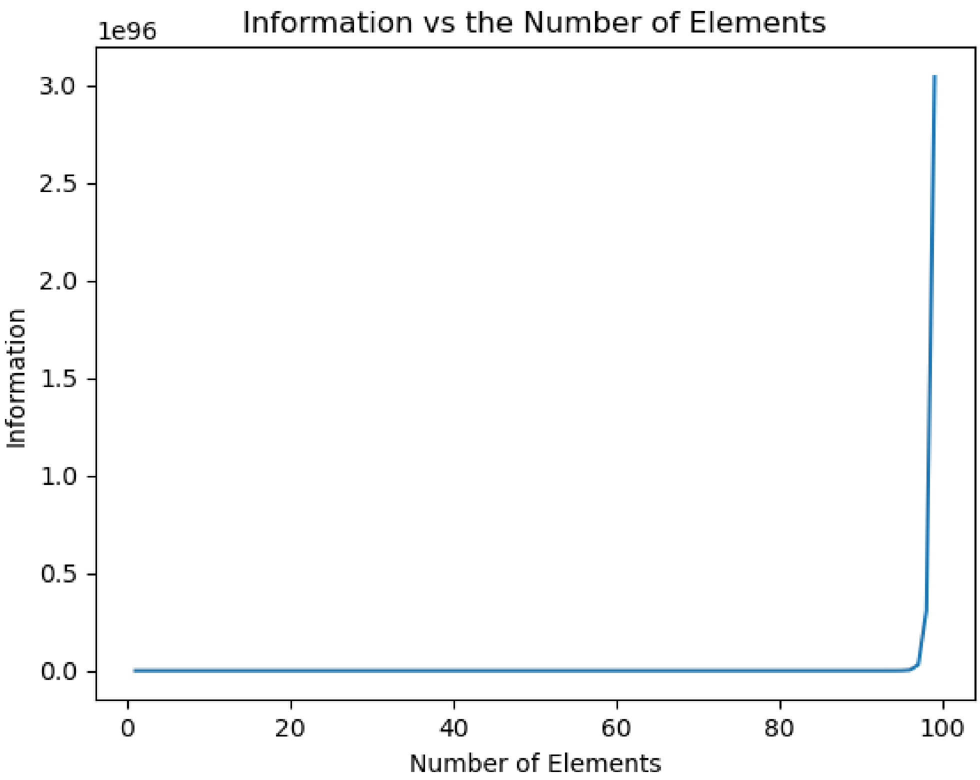
The above formula applies in the long term, that is, as time tends to infinity.

When one of the changes in symmetry is negative and the other is positive, that is, when one system is increasing its symmetry while the other is decreasing it via the interaction between the two, we have negative exerted power. Negative exerted power means that one system is ‘feeding’ on the other. If ΔC_{system} is positive and ΔC_{target} is negative, then our system is feeding on the target system. If the signs are reversed, then the target system is feeding on our system.

Reaching a singularity

We have seen that the compression of the information of a group is a supra-exponential function of the number of elements in the group. Here is a graph created by a Python program designed to compute information of a group and plot it against the number of elements of a group. For the plot, we have assumed the initial information of all elements to be the same.


Information vs the Number of Elements
1e7
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Information
1
2
3
4
5
6
7
8
9
Number of Elements


Information vs the Number of Elements
1e96
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Information
0
20
40
60
80
100
Number of Elements

When the information of the group grows vertically, we have reached a singularity. To change any one of those bits in the long term, we would have to change all of them simultaneously. Thus, we could say that it is a very ‘massive’ group. For all practical purposes, that information of the group is fixed.

Combinatorial Explosion

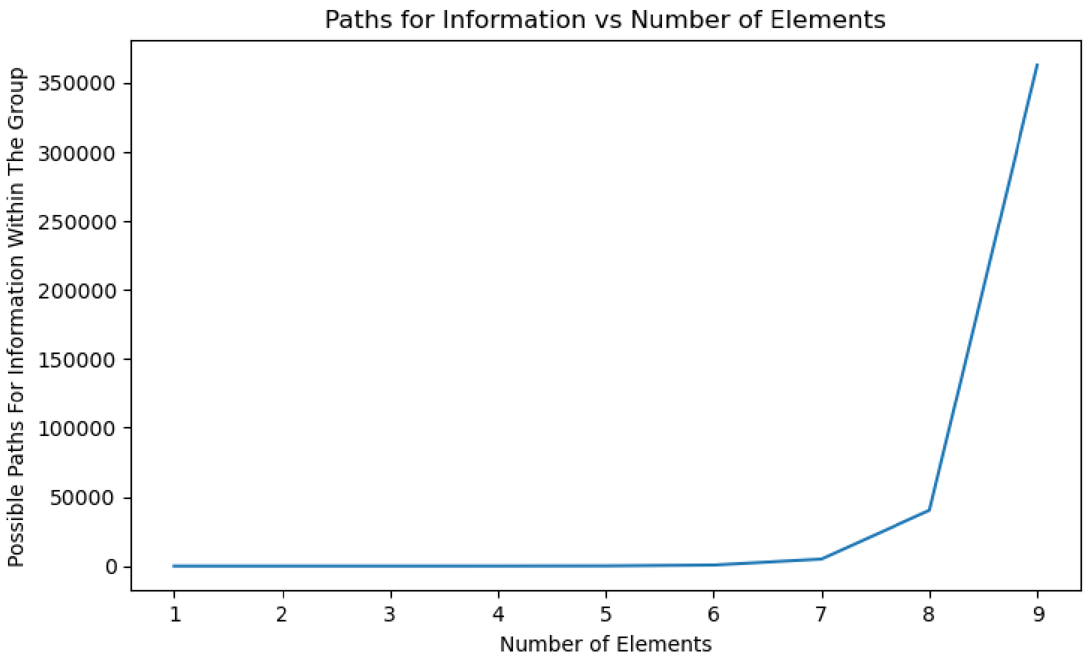
The advantages of structuring systems of variables as algebraic groups are:

