13) Practice 4

$a = 8 \quad t_{16} = 68$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$n = 12$

$S_n = \frac{12}{2}[16 + 11d] = 6[16 + 44]$

$= 6[60]$

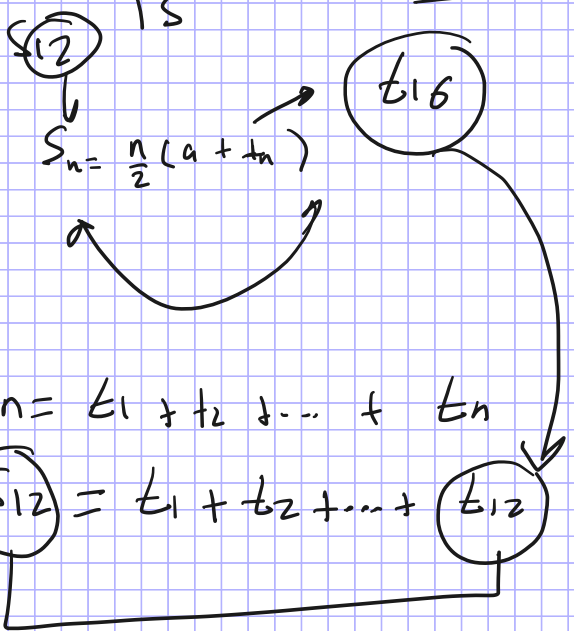
$S_{12} = 360$

$t_{16} = 68$

$t_n = a + (n-1)d$

$68 = 8 + (16-1)d$

$\frac{60}{15} = d \quad d = 4$



Can't

$S_n = \frac{n}{2}(a + l)$

$l =$

$S_n = t_1 + t_2 + \dots + t_n$

$S_{12} = t_1 + t_2 + \dots + t_{12}$

$t_n = ar^{n-1}$

18) $t_5 = 162 \rightarrow t_8 = -4374$

$t_n = ar^{n-1}$

$t_5 = 162 \quad r t_5 = t_6 \quad r^2 t_5 = t_7$

$r^3 t_5 = t_8$

$r^3 = \frac{t_8}{t_5}$

$\frac{-4374}{162} = r^3$

$r^3 = -27 \quad r = -3$

$\frac{t_8}{t_5} = \frac{ar^7}{ar^4}$

$\frac{-4374}{162} = r^3$

$r^3 = -27$

$r = -3$

$t_5 = 162 = a(-3)^{n-1}$

$162 = a(-3)^4$

$a = \frac{162}{81} = \boxed{2}$

$a = 2 \quad r = -3$

$t_1 = 2 \quad t_2 = -6 \quad t_3 = 18$

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$x,\ x+5,\ x+9,\ \ldots$

$\frac{x+5}{x} = r \qquad \frac{(x+9)}{(x+5)} = r$

$r = r$

$\frac{x+5}{x} \not\Rightarrow \frac{x+9}{x+5}$

$x^2 + 10x + 25 = x^2 + 9x$

$\boxed{x = -25}$

$-25+9 = -16$

$-25,\ -20,\ -16$

$\frac{-20}{-25} = \frac{4}{5}$ $\qquad \frac{14}{20}$

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a) $9 + 3 + 1 + \ldots$

$r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$

$-1 < r < 1$

$\left(\frac{1}{3}\right)^{\infty} \to 0$

$S_\infty = \frac{a}{1-r}$ $\qquad S_n = \frac{a(1-r^n)}{1-r}$

∞ $\qquad$ finite

$S_\infty = \frac{9}{1-1/3} = \frac{9}{2/3} = 9 \times \frac{3}{2} = \frac{27}{2}$

26) $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-4}$

$k=1 \quad \left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^3 = 8$

$a = 8$

$\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{0} + \cdots$

$a = \left(\frac{1}{2}\right)^{-3} \qquad r = \frac{1}{2}$

$\frac{\left(\frac{1}{2}\right)^{-2}}{\left(\frac{1}{2}\right)^{-3}} = \left(\frac{1}{2}\right)^{-2+3} = \frac{1}{2}$

$S = \frac{a}{1-r} = \frac{8}{1-1/2} = \frac{8}{1/2} = 16$

9) $t_5 = 2x+y \qquad t_7 = -8x+7y$

$t_n = a+(n-1)d$

$t_5 = a+4d$

$t_7 = a+6d = -8x+7y$

$2x+y = a+4d$

$-8x+7y = a+6d$

$10x-6y = -2d$

$d = \frac{10x-6y}{-2}$

$d = -5x + 3y$

$2x + y = a + 4(-5x + 3y)$

$2x + y = a + -20x + 12y$

$2x + y + 20x - 12y = a$

$\boxed{22x - 11y = a}$ $\quad d = -5x + 3y$

$t_n = a + (n-1)\,d$

$t_4 = 22x - 11y + 3(-5x + 3y)$

$= 22x - 15x - 11y + 9y$

$t_4 = \underline{7x - 2y}$ ✓

8) $t_4 = -11.98$ $\quad t_8 = 1.1$

$t_4 + d = t_5$ $\quad t_4 + 2d = t_6$ $\quad t_4 + 3d = t_7$

$t_8 = t_4 + 4d =$

$1.1 = -11.98 + 4d$

$\frac{1.1 + 11.98}{4} = d$

$t_4 = \boxed{a + 3d = -11.98}$

$t_8 = \boxed{a + 7d = 1.1}$

$t_8 = a + 7d = 1.1$

$a - 1.1 = -7d$

$\frac{a - 1.1}{-7} = d$

$t_4 = -11.98$

$a + 3d = -11.98$

$a + 11.98 = -3d$

$\frac{a + 11.98}{-3} = d$

$\frac{-21.79 - 1.1}{-7}$

$d = d$

$\frac{a - 1.1}{-7} = \frac{a + 11.98}{-3}$

$a + \frac{3(a - 1.1)}{-7} = t_4$

$\frac{-7a + 3(a - 1.1)}{-7} = \frac{-7a + 3a - 3.33}{-7} = \frac{-11.98}{1}$

$\frac{-4a - 3.33}{-7} = \frac{-11.98}{1}$

$-4a - 3.33 = 83.86$

$-4a = 87.19$

$a = -21.79$

$d = 3.27$

$t_{n2} = -21.79 + (2-1)(3.27)$

$-21.79 + (1)(3.27)$

$-21.79 + 3.27$

$t_2 = -18.52$

Geometric = Add

arithmetic = Sub

$t_n = a + (n-1)(d)$

$t_4 = a + 3d = -11.98$

$t_8 = a + 7d = 1.1$

$\square = \square$

$$\begin{array}{r} (a + 3d) = -11.98 \\ -\ (a + 7d) = 1.1 \\ \hline -4d = -13.08 \end{array}$$

$a + 3(3.27) = 11.98$

-21.79

$d = 3.27$

$t_n = a + (n-1)(d)$

$t_5 = a + (4)(d) = 2x + y$

$t_7 = a + (6)(d) = -8x + 7y$

$$\begin{array}{r} t_5 = a + 4d = 2x + y \\ -\ a + 6d = -8x + 7y \\ \hline -2d = 10x - 6y \end{array}$$

$\frac{-2d}{-2} = \frac{10x - 6y}{-2}$

$d = -5x + 3y$

$2x + y = a + 4(-5x + 3y)$

$2x + y = a - 20x + 12y$

$2x + y + 20x - 12y = a$

$22x - 11y = a$

$t_4 = 22x - 11y + (3)(-5x + 3y)$

$t_4 = 22x - 11y - 15x + 9y$

$t_4 = 7x - 2y$