ENGINEERING RELIABILITY

FAULT TREES AND RELIABILITY BLOCK DIAGRAMS

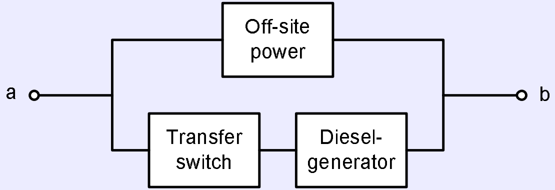
Harry G. Kwatny

Department of Mechanical Engineering & Mechanics
Drexel University

Outline

Reliability Block Diagrams

- A reliability block diagram (RBD) provides a success oriented view of the system.
- RBD's provide a framework for understanding redundancy.
- RBDs facilitate the computation of system reliability from component reliabilities.
- RBDs and fault trees provide essentially the same information.
- Below is the RBD for the backup power supply.

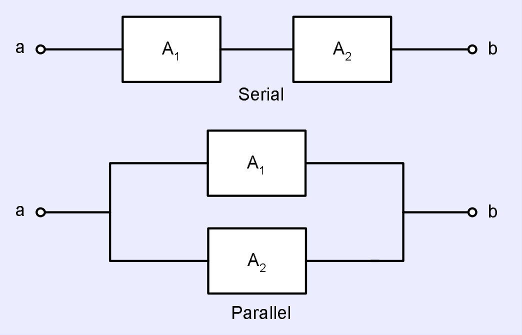


Reliability Block Diagrams – Definition

A reliability block diagram (RBD) is defined as follows:

- A reliability block diagram is a graph whose edges are the system components.
- There are a pair of nodes called *terminal nodes* – (a) and (b) in the backup power supply diagram.
- If there is a path between the terminal nodes which contains only edges with functional components, the entire system is functional. Otherwise it is not functional.

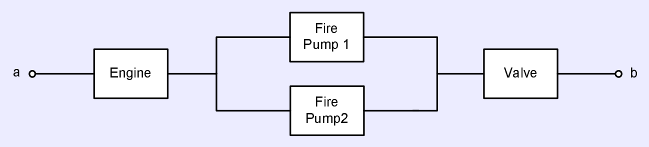
Serial & Parallel Configurations



- In the serial configuration, failure of either component, A_1 or A_2 causes system failure.
- In the parallel configuration, both components must fail in order for the system to fail – redundancy

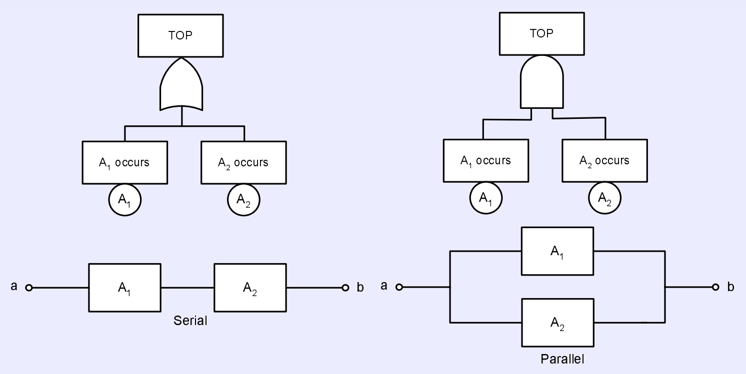
Example: Fire Pump System

- The reliability block diagram for the fire pump system is shown below.
- The redundancy of the pumps is clearly evident.



