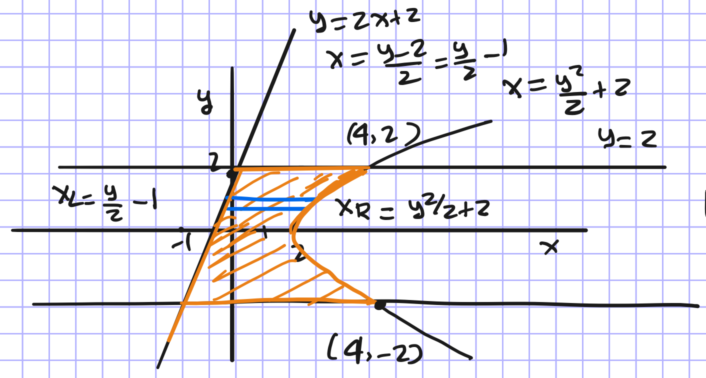
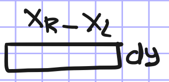
4)

$y = 2x+2 \qquad x = \frac{y^2}{2} + 2 \qquad y = -2 \qquad y = 2$





4)b) $A = \int_{-2}^{2} (x_R - x_L)\, dy$

$A = \int_{-2}^{2} \frac{y^2}{2} + 2 - \left(\frac{y}{2} - 1\right) dy$

$A = \int_{-2}^{2} \left(\frac{y^2}{2} + 2 - \frac{y}{2} + 1\right) dy$

$\frac{y^3}{6} + 2y - \frac{1}{2}\frac{y^2}{2} + y \Big|_{-2}^{2}$

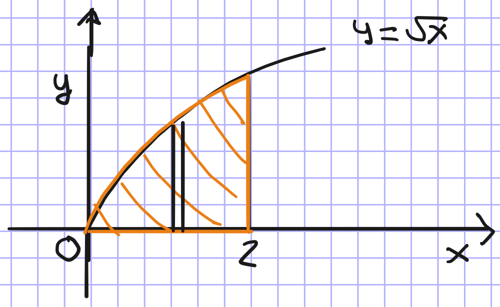
$\frac{8}{6} + 4 - \frac{4}{4} + 2 - \left[\frac{-8}{6} - 4 - 1 - 2 \right]$

$\frac{8}{6} + 4 - 1 + 2 + \frac{8}{6} + 7$

$A = \frac{16}{6} + 12 = \frac{8}{3} + \frac{12}{1} = \frac{8+36}{3} = \boxed{\frac{44}{3}}$ SHADED AREA

---

5)



a) $\int_0^2 \sqrt{x}\, dx = \int_0^2 x^{1/2}\, dx = \frac{2}{3} x^{3/2} \Big|_0^2$

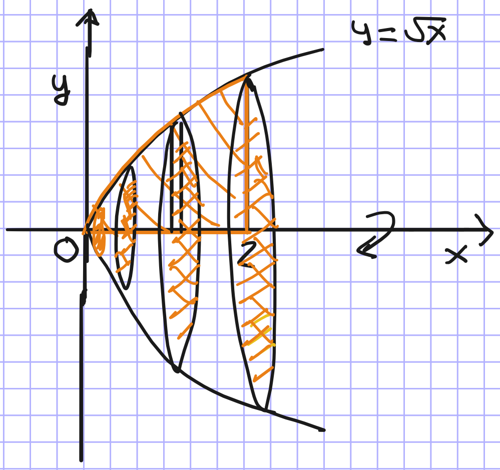
$= \frac{2}{3}\left(2^{3/2} - 0\right) = \boxed{\frac{2^{5/2}}{3}}$

$\frac{2}{3} \cdot \sqrt[2]{8}$

$\frac{2}{3}\sqrt{4 \times 2} = \frac{2\sqrt{4}\sqrt{2}}{3} = \boxed{\frac{4\sqrt{2}}{3}}$

