Calculus I Midterm 1 Review Package

Calculus I Review Topics for Midterm 1

- Limits
- Horizontal and Vertical Asymptotes
- Asymptotic Behaviour of Functions
- Continuity
- Definition of the Derivative
- Differentiability
- Derivative Rules: Sum, Power, Product and Quotient Rules
- Linear Approximation
- Tangent Line Equations

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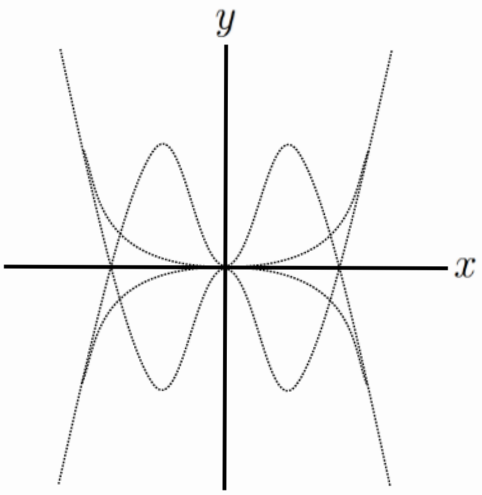
Math 100 Midterm 1 2022 Question 1

1. **[5 marks]** Trace the graph of $f(x) = 5x^3 - 5x^4$ on the axes below, using the dotted curves.



Math 100 Midterm (1+2) 2023 Questions

1. **[5 marks]** Let

$$f(x) = \frac{ax^2}{bx + cx^3},$$

where a, b and c are nonzero constants. Find the simple functions $s(x)$ and $l(x)$ such that $s(x)$ best approximates $f(x)$ for small values of x, and $l(x)$ best approximates $f(x)$ for large values of x.

Math 100 Midterm 1 2024S Questions

5. 5 marks ★★★☆ Hyperbolic functions are analogous to trigonometric functions, defined with respect to the unit hyperbola instead of the unit circle. They are particularly useful in the study of differential equations. They can also be defined in terms of exponential functions. One example is the hyperbolic cotangent function

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

(a) Find all horizontal asymptotes of $\coth(x)$.

(b) Find all vertical asymptotes of $\coth(x)$.

Recall: $\coth(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

(c) Using your answers to the previous parts, draw a rough sketch of $f(x)$ showing all asymptotes.

Math 100 Midterm 1 (2025F) + Questions

3 marks

1. Carefully find, with proof, all vertical and horizontal asymptotes of the curve

$$y = \frac{4x^2 + 4x - 48}{x^2 - 9}.$$

If there is no horizontal asymptote, or no vertical asymptote, explain why this is the case.

Math 100 Midterm 1 2022F Questions

2. **[5 marks]** Find all the horizontal and vertical asymptotes of $f(x) = \dfrac{x+2}{\sqrt{4x^2+3x+2}}$.

Math 100 Midterm 1 2022F Questions

3. [**5 marks**] Find all the values of m such that

$$f(x) = \begin{cases} 6x^3 - 2m & \text{if } x \leq -1 \\ 2x^2 + 5m & \text{if } x > -1 \end{cases}$$

is continuous.

Math 100 Midterm 1 2022F Questions

4. **[5 marks]** Let $f(x) = \frac{5}{x}$. Use a definition of the derivative to find $f'(x)$. No credit will be given for solutions using differentiation rules, but you can use those to check your answer.

Math 100 Midterm 1 2022F Questions

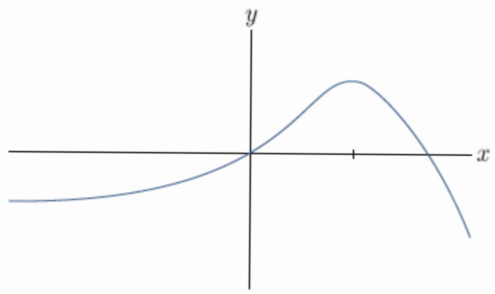
5. **[5 marks]** Suppose the function $f(x)$ is defined and continuous on all real numbers, and that