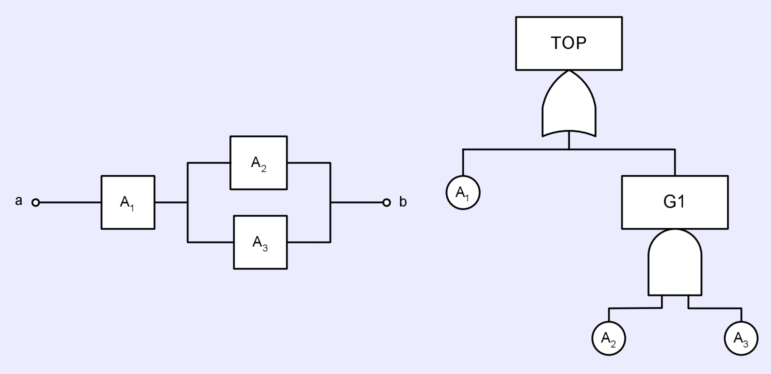
Reliability Block Diagrams & Fault Trees – 1

Engineering Reliability



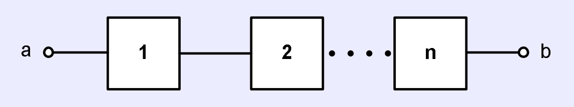
Notice that the fault tree takes a failure perspective, whereas the reliability block diagram takes a success perspective.

Reliability Block Diagrams & Fault Trees – 2



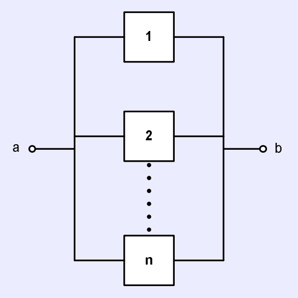
A simple serial parallel composition.

Series Structure



A system composed of n subsystems is called a series structure if the failure of any one component causes failure of the complete system.

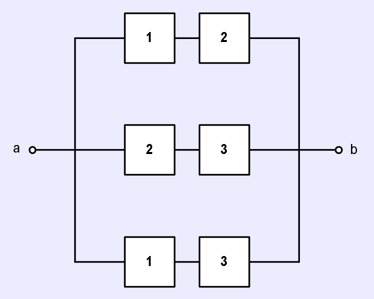
PARALLEL STRUCTURE



A system composed of n subsystems is called a **parallel structure** if it operates if any one or more of its components operates.

k-OUT-OF-n STRUCTURES

two-out-of-three structure



- A system that is functioning if and only if at least k of its n components is functioning is called a k-out-of-n structure.
- A 1-out-of-n structure is a parallel structure.

Definitions - Qualitative Analysis

- A system with n components is called a system of order n.
- x_i denotes the state of component or subsystem i,

$$x_i = \begin{cases} 1 & \text{the component is functioning} \\ 0 & \text{the component is failed} \end{cases}, \quad i = 1, \ldots, n$$

- x denotes the state of the entire system,

$$x = \begin{cases} 1 & \text{the system is functioning} \\ 0 & \text{the system is failed} \end{cases}$$

- The structure function is a function $\phi(x_1, \ldots, x_n)$ associated with a given system, such that

$$x = \phi(x_1, \ldots, x_n)$$

Structure Functions

- serial structure

$$\phi(x_1, \ldots, x_n) = x_1 \cdot x_2 \cdot \cdots \cdot x_n$$

- parallel structure

$$\begin{aligned} \phi(x_1, \ldots, x_n) &= 1 - (1 - x_1)(1 - x_2) \cdots (1 - x_n) \\ &= 1 - \prod_{i=1}^{n} (1 - x_i) \\ \phi(x_1, \ldots, x_n) &= \max_{i \in \{1, \ldots, n\}} x_i \end{aligned}$$

- k-out-of-n structure

$$\phi(x_1, \ldots, x_n) = \begin{cases} 1 & \sum_{i=1}^{n} x_i \geq k \\ 0 & \sum_{i=1}^{n} x_i < k \end{cases}$$

Coherent Structures

- A component is irrelevant if it has no effect on the functioning of the system, i.e., the i^{th} component is irrelevant if