Volume of Revolution

b) ↻ x axis



$$V=\pi\int_0^2 (\sqrt{x})^2\,dx$$

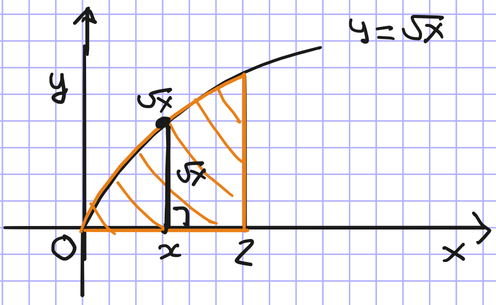
$$V=\pi\int_0^2 x\,dx$$

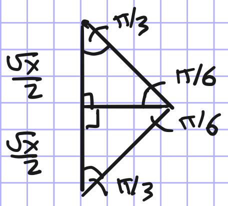
$$V=\pi\frac{x^2}{2}\Big|_0^2$$

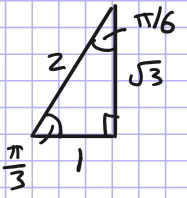
$$V=\frac{\pi}{2}(4-0)=\boxed{2\pi}$$

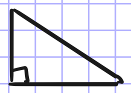
c)

Volumes by Slicing









$$A=\underbrace{\frac{\sqrt{x}}{2}}_{\text{half base}}\cdot\underbrace{\frac{\sqrt{x}}{2}\sqrt{3}}_{\text{height}}$$

$$A=\frac{x\sqrt{3}}{4}$$

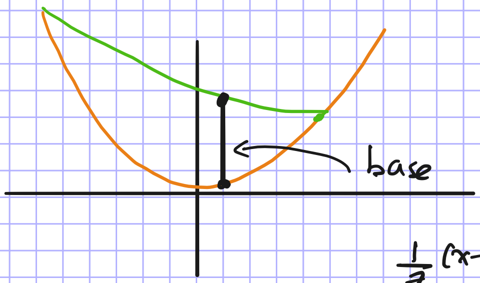
$$\int_0^2 A(x)\,dx$$

$$\int_0^2 \frac{x\sqrt{3}}{4}\,dx$$

$$\frac{\sqrt{3}}{4}\int_0^2 x\,dx$$

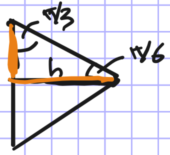
$$\frac{\sqrt{3}}{4}\,\frac{x^2}{2}\Big|_0^2=\frac{\sqrt{3}}{8}(4-0)=\boxed{\frac{\sqrt{3}}{2}}$$

$y = x^2$ $\quad$ $y = \frac{1}{7}(x-1)^2+1$



$$\frac{1}{7}(x-1)^2+1-x^2 = \text{base}$$

$$\frac{1}{14}(x-1)^2+\frac{1}{2}-\frac{x^2}{2}$$



$$h = \sqrt{3}\cdot\frac{b}{2}$$

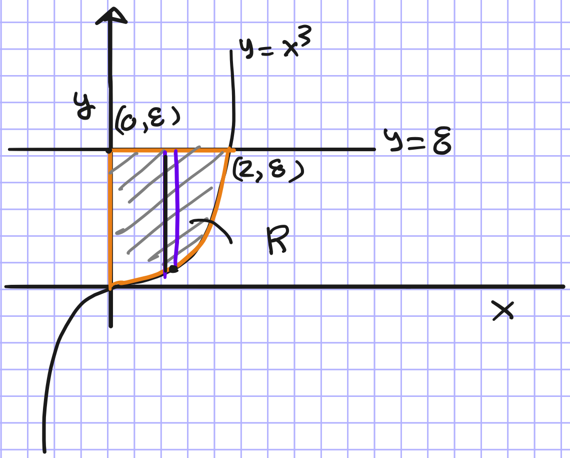
$$A = \boxed{\frac{1}{2}\left[\frac{1}{7}(x-1)^2+1-x^2\right]}\cdot\sqrt{3}\,\boxed{\frac{1}{2}\left[\frac{1}{7}(x-1)^2+1-x^2\right]}$$

$$\frac{\sqrt{3}}{4}$$

6) $y = x^3$ $\quad$ $y = 8$ $\quad$ y axis $\quad$ R

$y = x^3$ $\quad$ $x = 2$ $\quad$ x axis $\quad$ S

a)



