

From ‘The Will to Power’ to ‘The (Mathematical) Way to Power’: A Conceptualization of Power Through the Lens of Information Theory and Algebra

Modulo Four, NBE

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1 Purpose

The purpose of this document is to equip the individual with both a mathematical understanding of the metaphysics of power and a method to harness power—a way to actively expand themselves into the world. This integration blends philosophical insights with rigorous mathematical formulations to present a unified framework for understanding and attaining power.

2 Definition of Power

2.1 Active, Passive, and Symmetry

The distinction between active and passive is crucial in clarifying our concept of power. Power is our ability to make other systems take whatever form we want them to take, without having to adapt to external limitations. Symmetry, as explored through abstract algebra, plays a fundamental role in defining and quantifying this power.

2.2 Symmetry as a Fundamental Aspect of Power

Symmetry represents permanence under change, embodying the same bits expressed in new bits. By embedding symmetries into systems, we can visualize how information remains consistent despite transformations, thereby increasing the system's resilience and power.

3 Quantities: Physics vs. Information Theory

This section compares core quantities in physics with their counterparts in information theory, establishing a foundation for quantifying power through informational metrics.

3.1 Informational Mass

$$M_{\text{info}} = -\log_2 P_{\text{transition}} \quad (1)$$

Definition: Measures the resistance to state change in terms of probability. If a state change is unlikely to occur naturally, the informational mass is high.

Interpretation: In physics, higher mass means more resistance to changes in motion.

Example If a state transition has a 1% chance ($P = 0.01$), the informational mass is:

$$M_{\text{info}} = -\log_2 0.01 \approx 6.64 \text{ bits}$$

3.2 Mass to Information

Mass can be interpreted as a measure of resistance to change, analogous to how information is ingrained within a structure to resist external influences. The more mass (information) an object possesses, the more it resists alterations, thereby increasing its power.

3.3 Informational Distance

$$d_{\text{info}} = -\log_2 P_{\text{desired outcome}} \quad (2)$$

Definition: Measures how unlikely a specific outcome is.

Interpretation: Just as physical distance reflects how hard it is to reach a location, informational distance reflects the effort required to achieve an unlikely outcome.

Example If the probability of reaching a specific state is 5% ($P = 0.05$):

$$d_{\text{info}} = -\log_2 0.05 \approx 4.32 \text{ bits}$$

3.4 Informational Velocity

$$v_{\text{info}} = \frac{d_{\text{info}}}{\Delta t} \quad (3)$$

Definition: The rate of change of informational distance over time.

Interpretation: Describes how quickly the probabilities in a system are changing.

3.5 Informational Acceleration

$$a_{\text{info}} = \frac{\Delta v_{\text{info}}}{\Delta t} \quad (4)$$

Definition: The rate at which informational velocity changes.

Interpretation: Similar to physical acceleration, which measures changes in velocity.

3.6 Informational Force

$$F_{\text{info}} = M_{\text{info}} \cdot a_{\text{info}} \quad (5)$$

Definition: Combines the resistance to state change (informational mass) with the rate of change (informational acceleration).

Interpretation: Quantifies the effort needed to change the state probabilities.

3.7 Informational Work

$$W_{\text{info}} = F_{\text{info}} \cdot d_{\text{info}} \quad (6)$$

Definition: The total effort applied to change the state probabilities.

Interpretation: Similar to physical work, where force moves an object over a distance.

3.8 Informational Power

$$P_{\text{info}} = \frac{W_{\text{info}}}{\Delta t} \quad (7)$$

Definition: Measures how quickly effort is applied to achieve or maintain a state.

Interpretation: The less likely the outcome, the greater the informational power required to make it happen within a specific time frame.

Note on Time as Information Time can be viewed as the universe's ongoing processing of bits. From this perspective, we can consider how much information we personally process and shape relative to the total flow of information in any given moment. The greater the fraction of processed information aligned with our aims, the more deeply we imprint our influence on the unfolding of events.

4 Coherence

4.1 Ingraining Information

Ingraining information involves embedding desired data into a system such that it resists external perturbations. This process ensures that the system maintains its structure and coherence over time, thereby enhancing its power.

4.2 Entropy of Transitions

$$H(\text{Transition}) = - \sum_{x \rightarrow y} P(X_{t+1} = y \mid X_t = x) \log_2 P(X_{t+1} = y \mid X_t = x) \quad (8)$$

Definition: Measures the unpredictability of state transitions in a system.