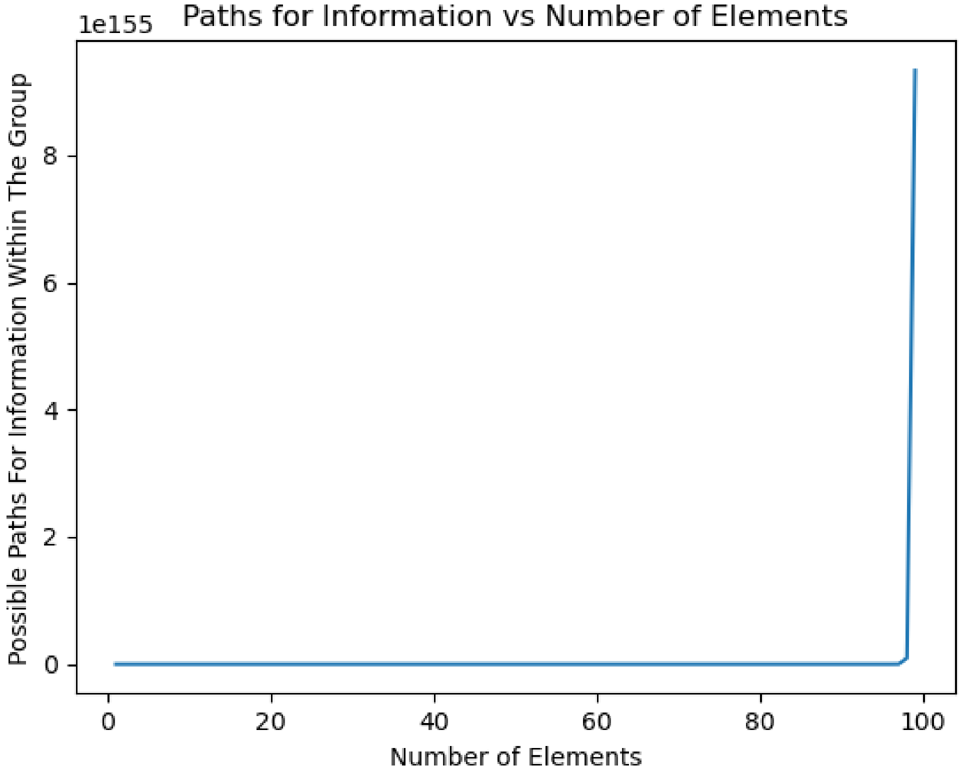
Groups allow us have a system cycling in time or logic so that the probabilities of all elements tend to 1.

Using automatic generators (those that do not require any action on our part, such as production in time, increase of probability in time, logical implication, or facilitation), we can assimilate the information of many outcomes into the information of the initial engineered outcome (that belonging to the identity element).

Given the symmetry in time or logic of the group, if an external agent wanted to change the whole group at any one time, assuming that the effort they would have to apply to change every element simultaneously is directly proportional to the negentropy of the group, then, the needed effort to change the group increases supra-exponentially with the number of elements of the group. But what if an external agent were to attack only one part of the system and hope that the change spreads? We are protected by combinatorial explosion.

Information is in the connections between the elements, in their relations. How many paths are there from each element to itself via the other elements of the group? The paths that the information can take within the group is $n!$, where n is the number of elements in the group. This is the number of permutations of n elements, known as the combinatorial explosion. Here are graphs of $n!$ against n:



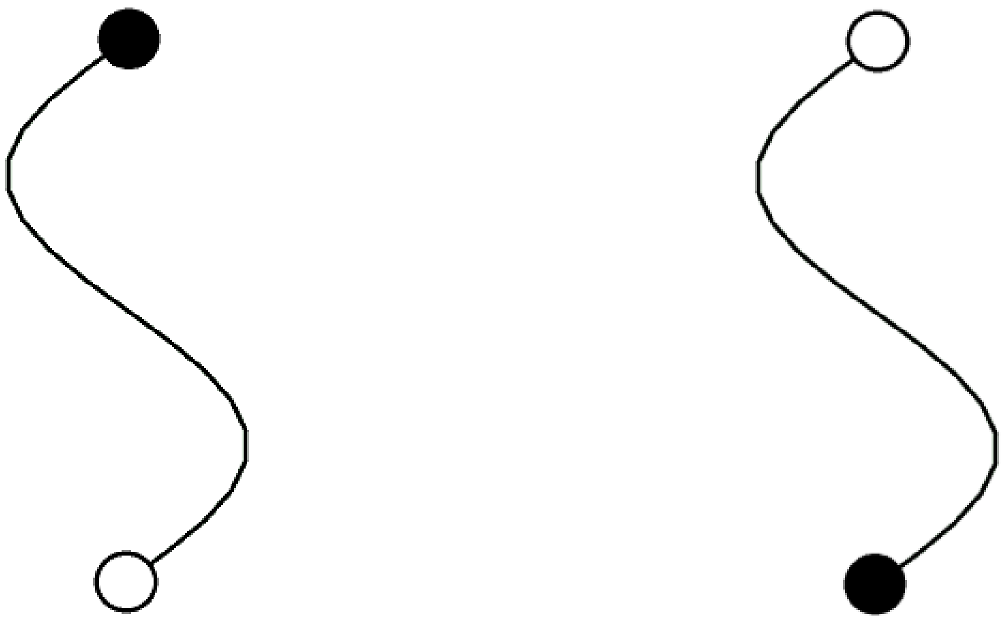


What that means is that each element is simultaneously being reinforced by information coming from many different paths, so that any attempt at an interference would be met by a lot of simultaneous information that contradicts the interference (imposes the original structure).

Visualising Symmetry

Symmetry means permanence under change, that is, the same bits expressed in new bits. Symmetry in time means a creates b which creates a. Symmetry in energy means a brings energy available to b, which brings energy available to a. Symmetry in logic means a implies b, which implies a. The fixed bits are the object. The fixed bits is the space the object occupies. The bits that change are the symmetries of the object.