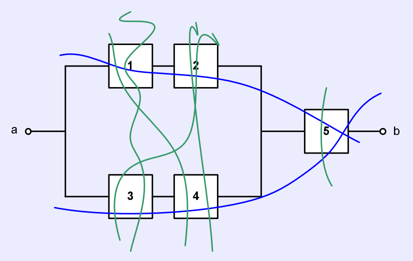
$$\phi(x_1, \ldots x_{i-1}, 0, x_{i+1}, \ldots, x_n) = \phi(x_1, \ldots x_{i-1}, 1, x_{i+1}, \ldots, x_n)$$

$$\text{for all } x_1, \ldots x_{i-1}, x_{i+1}, \ldots, x_n$$

  Otherwise it is called relevant.
- Assumption: the system will not run worse if a failed component is replaced by a functional component $\Rightarrow \phi(x_1, \ldots, x_n)$ is a nondecreasing function of each of the variables $x_1, \ldots, x_n$.
- A system is coherent if all of its components are relevant and its structure function is nondecreasing.
- By focusing on coherent structures we rule out certain pathologies.

Paths and Cutsets

- The set of components of a system of order n is
$$C = \{1, 2, \ldots, n\}$$
- A path set, P, is a subset of C which by functioning ensures that the system is functioning. A path set is minimal if it cannot be reduced without losing its status as a path set.
- A cut set, K, is a subset of C which by failing causes the system to fail. A cut set is minimal if it cannot be reduced without losing its status as a cut set.



DEFINITIONS - QUANTITATIVE ANALYSIS

- A system with n components is called a system of order n.
- A_i denotes the event that the component or subsystem i, $i = 1, \ldots, n$ is functioning at time t.
- A denotes the event that the entire system is functioning at time t.
- $P(A_i) = R_i(t)$ is the reliability of the i^{th} subsystem.
- $P(A) = R(t)$ is the reliability of the entire system.

Series Structure

- Note:
  - $A = A_1 \cap A_2 \cap \cdots \cap A_n \Leftrightarrow A^c = A_1^c \cup A_2^c \cup \cdots \cup A_n^c$
  - $A \subset A_i,\ i = 1, \ldots, n \Rightarrow R(t) \leq R_i(t)$
- A serial system reliability is no greater than the reliability of any subsystem!
- Suppose that the failure events A_i are mutually independent, then

$$R(t) = P(A) = P\left(\cap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i) = \prod_{i=1}^{n} R_i(t)$$

Parallel Structure

- $A = A_1 \cup A_2 \cup \cdots \cup A_n$
- If the subsystem failure events are independent:
  - $A^c = A_1^c \cap A_2^c \cap \cdots \cap A_n^c$
  - $P(A^c) = P(A_1^c) P(A_2^c) \cdots P(A_n^c)$
  - $P(A) = 1 - P(A^c) = 1 - \prod_{i=1}^n P(A_i^c) = 1 - \prod_{i=1}^n [1 - P(A_i)]$
- Consequently, for independent events:

$$R(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$

k-OUT-OF-n STRUCTURES

- Consider identical components,
- The probability failure of failure over a specified time period for a single component is p, so the component reliability is $r = 1 - p$,
- Let M denote the number of components that fail in the specified time period, then

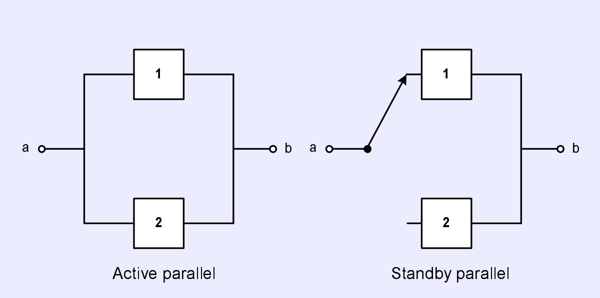
$$P(M = m) = \underbrace{C^n_m}_{\substack{\#\text{ ways to}\\ \text{get } m/n}} \underbrace{p^m}_{\substack{m\\ \text{failures}}} \underbrace{(1-p)^{n-m}}_{\substack{n-m\\ \text{survivers}}} = C^c_m (1-r)^m r^{n-m}$$

