Expanding Exerted Free Will by Increasing the Realm of Power while Preserving Coherence

Modulo Four, NBE

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Introduction

This document presents a self-contained framework for understanding how a system can expand its *free will* by increasing a quantity called the *realm of power*, while preserving *coherence* in its internal dynamics. It also discusses the significance of **group-theoretic** (or other *self-reinforcing* algebraic) structures for amplifying resistance to external changes, and explains how such structures, akin to Platonic objects, provide fixed, stable forms that underlie the stable structures of the world.

1. Definitions and Core Equations

1.1 Ingrained Bits, Substrates, and Agent Criteria

A system *ingrains* a bit of information in a physical or logical *substrate* (e.g., memory cells, stable configurations, or organizational processes). Once ingrained, forcing a change requires work (energy, informational force, etc.). The more bits a system can ingrain in this manner, the more *power* it exerts over that substrate.

An entity is considered an **agent** if its actions are *computationally irreducible* to anything else, meaning the only complete description of its behavior is the entity itself: no simpler external model can perfectly replicate or predict the agent's internal decision process. This irreducibility plays directly into the notion of free will, as it implies the agent's internal states are not trivially foreseeable by an outside observer.

- **Ingrained Coherent Bits**: The total bits of information the system firmly embeds, so that flipping or erasing them requires nontrivial external work.
- **Maximally Compressed Bits**: The minimal "core" specification needed to define those ingrained bits.

1.2 Realm of Power (R_{info})

We measure the system's *realm of power* as:

$$R_{\text{info}} \;=\; \frac{\text{Ingrained Coherent Bits}}{\text{Maximally Compressed Bits}}. \tag{1}$$

A large R_{info} indicates many bits under tight enforcement relative to how few bits are needed fundamentally to describe them. This ratio also reflects how *costly* it becomes for an external force to change the system's ingrained states.

1.3 Coherence (C)

We define *coherence* by:

$$C \;=\; \frac{\sum I(\text{Transition; Internal Structure})}{\sum H(\text{Transition})}, \tag{2}$$

where:

- $\sum H(\text{Transition})$ is the total *entropy* of the system's transitions, capturing their unpredictability,
- $\sum I(\text{Transition; Internal Structure})$ is the *mutual information* between those transitions and the system's internal design (its constraints and rules).

A high $C \in [0, 1]$ means the system's own structure strongly determines how it evolves over time, rather than succumbing to noise or external randomness.

1.4 Free Will ($\mathbf{Will_{info}}$)

We define the system's *free will* via:

$$\mathrm{Will}_{\mathrm{info}} \;=\; \Big(\frac{\Delta R_{\mathrm{info}}}{\Delta t}\Big)\, C, \tag{3}$$

where:

- $\Delta R_{\mathrm{info}}/\Delta t$ is the rate at which R_{info} changes,
- C is coherence as above.

A positive $\mathrm{Will}_{\mathrm{info}}$ implies the system expands (or at least retains) its realm of power while preserving internal consistency.

2. Generic Toy Example

We consider a simple system S in two configurations, S_1 and S_2, to illustrate R_{info}, C, and hence $\mathrm{Will}_{\mathrm{info}}$.

2.1 Initial Configuration S_1

- **Ingrained Coherent Bits** = 3.
- **Maximally Compressed Bits** = 3.

Thus,

$$R_{\mathrm{info}}(S_1) = \frac{3}{3} = 1.$$

Suppose the system toggles between states A and B with moderate predictability:

$$\sum H(\text{Transition}) = 0.6, \quad \sum I(\text{Transition; Internal Structure}) = 0.4.$$

Hence, by (2),

$$C(S_1) = \frac{0.4}{0.6} \approx 0.67.$$

So initially:

$$R_{\mathrm{info}}(S_1) = 1, \quad C(S_1) \approx 0.67.$$

2.2 Expanded Configuration S_2

Now let the system ingrain two additional bits, raising *total* ingrained bits to 5, while increasing its fundamental specification from 3 to 3.5. Then

$$R_{\text{info}}(S_2) = \frac{5}{3.5} \approx 1.43.$$

If the total entropy of transitions becomes 0.8 but the internal structure accounts for 0.56,

$$C(S_2) = \frac{0.56}{0.8} = 0.70.$$

We see that as the system gains new capabilities, it *still* preserves coherence.

Free Will Computation. From (3),

$$\text{Will}_{\text{info}} = \left(\frac{\Delta R_{\text{info}}}{\Delta t}\right) C.$$

Let $\Delta t = 1$. Then

$$\Delta R_{\text{info}} = 1.43 - 1.0 = 0.43, \quad C(S_2) = 0.70.$$

Hence,

$$\text{Will}_{\text{info}} \approx 0.43 \times 0.70 = 0.301.$$

This is positive, meaning S has grown its realm of power from 1.0 to 1.43 *and* stayed coherent, thus expanding its free will in an informational sense.

3. Group-Based Structures and Platonic Abstraction

3.1 Groups as Self-Reinforcing Algebraic Objects

In certain systems, especially those structured as a **group** (satisfying closure, associativity, identity, inverses), the ingrained bits become *self-reinforcing*:

- **Closure and Associativity** tie states together in a consistent algebraic web, so external changes to one part are promptly corrected via feedback from the other parts, *increasing the effort required* for permanent alteration.

- **Identity and Generators** mean that *all* states can be derived from a minimal subset (plus time or logic relations). Consequently, the *Maximally Compressed Bits* in Equation (1) remain comparatively small, thereby *raising* R_{info}.
- **Inverses** allow the system to *reverse or resist* perturbations, further amplifying the resistance to forced bit flips.

3.2 Platonic Objects and Stability

Groups, fields, rings, or other highly symmetric algebraic structures behave like **Platonic abstractions**: they are "invisible" or intangible in the sense that one cannot directly perceive or poke them physically, yet they *underlie* stable forms in reality.

In any group-based system, the *abstract* algebraic properties remain rigid: an outside attempt to deviate from the group rules must effectively *fight* the entire structure. Hence, groups (and similar self-reinforcing algebras) act as *fixed, stable realities*, offering formidable **resistance** to external disturbance.

Resistance via Abstraction. Because these algebraic properties cannot be directly broken without *undoing* the entire structure, they serve as intangible "fixed objects". This implies the power (work per unit time) necessary to change even a single ingrained bit *scales* with the logical or time-mediated informational connections to all others.

4. Conclusion

We have introduced:

- **Ingrained Coherent Bits** (the bits actively enforced in a substrate),
- **Realm of Power** R_{info} (ratio of ingrained bits to their minimal description),
- **Coherence** C (measure of how internally determined the system's transitions are),
- **Exerted Free Will** $\text{Will}_{\text{info}}$ (the product of R_{info}'s rate of increase and C).

When a system has **group-based** or otherwise self-reinforcing algebraic architecture, it acquires the same "rigidity" seen in Platonic objects whose symmetry sets are themselves groups. This rigidity makes external disruption far more costly, since a *single* bit's fate is intertwined with the entire algebraic structure. Meanwhile, expansions integrate new bits without a commensurate blow-up in the fundamental specification, *raising* R_{info} while preserving or even enhancing C. Consequently, the system's **exerted free will** (as an informational concept) grows.

A **non-group** system struggles to replicate this synergy, as each new element risks increasing the fundamental specification and/or lowering coherence. Finally, a **true agent** is one whose behavior cannot be reduced to a smaller external model, aligning with the irreducibility principle: to fully predict it, one must run the agent itself. Within this framework, expansions in R_{info} become authentic enhancements of the agent's self-determined influence, aided by the self,reinforcing properties of abstract algebraic objects.