Say that we have an element that produces another element, and which is in turn produced by the second element. We can see this as the following object rotating in time:



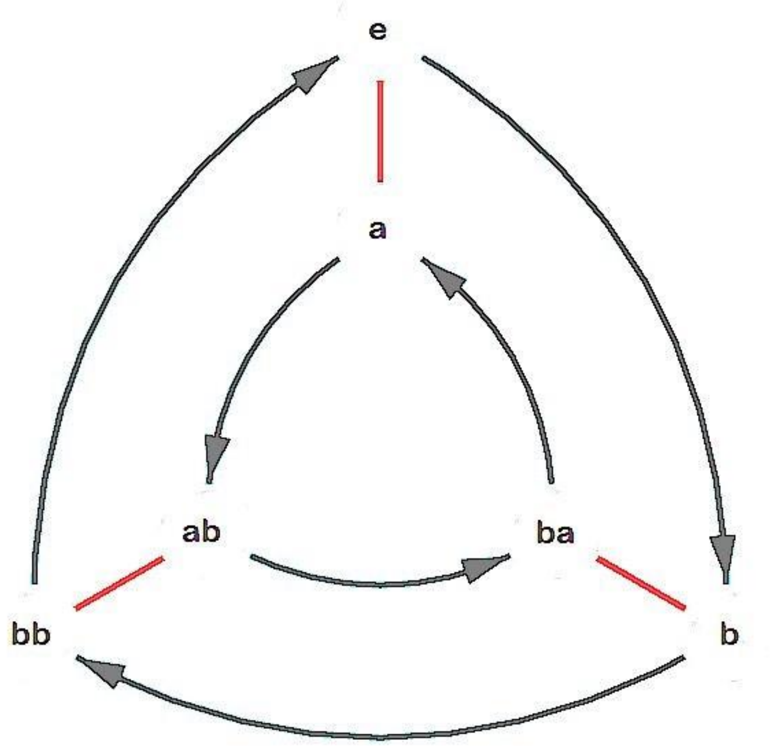
In the above image, the second shape is a rotation of the first shape by 180 degrees, or half a cycle.

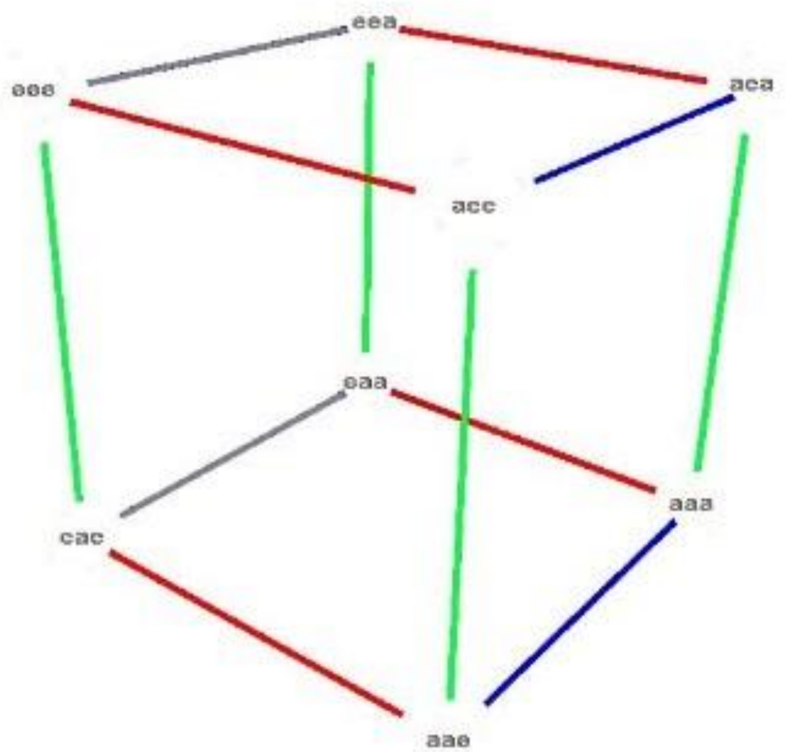
So the second position is just a rotation of the first position, so that the two variables can be compressed into just any one of them, plus a rotation. Thus, say the first variable (represented by the first position) has an a priori entropy of 1 bit, and so does the second variable (represented by the second position). Then, 2 bits can be compressed to 1 bit, therefore, the object has a symmetry of 2 bits per bit.

We would visualise the group representing the symmetries of the above image as:

It is a cyclic group of order 2, also known as C2, and the above image is its Cayley diagram.

Here are some Cayley diagrams for other groups (S3 and C2×C2×C2, from top to bottom):


e
a
ab
ba
bb
b


eea
eee
aca
acc
eaa
aaa
cac
aae

Building Objects of Power

Say the first variable has an entropy of one bit, and the second variable is another bit, and the rotation is in time. So 2 bits per bit, so the object has a mass of two.

We can use abstract algebra to create structures that contain this type of symmetry. Here is the example of a Cayley table for C2, the above group:

	e	***a***
e	e	a
a	a	e

The element *a* is the generator of time in this example. But it could be logical implication or shared available energy.

We can build these structures into bigger ones via the direct product method (multiply the structures, that is, connect them). Here is the Cayley table for C2 x C2 (V4):

	e	***a***	***b***	***ab***
e	e	a	b	ab
a	a	e	ab	b
b	b	ab	e	a
ab	ab	b	a	e

We can use Cayley tables such as the above as a means to understand how to build a group.

Direct product and Encryption

We can also build groups using other groups as building blocks. This is an operation called the direct product, which consists in connecting every element of one group to every other element of the other group. When we want to connect two groups, we know that the only structures that maintain the group properties is another group with the multiplied number of elements. The direct product of two groups is another group, of higher symmetry, because each of the elements in the metagroup is the combination of two (unlikely) elements of the groups. Thus, the energy of the elements of the metagroup is higher than the energies of the elements of the subgroups. The energy of the identity element in the subgroup is also ‘higher’ in theory but note that we have achieved it by summing the energies of the two groups, and not by doing something that really takes the multiplied energy of both. So we achieve a multiplication result from a sum energy investment, thus greatly increasing the effort efficiency of the metagroup, relative to the groups.