4.3 Conditional Entropy with Internal Information

$$H(\text{Transition} \mid \text{Int Inf}) = - \sum_{x \rightarrow y} P(X_{t+1} = y, \text{Int Inf}) \log_2 P(X_{t+1} = y \mid \text{Int Inf}) \quad (9)$$

Int Inf (short for Internal Information) represents the system's internal structure or constraints.

Definition: Quantifies the remaining uncertainty in transitions after considering the system's internal structure or constraints.

4.4 Mutual Information

$$I(\text{Transition}; \text{Int Inf}) = H(\text{Transition}) - H(\text{Transition} \mid \text{Int Inf}) \quad (10)$$

Definition: Measures how much the system's internal information reduces the uncertainty in transitions.

4.5 Probabilistic Gravity

Probabilistic gravity refers to the tendency of systems to develop habit-forming dynamics through the reinforcement of certain states. This concept illustrates how information embedding influences the likelihood of recurring outcomes, creating a form of informational attraction akin to gravitational pull in physical systems.

4.6 Coherence

$$C = \frac{\sum I(\text{Transition}; \text{Int Inf})}{\sum H(\text{Transition})} \quad (11)$$

Definition: Coherence quantifies the fraction of total uncertainty in the system's transitions that is explained by its internal information.

- $\sum I(\text{Transition}; \text{Internal Information})$: Represents the mutual information between transitions and the system's internal structure, measuring how much the internal information explains the transitions.
- $\sum H(\text{Transition})$: Represents the total entropy of transitions, capturing the overall uncertainty in the system.

Valid Range:

$$0 \leq C \leq 1$$

$C = 1$: The system's internal information perfectly explains all transitions. $C = 0$: The system's internal information does not explain the transitions at all.

Interpretation: Coherence evaluates how well the system's internal structure aligns with its observed transitions. A higher C indicates greater alignment, with $C = 1$ representing maximum coherence where all transitions are fully determined by internal information.

5 Interaction and Power

5.1 Dominant Interaction

Dominant Interaction quantifies the effectiveness of our interactions with an external system. It is defined as the ratio of the number of bits changed in the target system to the number of bits changed in our own system. This metric evaluates both the efficiency and the impact of our interactions on external systems.

$$D_I = \frac{\Delta B_{\text{target}}}{\Delta B_{\text{ours}}}$$

Definition: Dominant Interaction measures the proportion of changes induced in the target system relative to the changes made in our system, indicating the effectiveness of our interactions.

5.2 Interaction Efficiency

Interaction Efficiency assesses how effectively interactions are conducted over time. It is defined as the rate of dominant interactions per unit time.

$$\eta_I = \frac{D_I}{t}$$

- η_I : Interaction Efficiency.
- D_I : Dominant Interaction.
- t : Time over which the interaction occurs.

Definition: Interaction Efficiency quantifies the rate at which effective interactions occur, representing the dominant interactions achieved per unit of time.

5.3 Exerted Power

$$P_{\text{ex}} = \frac{\Delta C_{\text{target}}}{\Delta C_{\text{system}}} \quad (12)$$

Definition: Exerted power quantifies the ratio of the change in the compression (symmetry) of the target system to the change in symmetry of our system.

Interpretation: Positive exerted power indicates influence, while negative exerted power implies feeding dynamics.

5.4 Feeding Dynamics

Feeding occurs when one system increases its coherence by drawing resources, energy, or information from another system, resulting in the source system gaining coherence while the target system loses coherence.

6 Singularity and Combinatorial Explosion

6.1 Reaching a Singularity

When the information of a group grows supra-exponentially with the number of elements, the system reaches a singularity. At this point, altering any single bit necessitates simultaneous changes across all bits, making the system highly resistant to external modifications.

6.2 Combinatorial Explosion

Combinatorial explosion refers to the rapid increase in the number of informational paths within a system, enhancing its resilience. The number of permutations grows factorially with the number of elements, ensuring that any interference is met with robust informational reinforcement.

7 Symmetry in Power Structures

7.1 Visualising Symmetry

Symmetry in power structures represents permanence under change. By embedding symmetries into systems, we can visualize how information remains consistent despite transformations, thereby increasing the system's resilience and power.

7.2 Symmetry Quantification

Symmetry can be quantified as the ratio of difference to sameness. This measure evaluates how much information is preserved under various transformations, serving as a metric for the system's structural integrity.

