

GRAPHING RATIONALS

A

1. Which one of the following graphs does not have a vertical asymptote?

A. $f(x) = \frac{1}{x^2 + 1}$

B. $f(x) = \frac{1}{x^2 - 3}$

C. $f(x) = \frac{2}{x^2}$

D. $f(x) = \frac{3x+1}{x-5}$

2. The horizontal asymptote of the graph of the function $g(x) = \frac{-7x+2}{3x+2}$ is

A. $y = 0$

B. $y = -\frac{7}{3}$

C. $y = -\frac{3}{7}$

D. $x = -\frac{2}{3}$

$$\frac{-\cancel{7x} + \left(\frac{+2}{x}\right) = 0}{\frac{3\cancel{x}}{x} + \left(\frac{+2}{x}\right) = 0} = -\frac{7}{3}$$

3. Which function has a graph with a hole?

A. $y = \frac{x+4}{2x^2+8x}$

B. $y = \frac{x-4}{2x^2+8x}$

C. $y = \frac{4x+4}{2x^2+8x}$

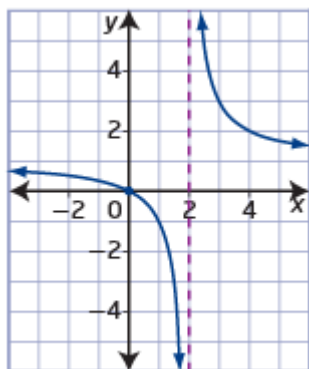
D. $y = \frac{x+4}{2x^2-8x} = \frac{x+4}{2x(x-4)}$

$$\frac{(x+\cancel{4})}{2x(x-\cancel{4})}$$

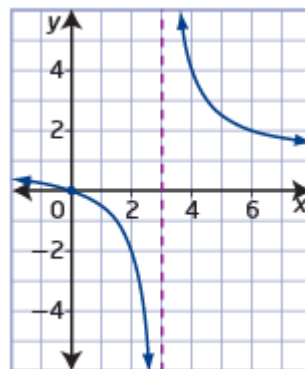
$$y = \frac{x}{x-3} \quad \text{cancel } x \quad \frac{2}{2-3} = 1$$

4. Which graph represents $y = \frac{x^2 - 2x}{x^2 - 5x + 6}$? $= \frac{x(x-2)}{(x-3)(x-2)} \Rightarrow y = \frac{x}{x-3}$

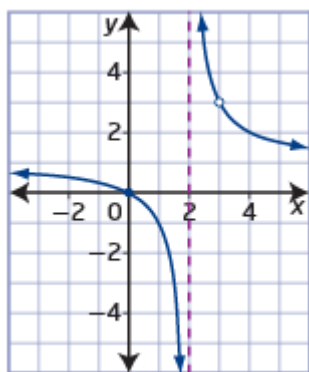
~~A.~~



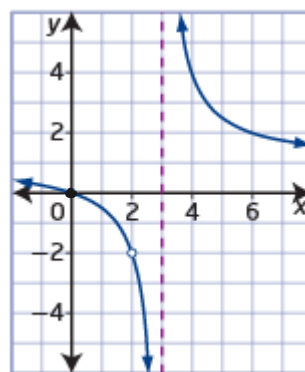
B.



C.



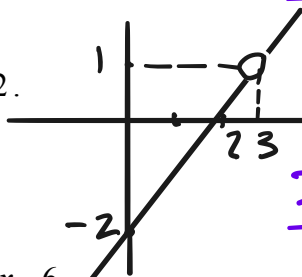
D.



(2, -2)
hole
 $x=0 \quad y=0$
VA $x=3$

5. Which statement about the graph of $y = \frac{x^2 - 5x + 6}{x - 3}$ is true?

- A. There is a vertical asymptote at $y = 3$.
B. There is a horizontal asymptote at $x = 2$.
C. There is a vertical asymptote at $x = 2$.
D. There is a hole at $(3, 1)$.