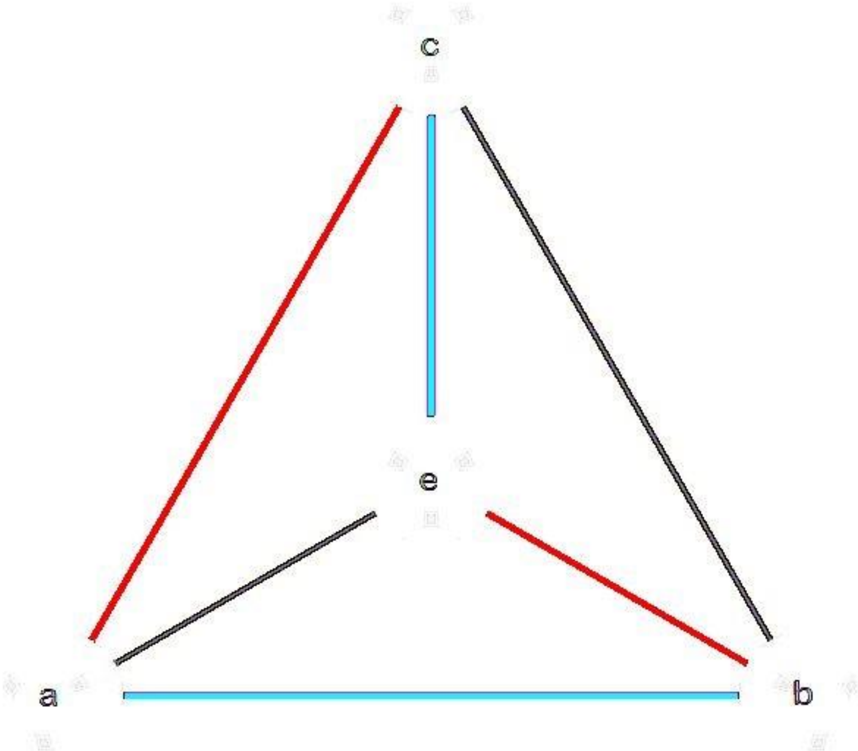
There are several ways for building on several groups, but we will only concern ourselves with the direct product. The reason the direct product is relevant is because it is the equivalent to connecting to cycles so that they feed back into each other, be it in time or in logic.

The direct product can be used as a means of encryption, given that no one can know what the metagroup (product group) is just by knowing one of the groups that compose it. This means that we could have people working on one group, be it physical or theoretical, and different people working on a different group, without any of those people knowing what the metagroup (the direct product of both quotient groups) really is. The product group cannot be deduced from any of the quotient groups alone.

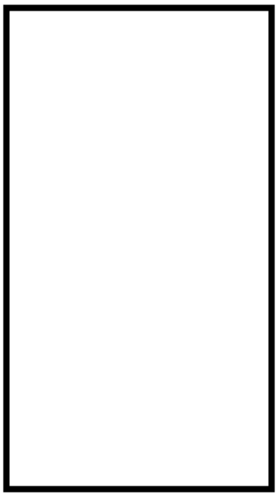
A Hybrid Logic-Time Group

As an example, say we have the following structure: *e* produces *a*, *a* produces *e*, *e* implies *b*, *b* implies *e*, *b* produces *c*, *c* produces *b*, *a* implies *c*, *c* implies *a*.

In that case, we have the following group, usually called V4, where the purple lines are the generator *produces*, the red lines are the generator *implies*, and the green lines are the generator *produces + implies.*



This group has a rectangle as an object of symmetry, where *produces* = *flip*, *implies* = *rotate 180* °, and *produces* + *implies* = *flip* + *rotate 180* °.



The information of all the elements of the group can be maximally compressed to the information of the element of the group with the least information. If someone ones to change any element of the group, they would have to change them all simultaneously because, as long as one of them exists, the whole group gets generated again.

Mass to information

If information is going to be ingrained into a physical structure for any length of time, then it needs to resist external influences. For a set of bits to resist the same influence simultaneously, they need to be affected by the same influence simultaneously, that is, there must be a connection between those bits.

Mass is a measure of resistance to change in motion. Therefore, we could see mass as a type of ingrained information. Mass reduces the entropy of the motion of an object. The mass of an object or a particle is a measure of the number of ingrained information about the motion of the particle or object. We consider several physical particles of any type to belong to the same object when they move simultaneously, that is, when the position at any time of each of the particles can be deduced from the position of any one of the particles that make up the object (if we know how they are bonded to each other), and therefore their trajectory can also be deduced from the trajectory of any of the particles in the object. The bonds between the particles make a priori independent probabilities into dependent ones.

Say the probability of a particle changing its trajectory is 0.1. Then, the entropy of a binary variable describing the trajectory of that particle is approximately 0.47 bits. Hence, the information ingrained by the particle being on that trajectory is $1/0.47 \approx 2.13$ bits per bit. Say an object is composed of 2 particles, then, the a priori probability of both particles being in their required positions is 0.1×0.1, and the entropy of such a variable is 0.08 bits, thus, the information ingrained by such an object is $1/0.08 \approx 12.4$ bits per bit. This is a measure of resistance, and moving two particles that are bonded together is much more difficult than moving two particles, each separately. To visualise this, imagine how many free-floating particles in the air one person moves in a day by walking around(many). Now imagine how much harder it is for a person to walk into a wall (it is much harder because the particles are bonded together).

Symmetry as Power

It seems reasonable to define negentropy as the ratio of maximum possible entropy to actual entropy, which coincides with our measure of information, or symmetry via information transmission, in bits per bit. This negentropy, or information as we have defined it, depends on making a priori independent probabilities be determined by one variable alone. What this means is that the growth of power depends on the erasure of the independence of other ‘quanta of power’ as Nietzsche would have it. The independence of quanta of power generates entropy, dissymmetry, disorder. The assimilation of several quanta of power into a given power structure generates order, symmetry, negentropy. As we have said previously, and we reiterate here, this negentropy depends not on things being identical in all possible ways, but in things being coherent with each other while somehow different, that is, it depends on symmetry, on permanence under change. Note that this is consistent with the etymological connection between power and potential, with potential being determined by the position of an entity within a conservative force field. Entropy acts as a conservative force (just like gravity), and negentropy acts as a measure of the potential energy of the structure within the field of entropy (just like height is directly proportional to potential energy in the gravitational field of Earth). Entropy is just the tendency of the quanta of power to be independent from each other unless they are forced to do otherwise.