7.2.1 Platonic Solids as Symmetric Structures

Platonic solids are renowned for having the highest degree of symmetry for a given number of vertices. Each Platonic solid is defined by faces that are congruent regular polygons, with the same number of faces meeting at each vertex. This uniformity ensures that Platonic solids possess the most symmetries possible for their respective vertex counts, making them exceptionally robust group-based systems.

However, while Platonic solids provide a foundational example of highly symmetric structures, they may be less robust compared to more complex algebraic structures such as fields or rings. Fields and other advanced algebraic systems offer greater flexibility and resilience by incorporating additional operations and properties that extend beyond mere symmetry. Nonetheless, the inherent symmetries of Platonic solids make them invaluable in constructing foundational models within group-based frameworks, serving as building blocks for more intricate algebraic constructions.

7.3 Symmetry as Power

Symmetry serves as a fundamental measure of power, encapsulating the system's ability to maintain coherence and resist external disturbances through structured information transmission.

8 Info-Algebras: Algebras as Informational Structures

Info-Algebras views algebras as information-theoretical structures, where the elements represent variable states and the generators serve as informational channels, defining how the state of one variable probabilistically influences the state of another.

8.1 Group Properties and Their Contribution to Resilience

8.1.1 Requirements for a Self-Sustaining System

1. **Starting Point for Generating Processes:** The system must begin in a state that allows the generating processes to be applied for the first time. This initial state acts as a foundation for the system's functionality and ensures that all other states can be reached from this starting point.
2. **Cyclic Flow of Information:** Once the generating processes are initiated, the system must support continuous cyclic flow of information. This means that information must pass from all elements to all other elements and eventually return to the starting elements. Such flow ensures that the system remains interconnected and coherent.
3. **Self-Containment:** The elements of the system must be sufficient to produce all the system's components. In other words, the system must be closed under its operations, ensuring it does not rely on external states or processes to sustain itself.
4. **Consistency and Predictability:** The system must handle multiple simultaneous processes without conflict. This requires that the order of operations does not affect the outcomes, ensuring consistency even under complex or overlapping interactions.

8.1.2 How These Requirements Correspond to Group Properties

1. **Identity (Starting Point for Generating Processes):** The identity element in a group is the starting point from which all generators are applied. It ensures that there is a consistent reference state from which the system can begin its processes. Without an identity element, the system would lack a defined origin for initiating transitions.

2. **Inverses (Cyclic Flow of Information):** The existence of inverses allows for the reversal of transitions, ensuring that every element can "return" to the starting state or progress cyclically through the group. In self-sustaining systems, this guarantees continuous flow and the ability to regenerate pathways, maintaining interconnection among all elements.
3. **Closure (Self-Containment):** Closure ensures that any operation between system elements results in an element that also belongs to the system. In a self-sustaining system, this means that the system produces all necessary components internally, reinforcing its autonomy and resilience.
4. **Associativity (Composition of Relations is Associative):** Associativity ensures that the sequence of operations does not affect the final outcome. In the context of information-theoretical relations, such as sharing information, associativity implies that the manner in which information flows through intermediaries does not alter the overall distribution or final state of the information within the system. This property is crucial for handling simultaneous or overlapping processes, as it allows multiple cycles to occur concurrently without causing inconsistencies or disruptions.

8.1.3 A Hybrid Logic-Time Group

A hybrid logic-time group incorporates both logical implications and temporal transformations, creating a robust algebraic structure that accounts for both reasoning processes and time-based state changes.

8.1.4 Note on the Types of Generators Possible

Generators in info-algebras are information-theoretical relations that define how information flows between elements. These generators can be binary, ternary, or of higher arity, depending on the complexity of interactions within the system. They serve as the foundational operations that, through composition, build the algebraic structure of the system.

8.1.5 Note on the Types of Algebraic Structures Possible

Info-Algebras extends beyond groups to include rings, fields, semigroups, monoids, modules, and vector spaces. Each algebraic structure provides different properties and capabilities for modeling informational dynamics and interactions within systems. For instance, rings and fields introduce additional operations such as addition and multiplication, offering more nuanced control over information manipulation and transformation.

8.1.6 Note on the Competition of Realities for Fitness

Realities, much like organisms, compete for coherence and resilience. Information-theoretical algebras provide a framework for understanding how different systems evolve and compete based on their internal symmetries and information structures. This competition drives the selection of more robust informational frameworks, akin to natural selection favoring more resilient organisms.