$$\frac{(x-3)(x-2)}{(x-3)} = \boxed{(x-2)}$$

$x \neq 3$

$$\frac{3-2}{3-3} = 1$$

6. Determine the x-intercept(s) of $f(x) = \frac{3x^2 - 3x - 6}{x^2 + 4x + 4}$.

- A. -2
B. -1, 2
C. -3, 6
D. 3, -6

$$\frac{3(x^2 - x - 2)}{(x+2)(x+2)} \Rightarrow \frac{3(x-2)(x+1)}{(x+2)(x+2)}$$

not x
we are looking for what value of x

$$\frac{3(x-2)(x+1)}{3} = 0$$

$$\boxed{(x-2)(x+1) = 0}$$

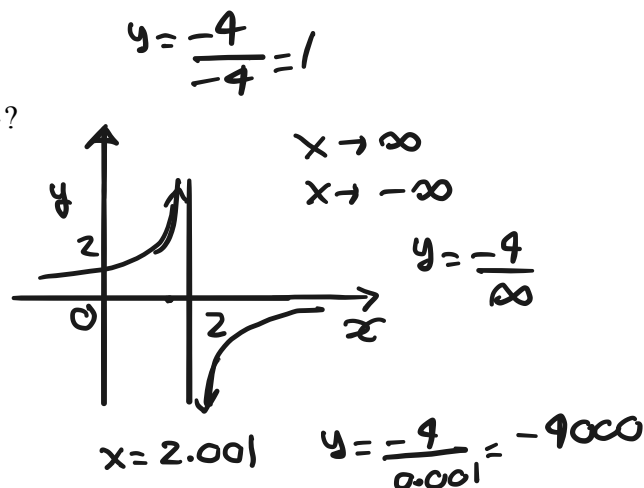
$$x = 2 \quad (x-2) = 0$$

$$x = -1 \quad (x+1) = 0$$

7. Which statement is true about the function $y = -\frac{4}{x-2}$?

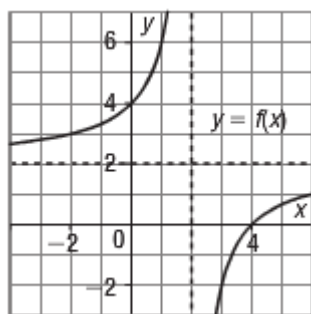
- ☒ As x approaches -2 , $|y|$ becomes very large.
☒ As $|x|$ becomes very large, y approaches -4 .
☒ As x approaches 2, $|y|$ becomes very large.
☒ As $|x|$ becomes very large, y approaches 4.

$$\begin{aligned} x &\rightarrow \infty & 1 - \infty &= \infty \\ x &\rightarrow -\infty \end{aligned}$$

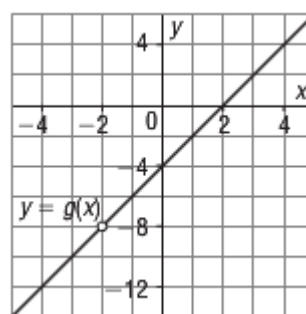


8. Which graph below represents the function $y = \frac{2x^2 - 8}{x - 2}$?

A.



B.

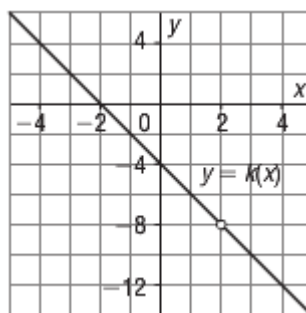


$$\frac{2x^2 - 8}{x - 2} \Rightarrow \frac{2(x^2 - 4)}{(x - 2)}$$

$$\frac{2(x-2)(x+2)}{(x-2)}$$

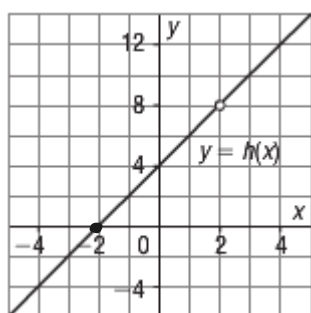
$$y = 2(x+2)$$

D.



$$\begin{aligned} x &\neq 2 \\ y &\neq 8 \\ x = -2 & \quad y = 0 \\ x = 0 & \quad y = 4 \end{aligned}$$

☒ C



9. The graph $f(x) = \frac{x^2 - 9}{x^2 - 2x - 3}$ has a "hole" at the point

- A. $(-3, 0)$
 B. $(3, 0)$
 C. $(-3, 1)$
☒ D. $(3, \frac{3}{2})$

$$y = \frac{(x-3)(x+3)}{(x-3)(x+1)} \Rightarrow \frac{(x+3)}{(x+1)} = y$$

$$x = 3$$

$$\frac{(3+3)}{(3+1)} = \frac{6}{4} = \frac{3}{2}$$

10. Consider the rational function $f(x) = -\frac{ax+5}{7x-b}$, where a and b are natural numbers. The vertical asymptote and the horizontal asymptote of the graph of $f(x)$, respectively, are

A. $x = -\frac{b}{7}, y = -\frac{a}{7}$ VA

B. $x = \frac{b}{7}, y = -\frac{5}{b}$

C. $x = \frac{b}{7}, y = \frac{a}{7}$

☒ D. $x = \frac{b}{7}, y = -\frac{a}{7}$

Handwritten work for Q10:

$$7x - b = 0 \Rightarrow x = \frac{b}{7}$$

$$-\frac{ax}{x} + \frac{5}{x} = 0 \Rightarrow \frac{7x}{x} - \frac{b}{x} = 0 \Rightarrow 7 - \frac{b}{x} = 0 \Rightarrow \frac{b}{x} = 7 \Rightarrow x = \frac{b}{7}$$

$$y = -\frac{ax}{7x} = -\frac{a}{7}$$

11. Which statement about the function $y = \frac{x}{x^2 - x}$ is true?

A. It has an x -intercept of 0.

B. It has a y -intercept of 0.

☒ C. It has a point of discontinuity at $(0, -1)$.