Symmetry Quantification

Symmetry can be quantified as the ratio of difference to sameness. Say we have a bit ingrained on a switch. If now we expand that bit into the same switch, at the same time, we do not have several bits ingrained in that switch, we still only have one, because the substrate, being exactly the same, does not multiply the probabilities of the outcomes. On the contrary, imagine we expand that bit into another switch, on a different position in space. Now, due to the difference in spatial position, we can multiply the a priori independent probabilities, and thus we have more bits ingrained on the initial switch. The same applies if, instead of changing position in space, we change any other variable, such as position in time. What this means is that, for the information on the switch to expand, it needs to expand into a difference. To expand information, we need to ingrain the same information again in a different part of the universe. Sameness requires difference. The same applies to laws or to matter. How many phenomena can I explain with a single law? Say I can explain phenomena that would occupy 500 pages with one single law, then, the law, or implicit information, is ingrained on all those different phenomena, which could in theory be otherwise, but they are not, that is, they represent a priori independent probabilities, which have been assimilated into a structure of power, that is, they have stopped being independent and have started sharing information.

What about matter? Say we have a brick. The reason why it is a more massive object than, say, a single atom, is because we can simultaneously determine the positions of many different molecules or atoms from the position of any one of them. This is due to the existence of spatial coherence between the positions of the atoms. If we know the position of one molecule, plus the value of the distance between them, then we know the position of all other molecules on the brick (they go together). This coherence, or symmetry, is what makes the brick more difficult to move than, say, one single atom floating around in the air. This is why complexity is not total order or total chaos, total identity or total difference, it is symmetry: permanence under change, sameness on difference. Symmetry is the perfect framework to measure the information of a structure, in bits per bit, that is, in explicit bits per implicit bits, or maximum bits per actual bits. Abstract algebra studies symmetry, and thus it is a useful tool for the creation of maximally coherent structures. Symmetry can be seen as a concept that does for general structures relative to their resistance to change what mass does for physical entities relative to their resistance to change their motion. Note that for a mass, the symmetry is directly proportional to the resistance. For a law, the symmetry is directly proportional to the resistance. For a power structure, the symmetry is directly proportional to the resistance.

When an agent of a structure resists external forces, it automatically determines new forces.

Organism and Reality

An organism is defined by its organisation. Whatever is produced by the organism that is not part of the feedback loop is excreted. An organism is constituted at least of the elements that allow the system to perpetuate itself through the environment. The excretions (mostly as heat but it could be anything else) are not part of the organism itself, although they are produced by it. Thus, not everything an organism produces is itself, and the group might cause external (non-group) variables to change. The important thing is that the identity element is capable of creating itself through the environment, by means of one or more other elements, and so are all other elements. The identity element is an initial state that contains the generators of the group as possible actions or processes to be carried out from that initial state. The other elements of the group are states that result from the generator actions or processes. The elements are positions, states, the generators are actions, processes. The operation tells us how to relate one generator to another, or one state to another. For a group that evolves in time, the elements do not need to be re-created identically, but, on the contrary, can be recreated into a new state as long as the new state allows for the continuation of the group, that is, as long as the new state is algebraically equivalent to the original element, relative to the rest of the group. Some of the first possible operations that come to mind for an algebra of real systems are '*simultaneously with*' and '*followed by*'. Note that '*simultaneously with'* is a commutative operation, whereas '*followed by*' is not necessarily commutative, although it can be. Note that the organism need not be physical as such, although the information needs to be somehow materialised. The organism could be a logically coherent structure, a set of activities, an organic or inorganic process of self-production, a hybrid between any of those, or something else. We can have several time cycles within one group, thus creating a system with several superposed internal frequencies.

Realities compete for fitness, just as organisms do. Realities need to be self-sustaining in order to be able to exist. To create a reality, the amount of energy that someone puts into it is not as important as the internal coherence of the structure. Most of the energy, unless informationally protected, gets dissipated and wasted. Unlikelihood (negentropy) tends to be swallowed by an ocean of entropy. For improbabilities to survive, they need to reflect each other in a coherent system that acts as a Darwinian competitor for the initial attractor. The power that we impose on the initial attractor works also for outside competitors, given that, to change one thing of the system they need to change the whole system (holographic-style design, algebraically symmetric). Memory, or the ability to engrain information so that it stays there for a long enough time is key to building systems. The system has to have enough memory so as to keep the engrained bit materialised long enough for it to be re-produced by the rest of the system. One state followed by another state is enough to bring a third variable closer to the desired state. Having only two states produce a third is the most effective way to build a system, thus we exploit the binary aspect of group theory. If we need three variables to produce a fourth, then we can just take the product variable and multiply it by another variable, thus exploiting the associative property. The entropy, when going from one attractor to another, first increases and then decreases. Inverses allow us to account for cyclic time or cyclic logical implication or causation., which is fundamental to the self-sustaining of the structure. Movement occurs relative to the static, thus our self-reinforced system, by being difficult to change for an outside force (due to its internal symmetries, its holographic style design) acts as a type of Archimedes lever for any outside system that is less coherent. To stabilise a variable, one needs many elements pointing back towards it. When we reach a singularity in the number of causal paths from one specific element to itself through the system, then that element can be thought of as fixed, something that just 'is'. The environment always offers some type of resistance, the key is to make the structure able to grow more than it gets dissipated by friction, so that we obtain a net growth per cycle. Improbability triumphs by exploiting the likelihoods in its environment.

Everything is an organism. Reality is an organism. The most symmetric organism wins.

Probabilistic Gravity

The a priori probability of an event occurring is a reflection of the state of all local (capable of influencing) variables. For example, why does a coin fall on heads on average 50% of the time and on tails the other 50% of the time? The minute variables that are affecting the toss balance out over time because there is nothing constraining them to be in a specific state, and that is why their influence is nil and we are left with maximum entropy of the variable.

If we force an outcome to occur, then, we are forcing all variables to be coherent with that state, that is, to reflect it, which then tends to reflect the enforced state back, even after it has ceased to be. The original outcome then forces the variables to reflect it again, and vice versa, and thus continuously in a loop that might end up amounting to nothing (the information being randomised along the way) or creating a whole habit dynamic. This is how a condensation dynamic emerges, or probabilistic gravity. I am mentioning this as a possible mechanism that might explain the habit-formation of nature, that is, the idea that things tend to repeat themselves once they happen. Outcome *x* makes outcome *y* more likely, which then makes outcome *x* more likely, and so on.