- The k/n system will survive if there are no more than $n - k$ failures, so

$$R = \sum_{m=0}^{n-k} C^n_m (1-r)^m r^{n-m}$$

DEFINITIONS

- Redundancy is the duplication of critical components in a system in order to improve reliability.
- Redundancy is normally a parallel connection of identical components – can be active or standby



Reliability of Active and Standby Systems

- Consider a redundant (parallel) combination of two identical components.
- Let T_1, T_2, T denote the failure times of component 1, component 2, and the system, respectively.
  - the units are identical, so $R_1(t) = R_2(t)$
  - assume the failures of the two units are independent events.
- The system reliability of the active parallel structure is

$$
\begin{aligned}
R_a(t) &= P(T_1 > t \cup T_2 > t) \\
&= P(T_1 > t) + P(T_2 > t) - P(T_1 > t)\, P(T_2 > t) \\
R_a(t) &= R_1(t) + R_2(t) - R_1(t)\, R_2(t)
\end{aligned}
$$

Reliability of Active and Standby Systems – 2

- The standby system does not start operating until the primary unit fails.
- The system can survive until time t if the primary unit survives until time t or the primary unit fails before time t, but the second unit survives until time t:

$$R_s(t) = P(T_2 > t \,|T_2 > T_1) = P(T_1 > t) + P(T_1 < t \cap T_2 > t)$$

- The two possibilities can be restated as $T_1 > t$ or T_1 occurs at $\tau < t$ and $T_2 > t - \tau$. Notice that

$$P(\tau < T_1 < \tau + d\tau) = f_1(\tau)\, d\tau$$
$$P(\tau < T_1 < \tau + d\tau \cap T_2 > t) = R_2(t-\tau) f_1(\tau)\, d\tau$$
$$P(T_1 < t \cap T_2 > t) = \int_0^t R_2(t-\tau) f_1(\tau)\, d\tau$$

$$R_s(t) = R_1(t) + \int_0^t R_2(t-\tau) f_1(\tau)\, d\tau$$

Example: Constant Failure Rate

- Components with constant failure rate λ have reliability

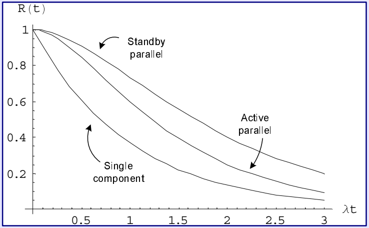
$$R(t) = e^{-\lambda t}$$

- An active parallel structure has reliability

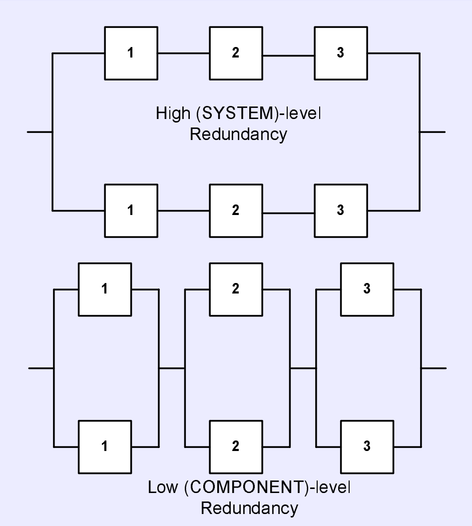
$$R_a(t) = 2e^{-\lambda t} - e^{-2\lambda t}$$

- A standby parallel structure has reliability

$$R_s(t) = (1 + \lambda t)\, e^{-\lambda t}$$



High- and Low-Level Redundancy



STRUCTURE FUNCTIONS REVISITED

- Consider a system with state vector $\mathbf{x} = \{x_1, \ldots, x_n\}$ and structure function $\phi_1(\mathbf{x})$, and a second system with state vector $\mathbf{y} = \{y_1, \ldots, y_m\}$ and structure function $\phi_2(\mathbf{y})$.
- if these systems are connected in series then the whole system is of order $n + m$, and its structure function is

$$\phi(\mathbf{x}, \mathbf{y}) = \phi_1(\mathbf{x})\,\phi_2(\mathbf{y})$$

- if these systems are connected in parallel then the whole system is of order $n + m$, and its structure function is

$$\phi(\mathbf{x}, \mathbf{y}) = \max\{\phi_1(\mathbf{x}), \phi_2(\mathbf{y})\}$$