8.1.7 Conclusion

The foundational properties of algebraic structures are essential for designing resilient and self-sustaining informational systems. By leveraging these properties, we can construct robust frameworks that maintain coherence and resist external disruptions.

8.2 Group Power Formula

$$P_{\text{group}} = \frac{\sum_{i,j \in G} W_{i*j}}{T_{\min}} \quad (13)$$

Definition: The total informational power required to disrupt a group G through its operations.

- W_{i*j} is the **informational work** needed to disrupt the result of combining elements i and j within the group.

- The summation $\sum W_{i*j}$ covers **all ordered pairs** of elements in G (i.e., all permutations of two elements).
- T_{\min} is the **duration of the shortest pairwise combination cycle**, representing the influence of the fastest generator.

9 Building Objects of Power

9.1 Direct Product and Encryption

We can build groups using other groups as building blocks through the direct product. This operation connects every element of one group to every other element of another group, creating a metagroup with higher symmetry and enhanced resilience.

Encryption Using Product Algebraic Structures: In algebraic structures like direct products or quotient groups, the obscurity of the full structure enhances security. For instance, let $G = H \times K$, where H and K are groups. An entity operating solely on H has access to only one component of G , making it impossible to reconstruct G without knowledge of K . Similarly, in quotient groups G/N , knowledge of the coset representatives does not reveal the underlying group G or the normal subgroup N without additional information. These principles make such algebraic structures valuable for cryptographic applications, where security depends on limiting access to the complete system.

9.2 Building Objects of Power

By leveraging abstract algebra, specifically group theory, we can construct objects of power that embody specific symmetries. These structures are designed to maximize coherence and resist external disturbances, thereby enhancing their power.

For example, using the associative property, we can take the composition of informational relations to create more complex and resilient structures. When the system reaches a singularity in the number of causal paths from one specific element to itself, that element can be considered fixed, solidifying its role within the system.

10 A Theory of Truth and Conduction

10.1 A Theory of Truth

Truth is defined as what is fixed and immutable within a system. It serves as the foundation for creating and maintaining desired outcomes by engineering states that align with the system's inherent symmetries. Truth is the object with the most symmetries, and it acts as the anchor around which power structures are built and maintained.

10.2 Conduction

Conduction involves the transmission of information through symmetrically structured channels, ensuring that messages are conveyed with minimal noise and maximal coherence. This mechanism preserves negentropy, allowing information to be effectively transmitted and utilized within the system. The fixed, immovable structures act as levers, conducting the forces of the agent to produce outcomes aligned with the system's symmetries.

11 The Observer and Bit Popping

11.1 Observer's Role in Information Compression

The observer plays a critical role in compressing information by *simultaneously* analyzing multiple pieces of information and identifying underlying rules that explain observed patterns with minimal data.

11.2 Bit Popping Mechanism

Cognitively, bit popping involves the liberation of bits through the observer's simultaneous recognition and compression of multiple information patterns. This process increases the system's entropy and releases energy, adhering to the information-energy equivalence principle, where erasing bits in the observer's memory releases energy into the system. In terms of the expansion of organisms, bit popping means the occupation of bit-holding substrate by the informational structure of the expander, which means the information previously held in that substrate, and therefore its independence, its entropy, is erased.

12 Symmetry and Free Will

12.1 Free Will Equation

$$\text{Will}_{\text{info}} = \left(\frac{\Delta R_{\text{info}}}{\Delta t} \right) C \quad (14)$$

Definition: The informational free will of a system quantifies its capacity to increase the quantity of information-holding substrate that contains its information, based on its internal structure, while maintaining coherence and resisting disruption.

- $\frac{\Delta R_{\text{info}}}{\Delta t}$ is the **rate of change** of the realm of power over time.
- C is the system's **coherence**, which must satisfy $0 \leq C \leq 1$.

Interpretation: A higher $\text{Will}_{\text{info}}$ indicates the system can expand or sustain its structured information without becoming incoherent. For free will to be positive:

- $\Delta R_{\text{info}} \geq 0$: The system must be actively increasing or maintaining its structured information.
- $0 \leq C \leq 1$: Coherence ensures that the system's transitions are consistent with its internal structure.

12.2 Symmetry and Determinism

Symmetry undermines classical determinism by introducing conditional knowledge that does not permit absolute predictions of future states. This balance between symmetry and free will allows systems to exert influence without being wholly determined by past states. Causality is generated by statistical regularities rather than absolute necessity, enabling free will to operate within the framework of informational symmetries.