$y = y$

$x^3 = 8$

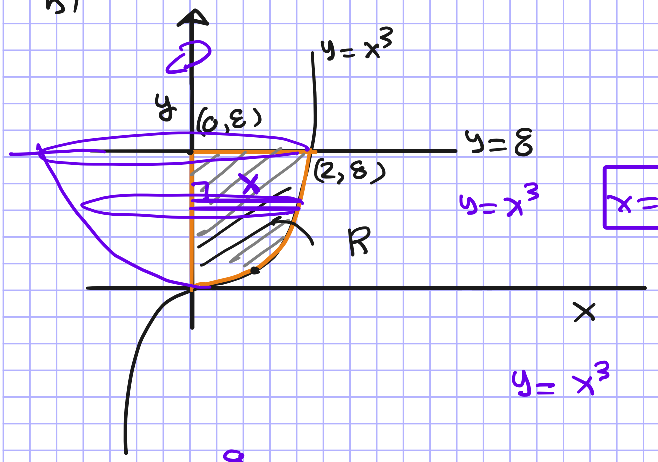
$$A = \int_0^2 \overset{y_{top}\ -\ y_{bot}}{8 - x^3}\, dx$$

$$A = 8x - \frac{x^4}{4}\Big|_0^2$$

a) $A = 16 - 4 - (0-0) = \boxed{12}$

Volume of Revolution

b)