The Observer and Bit Popping

The role of the observer is to superimpose conceptual structures so that the maximum number of bits is explained on the basis of as few bits as possible. For that purpose, several events must be considered simultaneously, either through memory in time, or by observing several occurrences at once. For example, consider the following sequence:

1011100011

Now consider the next sequence:

0100011100

An observer can see that the second sequence is only an inversion of the first sequence, and vice versa, so that both sequences could be compressed as:

1011100011 + invert

Imagine that this is repeated indefinitely in time. Then, all the sequences in that series could be compressed as:

1011100011 + invert indefinitely

The reader may argue that the rules occupy more bits than the sequences themselves, given that the entropy of the variable containing all possible rules is enormous, but that would imply that the observer is considering all possible rules. This is not what the observer is doing, however. The observer is only looking for the rules that match the sequences. This offers a much smaller number of possibilities, ideally only one, so that the entropy of the rule is zero after having looked at the sequences. (the observer uses the information of the sequences temporarily to reduce the entropy of the rule to zero and thus find it). Once found, a rule that occupies *X* number of bits to be *stored* implies a much larger number of bits (the sequences, in our example). Thus, the compression of the system is maximized. The erasure of the now liberated bits in the mind of the observer increases the entropy (randomisation) of the storage system (memory), which releases an amount of energy that can be calculated via de information-energy equivalence developed by Rolf Landauer (where *1 bit = k T ln2* joules of energy, where

k is Boltzmann's constant and T is the temperature of the environment). A theory is only as good as its measure of compression (explicit bits/implicit bits). Note that compression, in this context, implies coherence. As we increase the number of explicit bits covered by the theory or decrease the number of bits needed to store the rules, consistency increases. The same applies to logical structures. The more propositions we can extract from the simplest possible premises, the more coherent the logical structure will be and thus the more powerful. The number of bits popped (liberated, erased) is the quantity of meaning that has been extracted. Meaning can be quantified in bits, or joules.

A Theory of Truth

What is true is what is fixed, what cannot be changed. We use the truth, or fixed, to create an outcome by engineering another outcome (fixing it) and conducting it through the main fixed (the truth) to produce an output. We can see that as a process of input (our actions), process (the truth, the fixed), and output (the result). But nothing is absolutely fixed or absolutely movable, on the contrary, things are relatively fixed and relatively movable. The fixedness of a structure depends on its 'mass' (resistance to change). The 'mass' of a structure is determined by the number of ingrained bits that resist an external influence simultaneously. A structure that has a symmetry of 500 bits per bit is more fixed than a structure that has a symmetry of 2 bits per bit. Thus, if they come into conflict with each other, the more 'massive' structure prevails. The role of the observer is to notice what is fixed, what does not change. That is, the observer is trying to determine what the object of symmetry is.

The truth is the object with the most symmetries. The observer determines a priori truth, the agent determines a posteriori truth.

Conduction

Why would the observer want to know what the object of symmetry is? So that he can solve equations. That is, if an object is symmetric and we rotate one part of it, the other part also rotates. By knowing the structure of the object of symmetry, plus our input, we can deduce the output. If the observer wants to transmit something, he needs a channel.

A channel consists of some things that are already structured, so that the message will go through with minimum noise, instead of dissipating into the environment and being randomised. That is, a channel is a negentropy preserver, a symmetry relation between input and output. The observer wants to represent the object of symmetry in his cognition so that he can use it as a channel. The fixed, by being immovable, acts as a lever and conducts the forces of the agent to produce an outcome on the other side of the axis of symmetry.

Symmetry and Free Will

The problem with classical arguments for determinism of the type of Laplace`s Demon, which assert that *"We may regard the present state of the universe as the effect of its past and the cause of its future.",* is that the variables depend on each other. For example, one can say, if *x* and *y*, then *z*, where *z* is the logical overlap between *x* and *y*, as we do on syllogisms. But that does not allow us to deduce the state of the system at the next instant, because each variable depends on all local variables and all local variables depend on each variable, simultaneously. One does not know whether *x* is going to be or not, nor do we know if *y* is going to be or not, because *logic acts in a simultaneous abstract space, and not in time.* Just like symmetry. We can know that, if one side is up then the other is down, and vice versa, but we cannot determine which side is going to be up (in an isolated system). In other words, we can have conditional knowledge of a simultaneous type, like an equation, but that knowledge does not allow us to predict into the future with absolute certainty. Causality is generated by statistical regularities, but not determined on a logically absolute manner because, *given the simultaneous dependence of variables on each other, no state is logically closed due to knowledge of a previous state.* The reader might have notice that we have been considering *production in time* a valid symmetry throughout this book, but it is only the regularities on the environment that allow us to use the time symmetry on a practical level (for engineering purposes), without it being based on absolute logical necessity. Other symmetries, such as *logical implication* are indeed of the nature of logical necessity.

By exerting his will against other variables, the agent is free to determine the state of affairs as he pleases and cannot be determined by past events nor by external present configurations.

A Formula for Power

Power as perceived from the outside, that is, as resistance:

$$P_{resistance} = \frac{t}{\Delta bits \times H(F)}$$

where $P_{resistance}$ is power as resistance, t is the time it takes to permanently change a number *(delta)bits* of ingrained bits in the system and *H(F)* is the entropy of the binary variable F composed of all the outcomes s needed to produce the permanent change, that is, the magnitude of the outside influence. To put is otherwise, the power of a structure is:

Directly proportional to the effort it takes to permanently change it (inversely proportional to the entropy of the binary variable describing the outcomes needed to produce the permanent change).

Directly proportional to the time it takes to permanently change it.

Inversely proportional to how many bits are permanently changed in a given time with a given effort.

Notice that this is not a passive power, as all local variables are forced to be coherent with it. By not being determined by outside forces, the structure determines outside forces.

Power from the inside:

The power P of a structure is a measure of the number of ingrained bits that resist the same external influence event, avoiding the change of any one of the bits within the structure, whether the resistance manifests simultaneously as a blocking, or at different times as a recovery mechanism.

$$P = \frac{ingrained\ bits}{ingrained\ bit} = \frac{1}{H(T)}$$

Where *H(T)* is the *a posteriori* entropy of the structure, which, as the time tends to infinity, using abstract algebra, tends to zero, as the probability of all the required outcomes tends to 1.

Power is determined by the number of bits that imply each other, that is, the number of bits that need to be changed for any one single bit of the structure to be changed, it is *information*, measured in bits per bit. Power is a measure of negentropy, or potential, brought about by symmetry.

The Kube

The Kube is an example of the algebraic mechanism for the creation of structures of power.