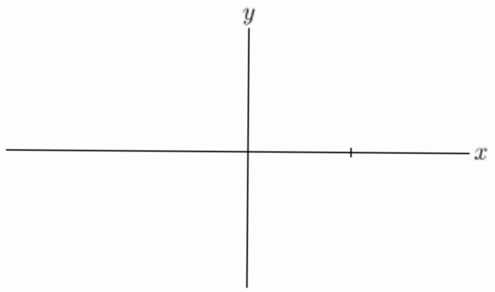
$$\lim_{h \to 0} \frac{f(5+h) - 3}{h} = -5.$$

Given this information, we can find the equation of the tangent line to the curve $y = f(x)$ at a particular point. What is the point, and what is the slope of the tangent line to the curve $y = f(x)$ at that point?

Math 100 Midterm 1 2022F Questions

6. **[5 marks]** Pictured below is the graph of a function. On the blank set of axes, sketch the graph of its derivative.





Math 100 Midterm (1+2) 2023S Questions

2. **[5 marks]** Find all the values of c such that

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq c \\ 4x - 1 & \text{if } x > c \end{cases}$$

is continuous.

Math 100 Midterm (1+2) 2023S Questions

3. **[5 marks]** Let $f(x) = 2x^2 + 3x - 1$. Use a definition of the derivative to find $f'(0)$. No credit will be given for solutions using differentiation rules, but you can use those to check your answer.

Math 100 Midterm I 2024S Questions

2. 3 marks ★☆☆☆ Let $f(x) = 3 - 2x^2$. Compute the derivative of f using the limit definition of the derivative. You will not receive marks for using derivative rules, but you may use these to check your answer.

Math 100 Midterm I (2025F) + Questions

4 marks 3. Find all values of c such that the following function is continuous :

$$f(x) = \begin{cases} 6 - cx & \text{if} \quad x > 2c, \\ x^2 & \text{if} \quad x \leq 2c. \end{cases}$$

Use the definition of continuity to justify your answer.

ubc math 180(100) limits review | questions

19. Find a and b such that $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$ is continuous.

ubc math 180(100) limits review | questions

20. Find m and b such that $f(x) = \begin{cases} x^2 & \text{if } x \leq 3 \\ mx + b & \text{if } x > 3 \end{cases}$ is differentiable.

Math 100 Midterm 1 2022F Questions

7. **[5 marks]** Find values for a and b such that such that the line $y = 2x - 2$ is tangent to the curve $y = ax^2 + b$ at the point $(1, 0)$.

Math 100 Midterm 1 2022F Questions

8. **[5 marks]** Let $f(x) = \dfrac{4x^3}{5\cos(x)}$. Find $f'(\pi)$.

Math 100 Midterm 1 2022F Questions

9. **[5 marks]** Let $f(x) = 2xe^x$. Find the equation of the tangent line to the curve $y = f(x)$ at $x = 0$.

Math 100 Midterm 1 2022F Questions

10. **[5 marks]** Let $f(x) = (x^{-4} + x^2)(x - x^3)$. Find $f'(1)$.

Math 100 Midterm 1 2022F Questions

11. For any positive integer n, the *Hassell model* of exponent n

$$f(x) = \frac{Rx}{(1+\frac{x}{M})^n}, \quad M > 0, R > 1$$

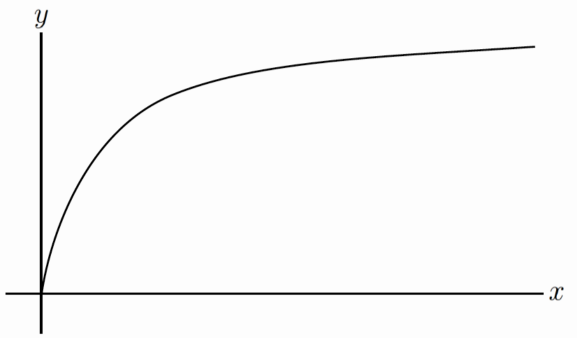
describes the size $f(x)$ of a population given the size $x \geq 0$ of the population in the previous generation. (The special case $n = 1$ gives the Beverton-Holt model from Assignment 1.)