$y = x^3$ $\boxed{x = y^{1/3}}$

$y = x^3$ $\boxed{x = y^{1/3}}$

$$V = \pi \int_0^8 x^2 \, dy$$

$$V = \pi \int_0^8 (y^{1/3})^2 \, dy$$

$$V = \pi \int_0^8 y^{2/3} \, dy$$

$$V = \pi \, y^{5/3} \cdot \frac{3}{5} \Big|_0^8$$

$$V = \frac{3\pi}{5} (8^{5/3} - 0)$$

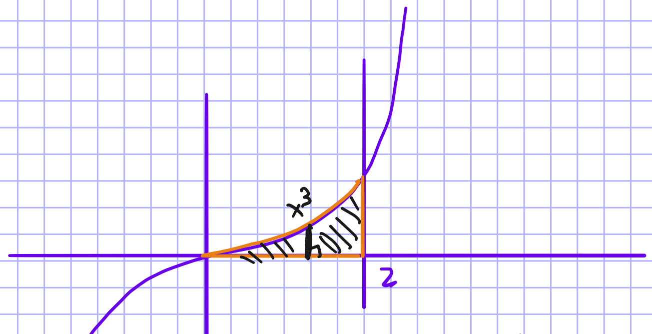
$$\downarrow$$

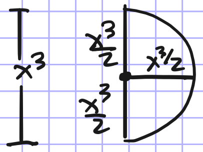
$$V = \frac{3\pi}{5} (8^{1/3})^5$$

$$V = \frac{3\pi}{5} (32) = \boxed{\frac{96\pi}{5}}$$

Volumes by Slicing

6/c) $y = x^3$ $x = 2$ x axis





$$A(x) = \frac{\pi \left(x^3/2\right)^2}{2}$$

$$\int_0^2 A(x)\,dx \qquad A(x)=\frac{\frac{\pi(\sqrt{x^3})^2}{2^2}}{2}$$

$$A(x)=\frac{\frac{\pi x^6}{4}}{2}=$$

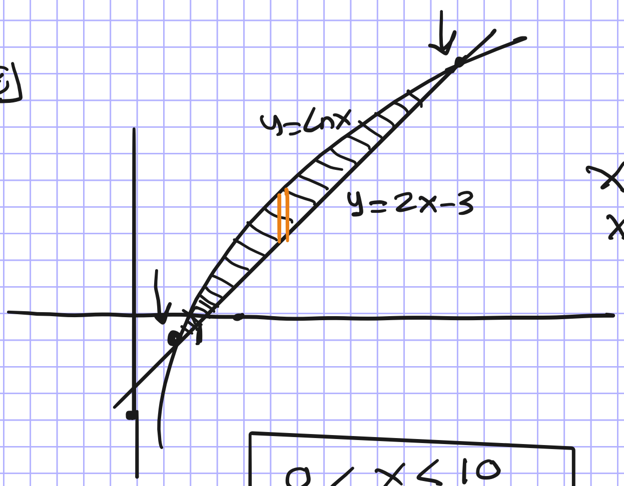
$$\int_0^2 \frac{\pi x^6}{8}\,dx$$

$$\frac{\pi}{8}\int_0^2 x^6\,dx$$

$$\frac{\pi}{8}\,\frac{x^7}{7}\Big|_0^2$$

$$\frac{\pi}{56}\left[2^7-0\right]=\boxed{\frac{2^7\pi}{56}}=\frac{128\pi}{56}=\boxed{\frac{16\pi}{7}}$$

8)



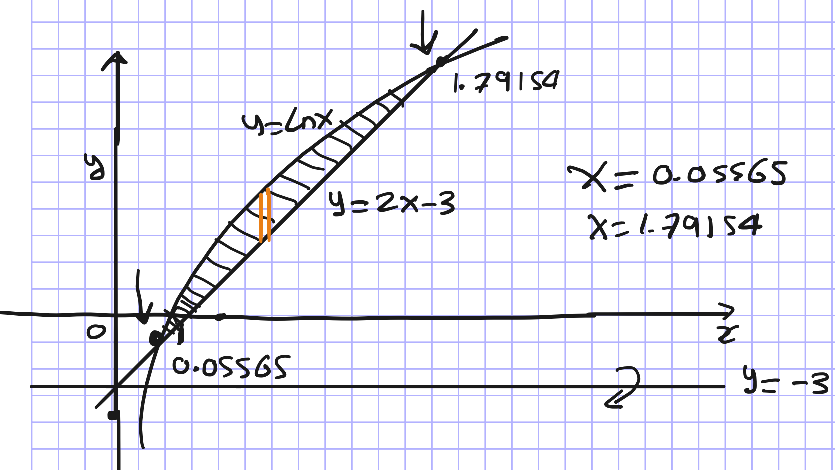
$x=0.05565$
$x=1.79154$

$$\boxed{\begin{array}{l} 0<x<10 \\ -10<y<10 \end{array}}$$

fnInt ((lnx-2x+3), x, 0.05565, 1.79154)

= 1.410619248

$x=0.05565$
$x=1.79154$



$r_{out} = \ln x + 3$

$r_{inner} = 2x - \cancel{3} + \cancel{3} = 2x$

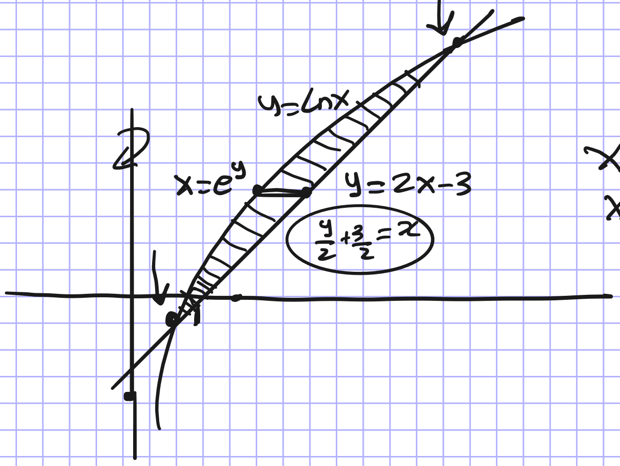
$$V = \pi \int_{0.05565}^{1.79154} \left((r_{out})^2 - r_{inner}^2 \right) dx$$

$$V = \pi \int_{0.05565}^{1.79154} \left[(\ln x + 3)^2 - (2x)^2 \right] dx = 18.783_{43616}$$

fnInt((ln(x)+3)^(2) − 4x², x, 0.05565, 1.79154)

