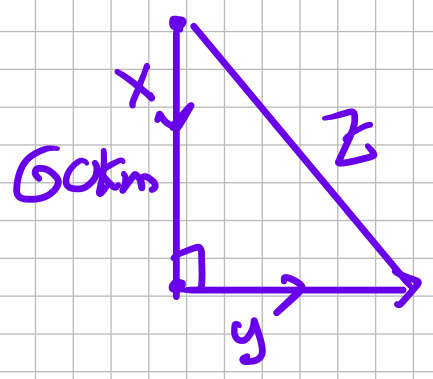
4)



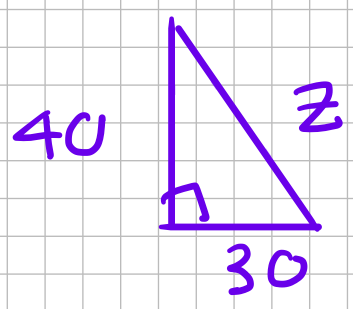
$\frac{dx}{dt} = -40 \quad \frac{dy}{dt} = 60$

$t = 1/2$

$x = 60 - 40t = 60 - 20 = 40$

$y = 60t = 30$

$z = 50$



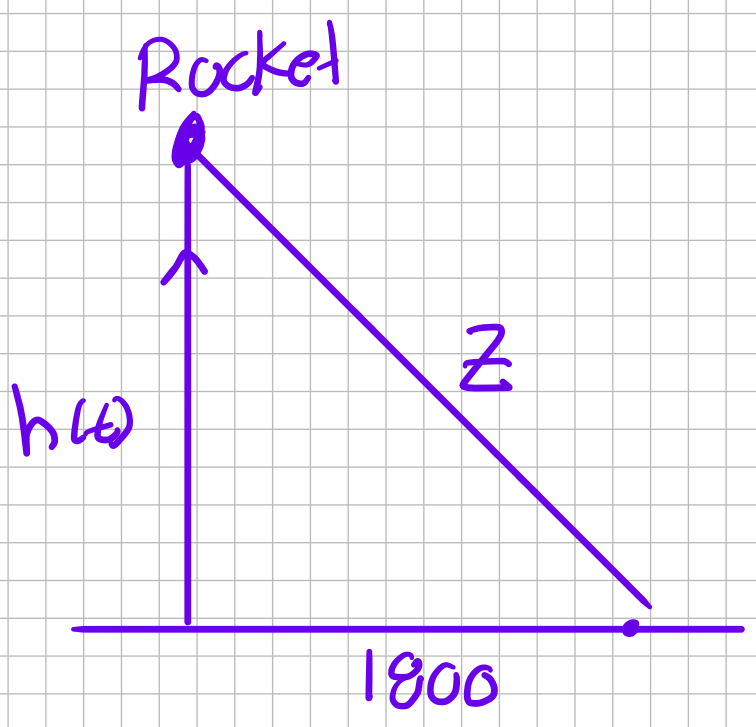
$z^2 = x^2 + y^2$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dz}$$

$$2(50)\frac{dz}{dt} = 2(40)(-40) + 2(30)(60)$$

$$\frac{dz}{dt} = \frac{-1600 + 1800}{50} = \frac{200}{50} = 4 \text{ km/hr}$$

6)



$h(t) = 400t - 16t^2$

$h(t) = 2400$

Find z $\quad z = \sqrt{1800^2 + 2400^2} = \underline{3000}$

$$z^2 = (1800)^2 + [h(t)]^2$$

$$2z\frac{dz}{dt} = 2h\frac{dh}{dt} \qquad \frac{dh}{dt} = 400 - 32t$$

$$2(3000)\frac{dz}{dt} = 2(2400)(400 - 32t)$$

What is t ?

$$h(t) = 2400$$

$$2400 = 400t - 16t^2$$

$$-16t^2 + 400t - 2400 = 0$$

$$-16[t^2 - 25t + 150] = 0$$

$$t = \frac{25 \pm \sqrt{625 - 4(150)}}{2}$$

$$t = \frac{25 \pm 5}{2} = 15, 10$$

$$-16(t-15)(t-10) = 0$$

Choose smaller time going UP!