(a) [**3 marks**] Let $n \geq 2$. Solve the equation $f(x) = x$.

(b) [**2 marks**] Let $n \geq 2$. Find the horizontal asymptote of $f(x)$.

(c) [**3 marks**] Let n_1 and n_2 be distinct positive integers. Find all points of intersection between the graphs of the Hassell models of exponent n_1 and n_2.

(d) [**3 marks**] Pictured below is a graph of the Beverton-Holt model. On the same set of axes, sketch the graph of a Hassell model of exponent $n \geq 2$. (You may assume that the parameters M and R are the same in both models.)



Math 100 Midterm I 2022F Questions

13. Let $f(x) = x^2$.

(a) [**3 marks**] Find the equation of the tangent line to $f(x)$ at the point (x_0, x_0^2). Your answer should be in the form $y = mx + b$, and may include the quantity x_0.

Answer:

(b) [**3 marks**] Imagine you were given a numerical value for x_0. Explain in 1-3 sentences how you would determine whether or not the tangent line described in part (a) passes through the point $(2, 3)$.

(c) [**1 mark**] Write down the equations of all tangent lines to $f(x)$ that pass through the point $(0, 0)$. Your answer(s) should be in the form $y = mx + b$. Your answer(s) do not have to be justified.

Answer:

(d) [**3 marks**] Let $q > 0$. How many tangent lines to $f(x)$ pass through the point $(0, q)$? (Remember to justify your answer.)

Answer:

(e) [**4 marks**] Let $q > 0$. Find the equations of all tangent lines to $f(x)$ that pass through the point $(0, -q)$. Your answer(s) should be in the form $y = mx + b$.

Math 100 Midterm (1+2) 2023S Questions

4. **[5 marks]** Find the slope of the tangent line to the curve

$$y = \frac{\sqrt{x} - 7}{\sqrt{x} + 7}$$

at $x = 9$.

Math 100 Midterm (1+2) 2023 $ Questions

11. Consider the following function

$$f(x) = x\left(\frac{e^{2\log(1+e^x)} - 1}{e^{2\log(1+e^x)} + 1}\right).$$

(a) **[2 marks]** Find the intercepts of $f(x)$.

(b) **[3 marks]** What polynomial best approximates $f(x)$ for large negative values of x?

(c) **[3 marks]** What polynomial best approximates $f(x)$ for large positive values of x?

Math 100 Midterm 1 2024S Questions

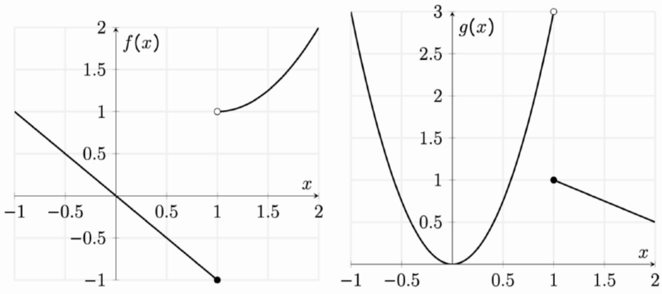
6. 3 marks ★★★☆ A function $f(t)$ has instantaneous rate of change

$$f'(t) = \sqrt{t} + \sqrt[3]{t-1}.$$

When $t = 9$, it is observed that $f(9) = 3$. Use a linear approximation to approximate $f(8.8)$.

Math 100 Midterm 1 2024 S Questions

7. 6 marks ★★☆☆ The graphs of the functions f and g are provided below. Answer each of the following questions:

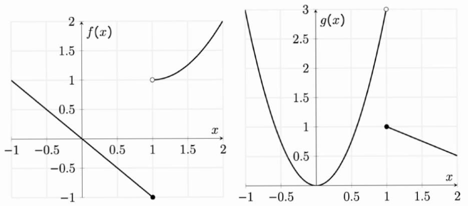


(a) If $h(x) = f(x) + g(x)$, what is $\lim_{x\to 1} h(x)$? Is $h(x)$ a continuous function for the values of x shown?