D. It has a vertical asymptote at $x = 0$.

Handwritten work for Q11:

$$y = \frac{x}{x(x-1)} \Rightarrow y = \frac{1}{x-1} \quad x \neq 0$$

12. The horizontal asymptote of the graph of $f(x) = \frac{2x+4}{3x-5}$ is

A. $y = 0$

☒ B. $y = \frac{2}{3}$

C. $y = -\frac{4}{5}$

D. $x = \frac{5}{3}$

Handwritten work for Q12:

$$\frac{1}{0-1} = \frac{1}{-1} = -1$$

$$\frac{2x}{x} + \frac{4}{x} = 0 \Rightarrow \frac{3x}{x} - \frac{5}{x} = 0 \Rightarrow 3 - \frac{5}{x} = 0 \Rightarrow \frac{5}{x} = 3 \Rightarrow x = \frac{5}{3}$$

13. The equation of a rational function can be written in the form $y = x - p, x \neq -3$. The graph of the rational function has a point of discontinuity at $(-3, 10)$. The value of p is

A. 13

B. 7

C. -7

☒ D. -13

Handwritten work for Q13:

$$10 = -3 - p \Rightarrow p = -3 - 10 \Rightarrow p = -13$$

14. Let f be a rational function given by $f(x) = \frac{ax^n}{bx^m}$. The graph has a horizontal asymptote of $y = 0$ when

- A. $n = m$
 B. $n < 0$
 C. $n > m$
 D. $n < m$

degree of num $<$ deg. of denom

15. The vertical asymptote of the graph of $f(x) = \frac{2x+4}{3x-5}$ is

- A. $y = \frac{3}{2}$
 B. $x = -\frac{5}{3}$
 C. $x = \frac{5}{3}$
 D. $y = \frac{2}{3}$

VA set denom = 0
 $3x - 5 = 0$

$3x = 5$

$\frac{3x}{3} = \frac{5}{3}$

$x = \frac{5}{3}$

16. The graph of a rational function has a horizontal asymptote at $y = 3$, a vertical asymptote at $x = -2$, and a y-intercept of 1. What is the equation of the function?

~~A.~~ $y = \frac{4}{x+2} + 3$ $x \rightarrow \infty \quad y = \frac{4}{\infty} + 3$
 B. $y = \frac{-4}{x+2} - 3$ $HA \quad y = 3 \quad |x| \rightarrow \infty \quad y \rightarrow 3$
 C. $y = \frac{-9}{x-3} - 2$ $HA \quad y = -2$
 D. $y = \frac{9}{x-3} - 2$ $HA \quad y = -2$

$$\frac{\frac{-4}{x}}{\frac{x}{x} + \frac{+2}{x}} + 3$$

$VA = 3$

$y = \frac{4x}{x+2} + 3$

$HA = 3$

$VA = -2$

$$\sin t = 3$$

17. ~~17~~

Consider the function $f(x) = \frac{ax^3 + bx^2 + cx + d}{2x + 5}$. If the numerator can be expressed in

the factored form $(2x + 5)(x^2 - p)$ and the graph of the function has a point of

discontinuity at $\left(q, \frac{9}{4}\right)$, the value of q is

- A. 4
- B. $\frac{5}{2}$
- C. -4
- D. $-\frac{5}{2}$

$$y = x^2 - p$$

$$(2x + 5)(x^2 - p)$$

$$(2x + 5)$$

$$\frac{9}{4} = \left(-\frac{5}{2}\right)^2 - p \Rightarrow p = -\frac{9}{4} + \left(\frac{25}{4}\right)$$

$$-p \cdot 2x + 5 = 0$$

$$\frac{2x}{2} = \frac{-5}{2} = -\frac{5}{2}$$

$$q = -\frac{5}{2}$$

$$\frac{16}{4} = 4$$

18. Which of the following rational functions describes a graph that has a horizontal asymptote of $y = -2$, a vertical asymptote of $x = 1$, and a y -intercept at $(0, 0)$?

- A. $f(x) = \frac{-2x}{x-1}$
- B. $f(x) = \frac{-x}{x-2}$
- C. $f(x) = \frac{x+2}{x-1}$
- D. $f(x) = \frac{x-2}{-x}$

19. Given $y = \frac{x+2}{x^2-3x-10}$, which statement is true?

- A. The equations of the vertical asymptotes are $x = -2$ and $x = 5$.
- B. There is a point of discontinuity in the graph of the function at $\left(-2, -\frac{1}{7}\right)$ and at $(5, 1)$.
- C. The range is $\{x \mid x \neq -2 \text{ or } 5, x \in R\}$.
- D. The non-permissible values are $x = -2$ and $x = 5$.

GRAPHING RATIONALS

A

1. **A**
2. **B**
3. **A**
4. **D**
5. **D**
6. **B**
7. **C**
8. **C**
9. **D**
10. **D**
11. **C**
12. **B**
13. **D**
14. **D**
15. **C**
16. **B**
17. **D**
18. **A**
19. **D**