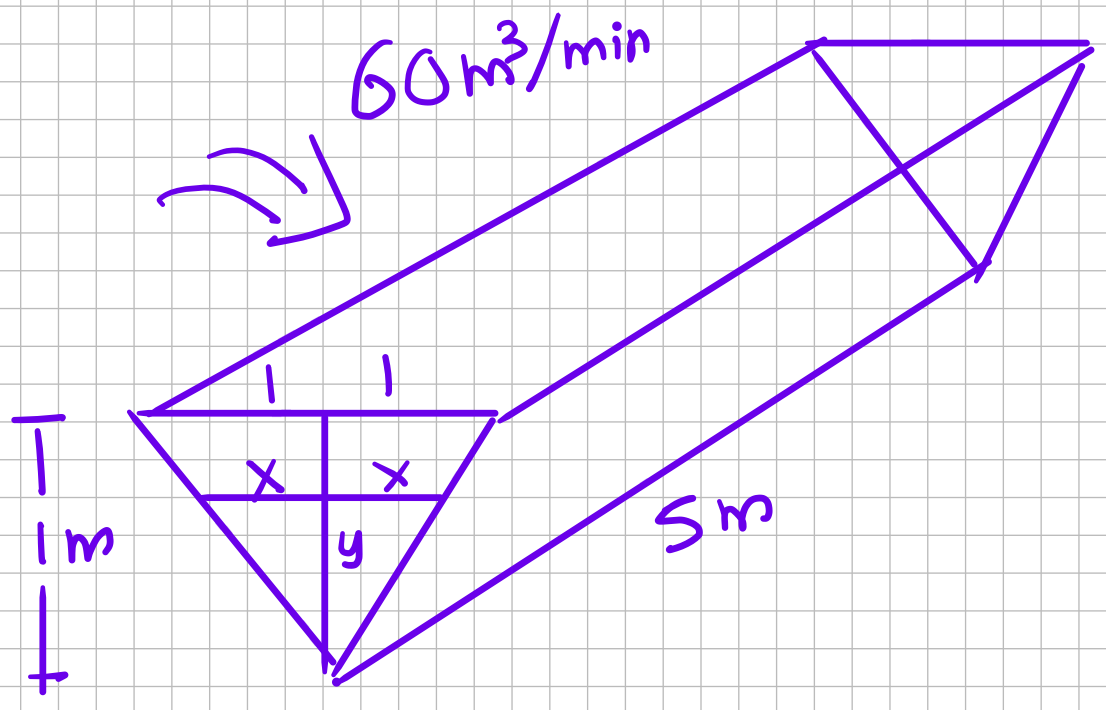
$t = 10$

$$\cancel{2}(3000)\frac{dz}{dt} = \cancel{2}(2400)(400 - 32t)$$

$$\frac{dz}{dt} = \frac{2400(400-320)}{3000} = 64 \text{ ft/s}$$

4.6 CONT.

3)



$y = 0.40\,m \qquad \frac{dy}{dt} = ?$

$\frac{dV}{dt} = 60 \frac{m^3}{sec}$

$\frac{x}{y} = \frac{1}{1}$ Similar Δ's

$x = y$

$V = \frac{1}{2} 2x \cdot y \cdot 5$

$V = 5y^2$

$\frac{dV}{dt} = 10y \frac{dy}{dt}$

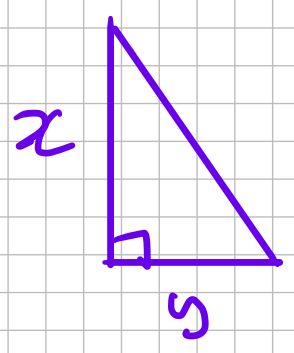
$\Downarrow$

$60 = 10(0.4)\frac{dy}{dt}$

$\frac{60}{4} = \frac{dy}{dt}$

$\frac{dy}{dt} = 15\ m/sec$

4)