(b) If $h(x) = f(x) - g(x)$, what is $\lim_{x\to 1} h(x)$? Is $h(x)$ a continuous function for the values of x shown?

(c) If $h(x) = f(x)/g(x)$, what is $\lim_{x\to 0} h(x)$? Note that $g(x)$ for $x < 1$ is quadratic.

7. 6 marks ★★☆☆ The graphs of the functions f and g are provided below. Answer each of the following questions:



(d) If $h(x) = g(x)/f(x)$, what is $\lim_{x \to 0} h(x)$?

(e) If $h(x) = f(x) \cdot g(x)$, which of the following options is true:

A) $h'(1.5) < 0$ **B)** $h'(1.5) = 0$
C) $h'(1.5) > 0$ **D)** $h'(1.5)$ does not exist

3. 3 marks Use the limit definition of the derivative to compute the derivative of the function

$$f(x) = \sqrt[11]{x}$$

at the point $x = 0$, or show that $f'(0)$ does not exist.

Remark: this question assesses your ability to use the limit definition of the derivative. Any answer not using it will receive 0 marks.

Math 100 MT1 Practice Questions (2025F)

5. 6 marks Suppose you know the function $g(x)$ is differentiable everywhere, and that the equation of the tangent line to its graph $y = g(x)$ when $x = 2$ is $y = -5x + 7$. Find the equation of the tangent line to $y = f(x) = (x^3 + x - 7)g(x)$ at $x = 2$.

Math 100 Midterm 1 (2025F) + Questions

3 marks 2. Suppose that $g(x)$ is an unknown function defined for all real numbers, and that you know that $g(3) = 2$ and $g'(3) = -1$. What is the linear approximation to g about $x = 3$?

Math 100 Midterm1 (2025F) + Questions

5 marks

4. Write down the equation of the line tangent to the parabola $f(x) = x^2$ at the point $x = a$. Then write down the equation of the line tangent to the hyperbola $g(x) = 1/x$ at the point $x = b$. Finally, find a line that is tangent to both the parabola $f(x) = x^2$ and to the hyperbola $g(x) = 1/x$.

Math 100 Midterm1 (2025F) + Questions

4 marks 5. Use the definition of derivative to compute the derivative of the function

$$f(x) = 25\sqrt{x}.$$

Math 100 Midterm1 (2025F) + Questions

2 marks 6. Differentiate the function

$$g(t) = \frac{3t-8}{5t+7}.$$

There is no need to simplify your answer.

Math 100 Midterm 1 (2025F) + Questions

3 marks 7. Find the derivative

$$\frac{d}{dx}\left(5\cos(x)\cdot\sqrt{x}\right).$$

There is no need to simplify your answer.

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16. Find a such that $\lim_{x\to -2} \dfrac{3x^2+ax+a+3}{x^2+x-2}$ exists.

20. Find the line that is tangent to *both* of the graphs $y = x^2$ and $y = x^2 - 2x + 2$.

14. Find all tangent lines of $y = 3x^2$ that pass through $(2, 9)$.

15. Find all tangent lines of $y = x^2 + x$ that pass through $(2, -3)$.

16. Find all tangent lines of $y = \frac{x}{x+1}$ that pass through $(1, 2)$.

Math 100+ Webwork 4

Problem 20. **(1 point)**
Given that

$$f(x) = x^8 h(x),$$

$$h(-1) = 3, \text{ and}$$

$$h'(-1) = 6$$

$h'(-1) = 6,$

calculate $f'(-1) =$ ______.

Use a linear approximation to estimate $f(0)$.

$f(0) \approx$ ______

Math 100 + webwork4

Problem 21. **(1 point)**
The volume and surface area of a sphere both depend on its radius.

$$V = \frac{4}{3}\pi r^3, S = 4\pi r^2.$$

(i) Find the rate of change of the volume with respect to the radius and the rate of change of the surface area with respect to the radius.

$\frac{dV}{dr} =$ ______

$\frac{dS}{dr} =$ ______

(ii) Find the rate of change of the surface area to volume ratio S/V with respect to the radius.

$$\frac{d(S/V)}{dr} = \underline{\qquad\qquad}$$

(iii) Eliminate the radius and express V as a function of S.