The Kube is a C2 x C2 x C2 group. It can be any such group, with any elements and any generators. We present here its fundamental structure and the process by which to create it.

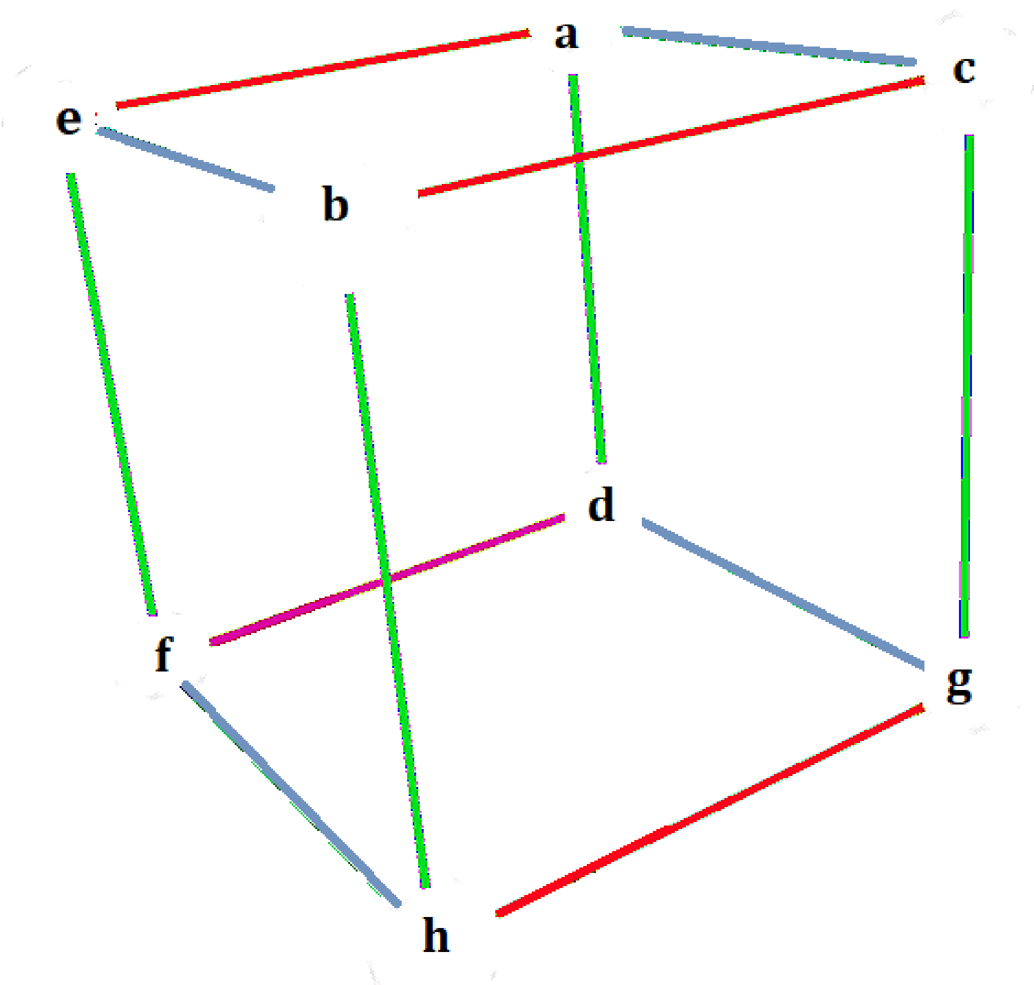
The Process

1. Create a C2 group, connected by generator 1 (g1).
2. Create another C2 group, connected by g1.
3. Connect the previous two C2 groups by g2 to obtain a V4 group.
4. Repeat steps 1 to 3 to obtain another V4 group.
5. Connect both V4 groups by g3 to obtain a C2 x C2 x C2 group.

Fundamental Structure

The Kube is composed of two V4 groups, each made up of two C2 groups, thus making a C2 x C2 x C2 group. Its quotient groups are C2and V4.

The following image is a diagram of the structure of any C2 x C2 x C2 group:



Now we will describe the process by which to create one of these groups in the real world, a structure we have named The Kube, by using the above image as a map.

Generators:

Red: Production in time.
Blue: Mutual information.
Green: Increase of the probability in time.

Other possible generators:
Logical implication.
Multiplication of available energy.

For this Kube:

Production in time (not necessarily symmetric going forwards in time).

Total mutual information per second (TMI/s):

The generator exists when $I(a|b)=0$ after a specific amount of time, which is just a special case of $I(a;b)>0$ in time where $I(a)\leq I(b)$.

e produces a and viceversa.

b produces c and viceversa.

f produces d and viceversa.

h produces g and viceversa.

Immediate mutual information (symmetric) (MI):

The generator exists when $I(a;b)>0$.

e shares information with b and viceversa.

a shares information with c and viceversa.

f shares information with h and viceversa.

d shares information with g and viceversa.

Mutual information per second (not necessarily symmetric going forwards in time): (MI/s):

The generator exists when $I(a;b)>0$ after a specific time.

e increases the probability of f and viceversa, in time.

a increases the probability of d and viceversa, in time.

b increases the probability of h and viceversa, in time.

c increases the probability of g and viceversa, in time.

The generators are channels with a channel capacity equal to the MI or MI/s between the elements.

All of the above can be quantified either as MI or MI/s.

The coherence *Coh* of a structure *S* is thus given by:

$$Coh = \frac{H(e) + H(a) \ldots + H(z)}{H(S)}$$

where

$$H(S) = \big(H(e) + H(a) \ldots + H(z)\big) - MI$$

and

$$MI = \sum_{i=a}^{z} \sum_{j \neq i} I(i;j) \qquad \textit{for each pair of elements } i,j \textit{ in the group.}$$

The coherence of a structure that depends on time is given by:

$$Coh = \frac{H(e) + H(a) \ldots + H(z)}{H(S)\ t}$$

where

$$H(S) = \big(H(e) + H(a) \ldots + H(z)\big) - MI/s$$

and

$$MI = \sum_{i=a}^{z} \sum_{j \neq i} I(i;j)/s + I(j;i)/s$$

Whereas before we quantified the maximum compression, now we are quantifying the actual compression, or coherence, of a structure.