13 A Formula for Power

13.1 Power as Resistance

Power, perceived as resistance, is directly proportional to the effort required to permanently change the system's ingrained bits and inversely proportional to the entropy of the binary variable describing the required outcomes.

13.2 Power from the Inside

Internal power is a measure of the number of ingrained bits that resist the same external influence event, maintaining the system's coherence over time. Power is determined by the number of bits that imply each other, acting as a measure of negentropy and potential brought about by symmetry.

14 Realm of Power

14.1 Definition and Formula

$$R_{\text{info}} = \frac{\text{Ingrained Coherent Bits}}{\text{Maximally Compressed Bits}} \quad (15)$$

Definition: The realm of power measures how much coherent information a system occupies relative to its fundamental information.

14.2 For Groups with Elements e and Generators g

$$R_{\text{info}} = \frac{\sum_{e \in G} I(e)}{\sum_{g \in G} I(g) + I(\text{identity})} \quad (16)$$

- $I(e)$ represents the **information content** of each element e in the group G .
- $I(g)$ and $I(\text{identity})$ represent the **information content** of the generators and the identity element, respectively.

15 From Building to Finding Information-Theoretical Algebras

This section provides a comprehensive methodology for both constructing and discovering hidden information-theoretical algebraic structures within complex systems using time series data. By integrating principles from information theory and algebra, the approach facilitates the design of robust, self-sustaining systems and the identification of underlying symmetries and interactions inherent in the system's dynamics.

15.1 Building Information-Theoretical Algebras

Constructing information-theoretical algebras involves defining a set of elements and operations that encapsulate the informational dynamics of the system. The following steps outline the process:

15.1.1 Step 1: Define the Element Set

1. **Select System Elements:** Choose the fundamental components of the system that will serve as the elements of the algebra. For example, in a financial system, these could be individual stocks or financial instruments; in a biological network, they might be genes or proteins.
2. **Characterize Element States:** Define the state of each element in a manner that captures its informational attributes. This could involve discretizing continuous variables or categorizing qualitative states.

15.1.2 Step 2: Establish Operations

1. **Determine Relevant Interactions:** Analyze the system to identify key informational interactions between elements.
2. **Define Operation Rules:** Formulate precise rules for how operations combine elements based on their informational interactions. Operations should reflect processes such as information fusion or conditional dependencies. In the context of Info-Algebraics, the primary operation is the **composition of relations**, where each operation represents the sequential application of information-theoretical relations.
3. **Incorporate Directionality (if applicable):** For directed interactions, ensure that operations account for the directionality of information flow.

15.1.3 Step 3: Verify Algebraic Properties

1. **Closure Verification:** Test all possible combinations of elements under the defined operations to ensure that the results remain within the set.
2. **Associativity Testing:** Verify that the operation is associative by checking multiple element groupings, i.e., $(a * b) * c = a * (b * c)$.
3. **Identity and Inverses:** Identify and confirm the existence of identity elements and corresponding inverses for each element in the algebra.
4. **Consistency Checks:** Ensure that the operations behave consistently with the informational dynamics of the system, maintaining coherence and predictability.

15.1.4 Step 4: Integrate Informational Constraints

1. **Incorporate Mutual Information:** Use mutual information measures to inform the strength and nature of interactions between elements.
2. **Adjust Operations Based on Information Flow:** Modify operation definitions to reflect the observed information flow patterns within the system, ensuring that the algebraic structure aligns with the system's informational dynamics.

15.2 Illuminating Information-Theoretical Algebras

Discovering existing algebraic structures within a system involves analyzing mutual information-based graphs to reveal symmetries and interaction patterns. The following steps outline the discovery process:

15.2.1 Step 1: Data Collection and Preprocessing

1. **Collect Time Series Data:** Acquire time series data representing the system's elements over a specified temporal range.
2. **Handle Missing Values:** Apply techniques such as forward filling, interpolation, or imputation to address gaps in the data.
3. **Normalize the Data:** Scale the data to ensure uniformity, preventing bias in mutual information calculations.

15.2.2 Step 2: Data Discretization

1. **Select Discretization Technique:** Transform continuous time series data into categorical bins using methods like quantile-based binning or uniform binning.
2. **Determine Number of Bins:** Choose the number of bins to balance information granularity with computational efficiency.