$V = \underline{\qquad\qquad}$

(iv) Find the rate of change of the volume with respect to the surface area.

$$\frac{dV}{dS} = \underline{\qquad\qquad} \text{ cm}$$

Math 100+ Webwork 4

Problem 24. (1 point)

The numbers involved in this problem are not completely precise (since the exact numbers are randomized), but the ranges **are** *set up to be fairly accurate for 2024. In other words, your final answer is pretty close to the* *real-world answer.*

Suppose the national debt of the United States of America is currently 35.3 trillion dollars, and currently increasing at a rate of 10 billion dollars a day. Suppose also the population of the USA is currently 350 million, and is currently increasing at a rate of 1.2 million people per year. (You can find updated values with a quick internet search).

Given this information, at what rate is the national debt per capita currently changing? Give your answer in terms of dollars (per person) per day.

Current rate of change of national debt per capita is ____________ dollars per day.

Math 100+ Webwork 4

Problem 15. (1 point)
For what value(s) a is the line tangent to the graph of

$$f(x) = 4x^3 - 6x^2 - 70x + 24$$

at $x = a$ parallel to the line $y = 2x + 1$?

$x =$ __________

(If there is more than one value enter your answer as a comma separated list, eg. `2,3,4`)

Math 100+webwork 4

Problem 18. (1 point)
The following table gives the values for functions f and g and their derivatives for integer values of x between 1 and 5.

x	1	2	3	4	5
$f(x)$	2	-3	2	-4	-4
$f'(x)$	-1	-5	-2	3	4
$g(x)$	1	3	2	1	-2
$g'(x)$	3	-1	1	-1	3

Let

$$p(x) = f(x)g(x), \qquad q(x) = \frac{f(x)}{g(x)} \qquad r(x) = xf(x) + \frac{g(x)}{x}$$

Find

a) $p'(2) =$ ________

b) $q'(3) =$ ________

c) $r'(4) =$ ________

Math 100+ webwork 4

Problem 2. **(1 point)**
Differentiate each function below by first using algebra to manipulate them into the form $b \cdot a^x$, where a and b are constants.

1. $\frac{d}{dx}\left[9^{-x}\right] =$ ________

2. $\frac{d}{dx}\left[5^{x+2.7}\right] =$ ________

3. $\frac{d}{dx}\left[4^{2x}\right] =$

$\frac{d}{dx}$ [ ] = ______

Math 100+webwork 4

Problem 3. (1 point)
Let $f(x) = 3e^{x+3} + e^{-1}$.

$f'(0) =$ ______

Note: the chain rule is not necessary here (we haven't learned it yet).

Math 100+webwork 4

Problem 5. **(1 point)**
Differentiate the following function:

$$R(x) = \frac{\sqrt{5}}{x^4}$$

$R'(x) =$ ________

Math 100+webwork 4

Problem 11. **(1 point)**
Compute the derivatives of the following functions, where K, k, a, A, n are positive constants:

(a) The Michaelis-Menten kinetics function

$$v = \frac{Kx}{k+x}.$$

$\frac{dv}{dx} =$ ________

(b) The Hill function

$$y = \frac{Ax^n}{a^n + x^n}.$$

$\frac{dy}{dx} =$ ______

Math 100+ webwork 4

Problem 8. **(1 point)**

Differentiate:

$$Y(u) = \left(\frac{1}{u^2} + \frac{7}{u^3}\right)(u^5 + 8u^2)$$

$Y'(u) =$ ______

Problem 12. **(1 point)**

Compute the derivative of the given function.

$$f(x) = (5x^2 + 5x + 4)\frac{-11x - 1}{5x^2 + 5x + 4}$$

$f'(x) =$ ____________.

Problem 13. **(1 point)**

Find the derivative of $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}}$.

Note that you can do this two ways: by simplifying first, or by using the Quotient Rule. Both approaches should obtain the same answer.

$F'(x) =$ ____________

Math 100 + Webwork 4

Problem 14. **(1 point)**

Let $f(x) = \dfrac{x}{x + \frac{3}{x}}$. Find $f'(x)$.