System Level vs Component Level Redundancy

- Suppose the two systems in the parallel structure are identical, i.e., we have a system level redundant configuration. In this case,

$$\phi_S(\mathbf{x}, \mathbf{y}) = \max\{\phi_1(\mathbf{x}), \phi_1(\mathbf{y})\} \leq \phi_1(x_1 y_1, \ldots, x_n y_n)$$

- Consider a component level redundant configuration of system one, the structure function is

$$\phi_C(\mathbf{x}, \mathbf{y}) = \phi_1(\max\{x_1, y_1\}, \ldots, \max\{x_n, y_n\})$$

But, for coherent systems,

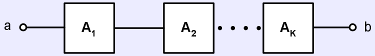
$$\phi_1(\max\{x_1, y_1\}, \ldots, \max\{x_n, y_n\}) \geq \phi_1(x_1 y_1, \ldots, x_n y_n)$$

So,

$$\phi_C(\mathbf{x}, \mathbf{y}) \geq \phi_S(\mathbf{x}, \mathbf{y})$$

- Thus, in general, we get a better (more reliable?) system through component redundancy rather than system redundancy.

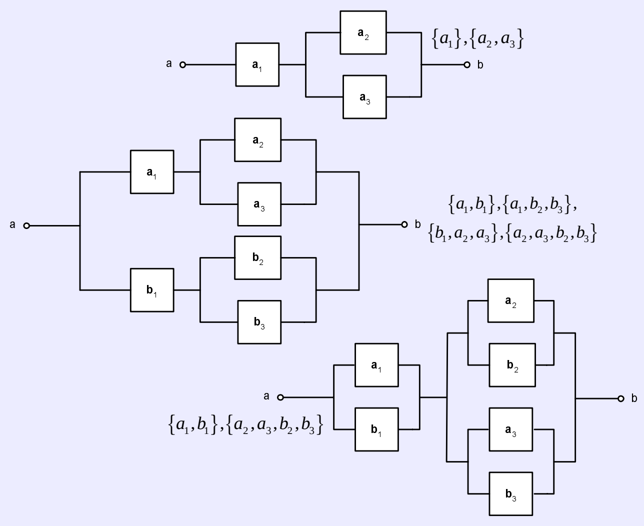
System Level vs. Component Level Redundancy – 2



$$\begin{bmatrix} A_1 \cap B_1 & A_1 \cap B_2 & A_1 \cap B_3 & \cdots & A_1 \cap B_K \\ A_2 \cap B_1 & A_2 \cap B_2 & A_2 \cap B_3 & \cdots & A_2 \cap B_K \\ A_3 \cap B_1 & A_3 \cap B_2 & A_3 \cap B_3 & \cdots & A_3 \cap B_K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_K \cap B_1 & A_K \cap B_2 & A_K \cap B_3 & \cdots & A_K \cap B_K \end{bmatrix}$$

- Consider a system with K minimal cutsets, $A_1, A_2, \ldots, A_K$.
- For a system level redundant configuration, place an identical system with cutsets labeled $B_1, B_2, \ldots, B_K$ in parallel with the original. The redundant system cutsets are all combinations $A_i \cap B_j$, $i, j = 1, \ldots, K$.
- A component level redundant system has cut sets $A_i \cap B_i$, $i = 1, \ldots, K$.
- Consequently, $R_C \geq R_S$.

Example: System vs Component Redundancy



Redundancy Limitations

- Common-mode failures are caused by dependencies that cause redundant components to fail simultaneously.
  - common power supply
  - shared environmental stresses
  - common maintenance issues
- Load sharing can cause reliability degradation in active parallel systems.
  - failure of one unit can cause increased stress on remaining units, e.g., engines, pumps, etc.,
- switching failures can occur in standby parallel systems