15.2.3 Step 3: Mutual Information Calculation

1. **Compute Mutual Information:** Calculate the mutual information between each pair of system elements to assess their interdependencies.

$$MI(X; Y) = \sum_{x \in X} \sum_{y \in Y} P(x, y) \log \left(\frac{P(x, y)}{P(x)P(y)} \right) \quad (17)$$

where X and Y are discretized variables representing system elements.

15.2.4 Step 4: Graph Construction

1. **Build Undirected Mutual Information Graph:** Create an undirected graph where nodes represent system elements and edges denote significant mutual information values between pairs.
2. **Build Directed Mutual Information Graph:** Develop a directed graph where edges indicate the influence between elements at different time steps.

15.2.5 Step 5: Algebraic Group Detection

1. **Neighbor Identification:** For each node, determine its neighbors based on mutual information thresholds or top mutual information partners.
2. **Subgroup Formation:** Explore combinations of neighboring nodes to form potential algebraic groups by evaluating connectivity patterns.
3. **Structural Validation:** Ensure that the formed subgroups adhere to the defining properties of the target algebraic structures, such as closure, associativity, and the existence of identity and inverse elements.
4. **Recording Groups:** Catalog validated groups for further analysis and visualization.

15.2.6 Step 6: Visualization

1. **Node Coloring:** Use different colors to distinguish between various roles within the algebraic groups, such as identity elements and group members.
2. **Edge Styling:** Apply varied edge colors and styles to represent different types of relationships, such as directional flows or the strength of mutual information.
3. **Graph Layouts:** Employ layout algorithms (e.g., spring layout, circular layout) to highlight the structural properties of the groups.

15.3 Implementation Considerations

15.3.1 Scalability

- **Handling Large Element Sets:** Design the algebraic framework to accommodate scalability, ensuring that it remains computationally feasible as the number of elements increases.
- **Efficient Computation of Operations:** Optimize the algorithms used for defining and verifying operations to handle complex interactions efficiently.

15.3.2 Robustness

- **Error Handling:** Implement mechanisms to detect and correct inconsistencies or violations of algebraic properties during operation definitions.

15.3.3 Validation and Testing

- **Simulated Environments:** Test the constructed algebra in simulated environments to evaluate its effectiveness in capturing the system's informational dynamics.
- **Real-World Data:** Apply the algebraic framework to real-world datasets to validate its applicability and accuracy in reflecting actual system behaviors.

15.4 Best Practices

- **Scalability Planning:** Design the algebraic framework with scalability in mind, allowing for future expansion without compromising structural integrity. This can be achieved **e.g., via direct products or other algebraic operations**, enabling the merging of simpler algebras into more complex structures while preserving their foundational properties.

15.5 Conclusion

Building and illuminating information-theoretical algebras within complex systems requires a meticulous blend of algebraic principles and information-theoretical insights. By systematically defining system elements, establishing appropriate operations, and ensuring adherence to algebraic axioms, robust and meaningful algebraic structures can be constructed. Simultaneously, leveraging mutual information measures and graph-based analyses facilitates the discovery of existing algebraic structures, enhancing the understanding of the system's underlying symmetries and interactions. This dual approach not only deepens the comprehension of complex systems but also provides a foundation for designing systems with desired informational and functional properties. Future endeavors may explore more advanced algebraic constructs and integration with other computational frameworks to further expand the capabilities of information-theoretical algebras in various domains.

16 Closing Remarks

16.1 Contact Information

If you have any comments about the contents of this book or its commercialization, or desire to share ideas and/or cooperate towards the advancement of the concepts presented, please contact the author via email at: `modulo4@proton.me`

16.2 Final Thoughts on the Algebra of Power

This work is experimental and part of a broader philosophical project. Your review and feedback are greatly appreciated to further refine and expand the concepts introduced. The overarching aim is to create an “algebra of power”: a self-contained, resilient structure encoding the information the user intends to enforce, ensuring that:

- No one other than the user can meaningfully alter or disrupt the system from the outside,
- It remains coherent with the user's internal aims and real-world behavior,
- The user can expand this realm of power (via direct product or other algebraic expansions) without losing internal consistency.

By applying the following steps—defining states, enforcing group properties, ingraining information, measuring coherence, leveraging feeding dynamics, and merging algebras—the user builds a robust, self-sustaining system of power that can steadily grow while preserving their free will in the Info-Algebraic framework.