$f'(x) =$ ____________________

Math 100 + Webwork 4

Problem 17. **(1 point)**

A parabola has equation $y = ax^2 + bx$, where a and b are constants. Its tangent line at $(1, 0)$ has equation $y = 2x - 2$.

What are a and b?

$a =$ ____ $b =$ ____

DIFF. RULES REVIEW

- $\frac{\mathrm{d}}{\mathrm{d}x}[a \cdot f(x)] = a \cdot f'(x)$
- $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g^2(x)}$
- $\frac{\mathrm{d}}{\mathrm{d}x}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

- $\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = f'(x)+g'(x)$

Math 100t Webwork 4

Problem 23. **(1 point)**
By applying the Product Rule twice, one can prove that if f, g, and h are differentiable, then

$$(fgh)' = f'gh + fg'h + fgh'.$$

Now, in the above result, letting $f = g = h$ yields

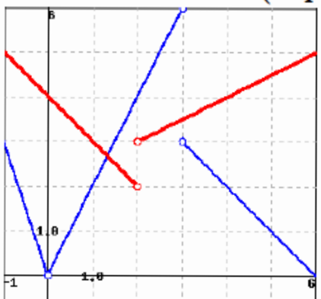
$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x).$$

Use this last formula to differentiate $y = e^{3x}$.

$y' =$ ________

Math 100 + webwork 4

Problem 19. **(1 point)**



Note: You can click on the graph to obtain a larger version in a new browser window.

The graphs of the function f (given in blue, thinner) and g (given in red, thicker) are plotted above. Suppose that $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$. Find each of the following:

$u'(1) =$ ______

$v'(1) =$ ______

Math 100+ Webwork 3

Problem 22. **(1 point)**
Suppose the function $f(x)$ is defined and continuous on all real numbers and you know that

$$\lim_{h \to 0} \frac{f(5+h) - 2}{h} = -5.$$

It follows that $y =$ ______ is the equation of the tangent line to $f(x)$ at the point (__,__).

Note: There is more that one correct solution to this problem, but WeBWorK will only mark as correct the one that is the "most obvious", i.e., the one that can be solved directly with the constants available to you.

Math 100+ Webwork 3

Problem 23. **(1 point)**
In this question, you'll practice using linear approximations.

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1. Some function $f(x)$ passes through the point $(1, 9)$. Its tangent line at that point is given by the equation
$$y = 9 - 4(x - 1).$$

Use a linear approximation to give an approximate value of $f(3)$.

$f(3) \approx$ ______

2. Suppose some function $g(x)$ has $g(-8) = -2$ and $g'(-8) = -9$. Give the formula for the linear approximation of $g(x)$ about the point $x = -8$.

Linear approximation: $g(x) \approx$ ______

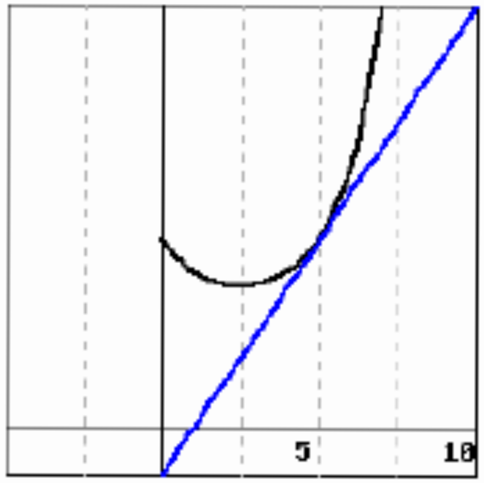
3. Suppose some function $h(x)$ has $h(-9) = 1$ and $h'(-9) = 3$. Use a linear approximation to approximate $h(-10.5)$.

$h(-10.5) \approx$ ______

Math 100+ webwork 3 Question

Problem 25. **(1 point)**
The figure below shows $f(x)$ and its linear approxima-

tion at $x = a$, $y = 4x - 4$. (The linear approximation is shown in blue.)



(Click on the image to view an enlarged graph.)