$= \boxed{18.783_{43616}}$

c) 2 y axis



x = 0.05565
x = 1.79154

$\boxed{y = \ln x}$

$$V = \pi \int_{-2.8887}^{0.58308} \left(\frac{y}{2} + \frac{3}{2}\right)^2 - (e^y)^2 \, dy$$

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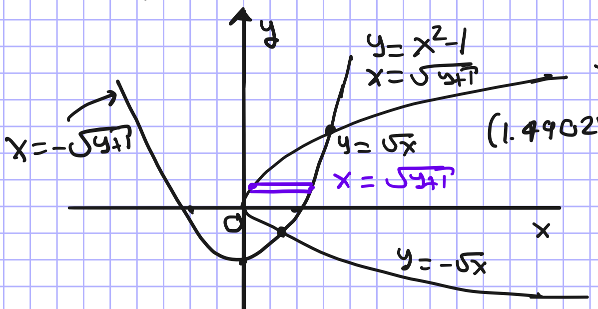
9) $y = x^2 - 1 \qquad x = y^2$



(1.4902161, 1.2207441)

$y_1 = x_2 - 1$

$y_2 = \sqrt{x}$

$0 < x < 4$

$0 < y < 4$

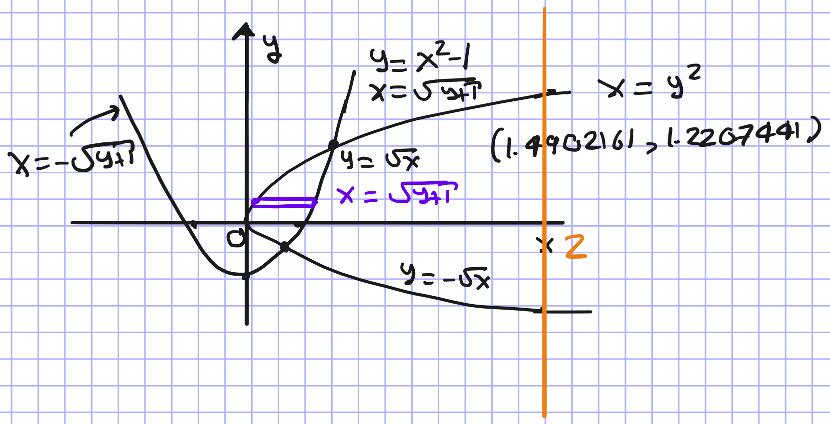
$y_1 = \sqrt{x}$ $\quad y_2 = x^2 - 1$

$y_1 = -\sqrt{x}$ $\quad y_2 = x^2 - 1$

$$\boxed{\begin{array}{l} 0 < x < 4 \\ -4 < y < 0 \end{array}}$$

a) $$\int_{-0.724492}^{1.2207441} \left(\sqrt{y+1} - y^2\right) dy = \underline{1.3767\ldots}$$

b)