Now we will specify what MI/s (mutual information per second) means.
We know that MI (for two binary variables) is given by:

$$I(X;Y) = p(x,y)log_2\frac{p(x,y)}{p(x)p(y)} + p(nx,ny)log_2\frac{p(nx,ny)}{p(nx)p(ny)} + p(nx,y)log_2\frac{p(nx,y)}{p(nx)p(y)} + p(x,ny)log_2\frac{p(x,ny)}{p(x)p(ny)}$$

where *nx* and *ny* means *not x* and *not y*, respectively.
Now we introduce time:

$$I(X;Y)/s = ((\frac{p(x,y)}{t})log_2\frac{\left(\frac{p(x,y)}{t}\right)}{\frac{p(x)p(y)}{t\hat{}2}} + (p(nx,ny)/t)log_2\frac{(\frac{p(nx,ny)}{t})}{(p(nx)p(ny)/t\hat{}2)})$$

$$+ (p(nx,y)/t)log_2\frac{(\frac{p(nx,y)}{t})}{(p(nx)p(y)/t\hat{}2)})$$

$$+ (p(x,ny)/t)log_2\frac{(\frac{p(x,ny)}{t})}{(p(x)p(ny)/t\hat{}2)})$$

Where *t* is the time space to which the probabilities apply.

The above formula can be simplified as:

$$I(X;Y)/s = \frac{p(x,y)}{t}log_2\frac{p(x,y)t}{p(x)p(y)} + \frac{p(nx,ny)}{t}log_2\frac{p(nx,ny)t}{p(nx)p(ny)}$$

$$+ \frac{p(nx,y)}{t}log_2\frac{p(nx,y)t}{p(nx)p(y)} + \frac{p(x,ny)}{t}log_2\frac{p(x,ny)t}{p(x)p(ny)}$$

Or, top put it more succinctly:

$$\frac{I(X;Y)}{s} = \sum_{x\in X,y\in Y}\frac{p(x,y)}{t}log_2\frac{p(x,y)t}{p(x)p(y)}$$

The above formula is what we have called MI/s (for a binary variable).

For a hybrid structure (time and logic): *HMI(S)=MI/s(S)* + *MI(S)* (Hybrid mutual information of a structure equals mutual information per second plus mutual information).

Falsifiability

For us to know whether the ideas presented in this book offer a useful model of reality, there are some tests we can run. We can measure the internal mutual information of two different structures that are not compatible with each other (due to some logical incoherence between the structure of both, their position in space, or any other variable). Then, we make them come into mutual contact and we see whether the structure that has more internal mutual information (coherence) is indeed more resilient, which is what our ideas predict. We can also see things emerge when we match up components with mutual information that would otherwise not have resisted the friction from random variables on the outside. We can check whether the amount of internal mutual information is positively correlated with the ability of an organism to come into existence (be formed in the first place) and to stay within existence (to not be randomised). If the structures with mor internal mutual information prove to indeed be more resilient or to outlive other structures, then our hypothesis would be correct and of value for systems engineering.

Cosmovision

This is a view of the world where the fixed wins. The fixed determines anything that comes into contact with it, as the volatile has no choice but to adapt to the fixed. We have seen that things are not absolutely fixed or absolutely volatile bit, rather, they are more or less fixed than other things (fixedness is relative). Our objective was to build structures that are as fixed as possible, so that they can impose themselves and withstand attack. Fixedness is a connection between different sets of bits that would, in principle, be able to operate separately (they can hold more entropy than they do hold). When we compress bits into less bits, we get informational density, and thus fixedness. We saw that this may have applications for cognitive science (the observer and bit popping), systems engineering, our understanding of reality and organisms, and our vision of what truth is or how it is identified (epistemology).

Further Research

The next step is to develop programs that can be used to measure the coherence of a group-shaped system starting from the probabilities of each variable given another pair of variables. This could be made into a Cayley table where, apart from the elements being combines and their product, we also observe the mutual information between the variable formed by the two factor elements and the product variable. Then, we would add the information of the elements in the columns and rows titles and subtract from that the information of the elements inside the multiplication table (the products). The first number is the amount of a priori bits, and the second number is the number of final bits. The first divided by the second gives us the number of bits per bit, or coherence of the structure (degree of redundancy of the information). Further development of that type of software could be useful for the integration of information theory into systems engineering, with the intention of increasing robustness, resilience to external influence, and self-regeneration of the system. It could also be useful for systems analysis given that, when tackling an external system, attacking its internal coherence would be a priority for overriding it. Understanding the internal coherences of systems is key to understanding how variables that an observer would categorise as separate are actually part of the same informational substrate (they share mutual information). imply or reinforce each other, giving rise to a view of organisms (whether natural or artificial) and realities that is based on the coherence (the number of bits per bit) of the informational substrate, that is, in the coherence of the system, its “fixedness”.

Conclusion

From the point of view of information theory, we can also measure the interaction between two systems (number of bits per bit lost by each of the systems due to the interaction), and determine which system is imposing itself and which is being weakened. What should be looked at when determining the strength of a system is not how many bits have been changed, but how many bits have been decohered. By monitoring changes in coherence, we know whether a system is growing (increasing the number of bits that are coherent, that is, expanding into new substrate) or shrinking/fragmenting (decohering into several smaller systems or losing substrate for its information). This offers a new understanding of power, growth, competition (for substrate on which to ingrain one's information) and organism.

The intention of this writing is to offer a view of reality that allows for the quantitative measurement of power (understood as the number of bits dominated by the same underlying bits), interactions between systems (and the consequences of those interactions for each of the systems involved), as well as a framework for the effective construction of new systems.

The idea at the core of this writing that mutual information (as defined by Shannon) can be seen as symmetries of the same underlying object, which can be understood from the point of view of abstract algebra. This work is an initial presentation of some of the possibilities behind that idea. By illuminating the underlying objects (the fixed, or the implied bits), and its many symmetries (all of the phenomena that are coherent with those implied bits, which can also be quantified as bits), we can see the world as a collection of objects and their many symmetries. All objects (physical or abstract) are essentially a piece of information, whether they are physical laws, theories, biological entities, or stones, and their range of expansion and influence can be quantified on the basis of the physical substrate their information occupies, per piece of information that they fundamentally are. That is the coherence, or power, of a system (bits per bit).