(a) What is the value of a?
$a =$ ______

(b) What is the value of $f(a)$?
$f(a) =$ ______

(c) Use the linear approximation to approximate the value of $f(5.1)$.
$f(5.1) =$ ______

(d) Is the approximation an under- or overestimate?

The approximation is an [?/under/over]estimate.

The approximation is an [?/under/over]estimate.

Problem 26. (1 point)
Suppose f is a function which is defined for all real numbers and satisfies the following two properties:

1. $f(x+y) = f(x)f(y)$ for all real numbers x, y, and

2. $\lim_{x \to 0} \frac{f(x)-1}{x} = 7.$ $\lim_{h \to 0} \frac{f(h)-f(0)}{h} = 7$

$f'(0) = 7$

$\lim_{h \to 0} \frac{f(h)-1}{h} = 7$

Use the definition of the derivative to find the relationship between $f'(x)$ and $f(x)$.

$f'(x) =$ 7 $\cdot f(x)$.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{f(x)f(h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{f(x)[f(h)-1]}{h}$

$= f(x) \lim_{h \to 0} \left[\frac{f(h)-1}{h}\right]$ $= 7$

Problem 27. (1 point)
For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 6$? Use the graph below to help check your answer.

$a =$ _____ $b =$ _____

Math 100 + webwork 3 Question

Problem 28. **(1 point)**
In this question, we'll explore some surprising behaviour of a particular derivative. Along the way, you'll practice using the definition of the derivative and computing limits.

Let

$$f(x) = \begin{cases} x^{1.2}\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

We'll investigate the derivative of $f(x)$ near the origin.

1. Fill in the limit you would compute to find $f'(0)$, using the limit definition of the derivative.

Note: your answer should depend on h, and have no x's in it. Do not evaulate the limit yet.

$f'(0) = \lim_{h \to 0}$ ______

2. Find $f'(0)$ by evaluating the limit above.

Hint: think about the behaviour of the two factors, and remember $\left|\sin\left(\frac{1}{x}\right)\right| \leq 1$ *– that is, it never gets very big.*

After thinking in that direction, if you still aren't sure how to evaluate the limit, a substitution such as $y = \frac{1}{h}$ *might make the limit look more familiar. Note that after performing this substitution, you would no longer be computing a limit at 0, but a limit at* $\pm\infty$.

$f'(0) =$ ______

3. You may use without justification that whenever $x \neq 0$, it is true that $f'(x) = 1.2x^{0.2}\sin(1/x) - x^{-0.8}\cos(1/x)$.

Does the limit $\lim f'(x)$ exist? If so, compute it and

- $f'(x)$ is both defined and continuous at $x = 0$

enter your answer in the box below. If not, enter 'DNE'.

$\lim_{x \to 0} f'(x) =$____

4. Which of the options below best describes $f'(x)$?

- $f'(x)$ is not defined at $x = 0$
- $f'(x)$ is defined at $x = 0$ but not continuous at $x = 0$

Math 100 Old exam Question

25. 6 marks In this question g is an unknown function defined for all real numbers. You are given that $g(2) = 3$ and $g'(2) = -7$:

(a) What is the linear approximation to g about $x = 2$?

(b) Let f be the function defined by $f(x) = 2^x \cdot g(x)$. Is f differentiable at $x = 2$? If so determine $f'(2)$; if not explain why not.

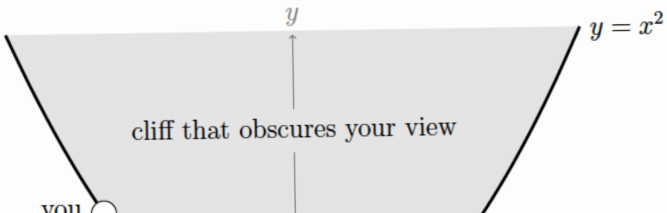
Math 100 Old exam question

30. 6 marks Suppose $f(x)$ and $g(x)$ are differentiable functions such that the equation of the tangent line of $f(x)$ at $x = -2$ is $y = 1 - 2x$ and the tangent line to $g(x)$ at $x = -2$ is $y = 4x - 3$. Find the equation of the tangent line of $f(x)/g(x)$ at $x = -2$.