$r_{out} = 2 - y^2$

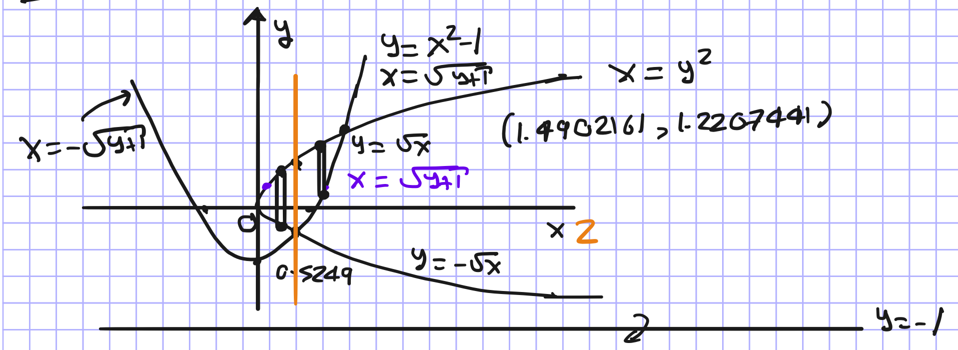
$r_{inner} = 2 - \sqrt{y+1}$

$(-0.724492 \le y \le 1.2207441)$

$$V = \pi \int_{-0.724492}^{1.2207441} \left[(2 - y^2)^2 - (2 - \sqrt{y+1})^2\right] dy$$

$$V = \boxed{11.50143273}$$

c)



$r_{out} = \sqrt{x} - -1 = \sqrt{x}+1$ $\qquad 0 < x < 0.5249$

$r_{in} = -\sqrt{x} - -1 = -\sqrt{x}+1$

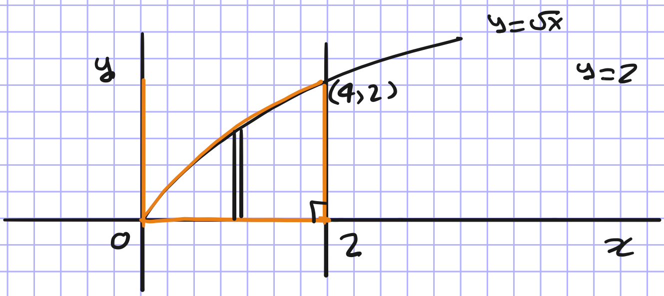
$$V_1 = \pi \int_0^{\boxed{0.5249}} \left[(\sqrt{x}+1)^2 - (-\sqrt{x}+1)^2\right] dx \qquad 0 < x < 0.5249$$

$$V_2 = \pi \int_{\boxed{0.5249}}^{1.4902} (\sqrt{x}+1)^2 - (x^2 \cancel{-1} \cancel{+1})^2 \, dx \qquad 0.5249 < x < 1.4902$$

$V = V_1 + V_2$

10) $f(x) = \sqrt{x}$ $\qquad y = 0$ $\qquad x = 2$

a) ∂ x axis

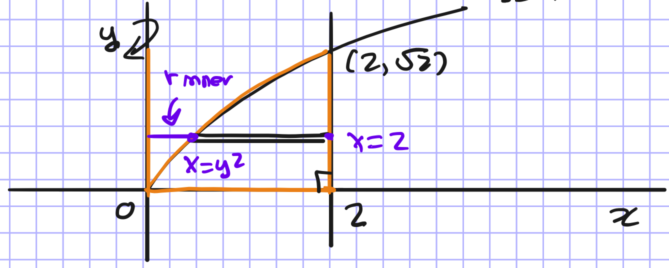


$2 = \sqrt{x}$
$4 = x$

a) $V = \pi \int_0^2 (\sqrt{x})^2 \, dx = \pi \, x \Big|_0^2 = \pi(2-0) = \boxed{2\pi}$