Math 100 old exam question

34. 6 marks You are walking West to East along the South edge of a cliff that has profile $y = x^2$ viewed from above, as shown in the diagram below. You begin at $x = -3$. At what x-value can you first see your friend waiting for you at $(4, 7)$?

That is: what is the first x-value where the straight line from you (at the edge of the cliff) to your friend does *not* cross the cliff?



35. 6 marks Find the equation(s) of the tangent line(s) to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $y = \frac{1}{2}x - 1$.

36. 6 marks Use the definition of the derivative to compute $f'(x)$, where

$$f(x) = \frac{x-3}{x+1}.$$

You *must* use the limit definition of the derivative. Any solutions presented with any other methods will receive zero marks.

Math 100 Old exam question

37. 6 marks Consider the function

$$g(x) = \frac{e^x + 1}{e^x - 1}$$

(a) Describe all points at which $g(x)$ is continuous.

(b) Find all horizontal asymptotes of $g(x)$.

(c) Find all vertical asymptotics of $g(x)$.

Math 100 old exam question

39. 6 marks Find the value of a such that the function

$$f(x) = \frac{ax^2 - 5x - 12}{|x - 4|}$$

has no vertical asymptote at $x = 4$. For this value of a, compute $\lim_{x \to 4} f(x)$ and use this to produce a sketch of $f(x)$ near $x = 4$.

Math 100 MT1 Practice Questions (2025F)

1. 2 marks A particle is moving along a straight path. At time t, its position along the path is given by

$$s(t) = 4t^3 - 9t^2.$$

What is the average velocity of the particle from the time $t = 2$ to $t = 2.005$?

Leave your answer in calculator-ready form.

Math 100 MT1 Practice Questions (2025F)

2. [3 marks] Give the interval(s) on which this function is continuous:

$$f(x) = \frac{1}{\sqrt{1 - e^{5x}}} + \frac{1}{\sqrt{6 - x}}\ .$$

UBC Math 100 midterm 1A (2014)

9 marks

1. Determine whether each of the following limits exists, and find the value if they do. If a limit below does not exist, determine whether it "equals" ∞, $-\infty$, or neither.

(a)

$$\lim_{t \to -1} \frac{t^2 + 3t + 2}{t^2 - 1}$$

(b)

$$\lim_{x \to 3} \frac{\sqrt{x^2 + 16} - 5}{x - 3}$$

(c)

$$\lim_{t \to 2} \frac{t - 2}{\sqrt{|t - 2|}}$$

UBC Math 100 midterm 1A (2014)

4. Let $f(x) = \dfrac{\sqrt{x^2+3}-3x}{x-1}$

4 marks

(a) Determine the horizontal asymptotes of the graph $y = f(x)$.

3 marks

(b) Determine the vertical asymptote(s) of the graph $y = f(x)$. For each vertical asymptote $x = a$, determine whether each of the one-sided limits "equals" $+\infty$ or $-\infty$ as x approaches a.

UBC Math 100 midterm 1A (2014)

12 marks 2. (a) If $f(x) = 2e^x + x^{2e}$, find $f'(x)$.

(b) Let $f(t) = \dfrac{e^t + t^n}{t^2 - 1}$, where n is a positive integer. Determine where $f(t)$ is differentiable.

(c) The line tangent to $f(x) = Ax^2 + 2\sqrt{x}$ at $x = 1$ passes through the origin. Find A.

(d) Let

$$g(x) = (x+1) \cdot f(x) + \frac{x^2}{f(x)}$$

where $f(1) = 3$, $f'(1) = 0$. Compute $g'(1)$.

5 marks 5. Let

$$f(x) = \begin{cases} c + \cos(x) & x \leq 0 \\ xg(x) & x > 0 \end{cases},$$

where c is a constant and $g(x)$ is a continuous function on the interval $x > 0$ such that $0 \leq g(x) \leq 1$ for all $x > 0$. Find the value of c that makes $f(x)$ continuous everywhere.