b) ∂ y axis

$x = y^2$
$y = \sqrt{x}$



$r_{out} = 2$

$r_{inner} = y^2$

$$V = \pi \int_0^{\sqrt{2}} \left[(2)^2 - (y^2)^2\right] dy$$

$$V = \pi \int_0^{\sqrt{2}} (4 - y^4) \, dy$$

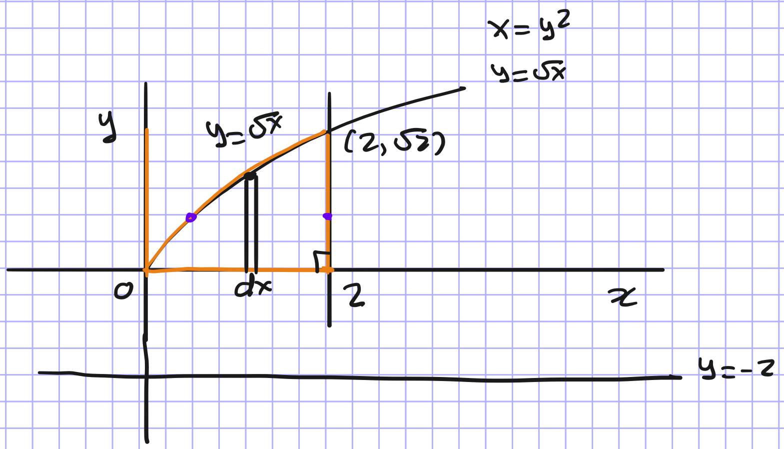
$$V = \pi\left[4y - \frac{y^5}{5}\right]_0^{\sqrt{2}}$$

$$\boxed{V = \pi\left[\frac{4\sqrt{2}}{1} - \frac{(\sqrt{2})^5}{5}\right]}$$

$$\pi\left[\frac{4\sqrt{2}}{1} - \frac{4\sqrt{2}}{5}\right]$$

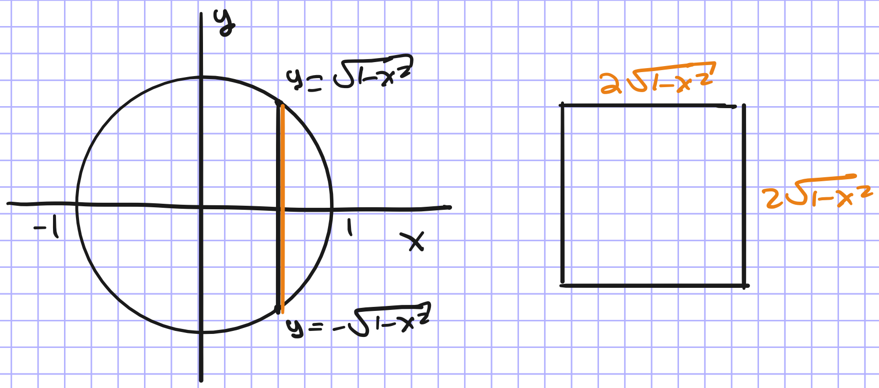
$$V = \pi\left[\frac{20\sqrt{2} - 4\sqrt{2}}{5}\right] = \frac{\pi(16\sqrt{2})}{5}$$

10/6 ② $y = -2$



$$V = \pi\int_0^2 (\sqrt{x} - -2)^2 - (0 - -2)^2 \, dx$$

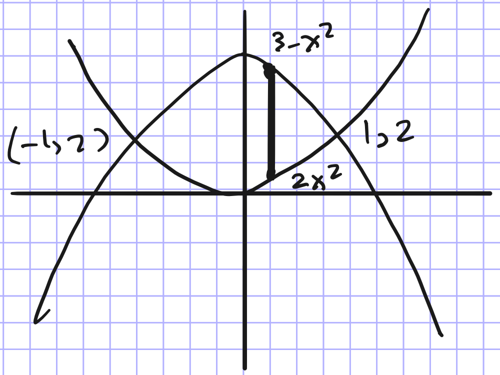
$$V = \pi\int_0^2 \left[(\sqrt{x}+2)^2 - 4\right] dx$$

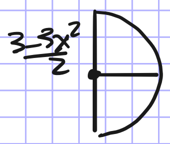


$$A = \int_{-1}^{1} 2\sqrt{1-x^2} \cdot 2\sqrt{1-x^2}\, dx$$

Ex. 2 P.7

$y = 2x^2$ $y = 3 - x^2$





$y = y$

$2x^2 = 3 - x^2$

$$\text{Diam} = 3 - x^2 - 2x^2 = 3 - 3x^2$$

$$V = \int_{-1}^{1} A(x)\, dx$$

$$A(x) = \frac{\pi \left(\frac{3-3x^2}{2}\right)^2}{2}$$

$$A(x) = \frac{\pi (3-3x^2)^2}{8}$$

$$V = \int_{-1}^{1} \frac{\pi (3-3x^2)^2}{8}\, dx$$