$\frac{dx}{dt} = 2 \text{ cm/min}$

$\frac{dy}{dt} = -1 \text{ cm/min}$

after 2 minutes

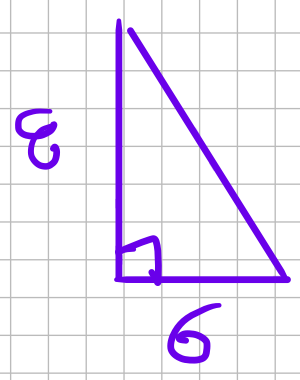
how fast is area changing

$t = 2$

$x = 8 + 2t$

$y = 6 - t$

$t = 2$	$x = 12$
$t = 2$	$y = 4$

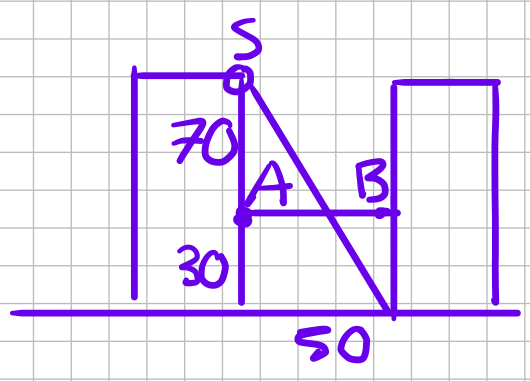


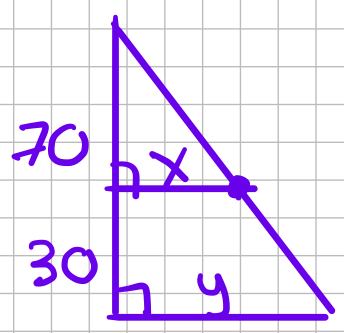
$A = \frac{1}{2} x y$

$\frac{dA}{dt} = \frac{1}{2} \frac{dx}{dt} \cdot y + \frac{x}{2} \frac{dy}{dt}$

$= 4 + 6(-1) = \underline{\underline{-2}} \ \frac{cm^2}{min}$

5]





$\frac{dx}{dt} = 2 ft/sec$

$x = 25 \quad \frac{dy}{dt} = ?$

$$\frac{70}{x} = \frac{100}{y}$$

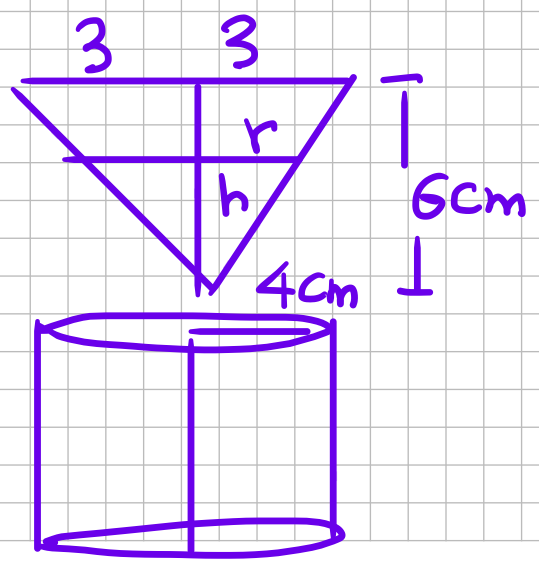
$$70y = 100x$$

$$y = \frac{100}{70}x$$

$$\frac{dy}{dt} = \frac{100}{70}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{10}{7} \cdot 2 = \frac{20}{7} \ ft/sec$$

6]



$h = 4 \quad \frac{dh}{dt} = -1 \frac{cm}{min}$

First find $\frac{dV}{dt}$ leakage into Cylindrical pot

CONE

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \frac{3}{6} \qquad \frac{r}{h} = \frac{1}{2}$$

Similiar Δ's

$$\boxed{r = \frac{h}{2}}$$

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{\pi h^3}{12} = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$

$$= \frac{\pi}{4}(4)^2(-1)$$

$$\frac{dV}{dt} = -4\pi$$ CONE LEAKING into Cylinder

Cylinder

$$V = \pi(4^2)h$$

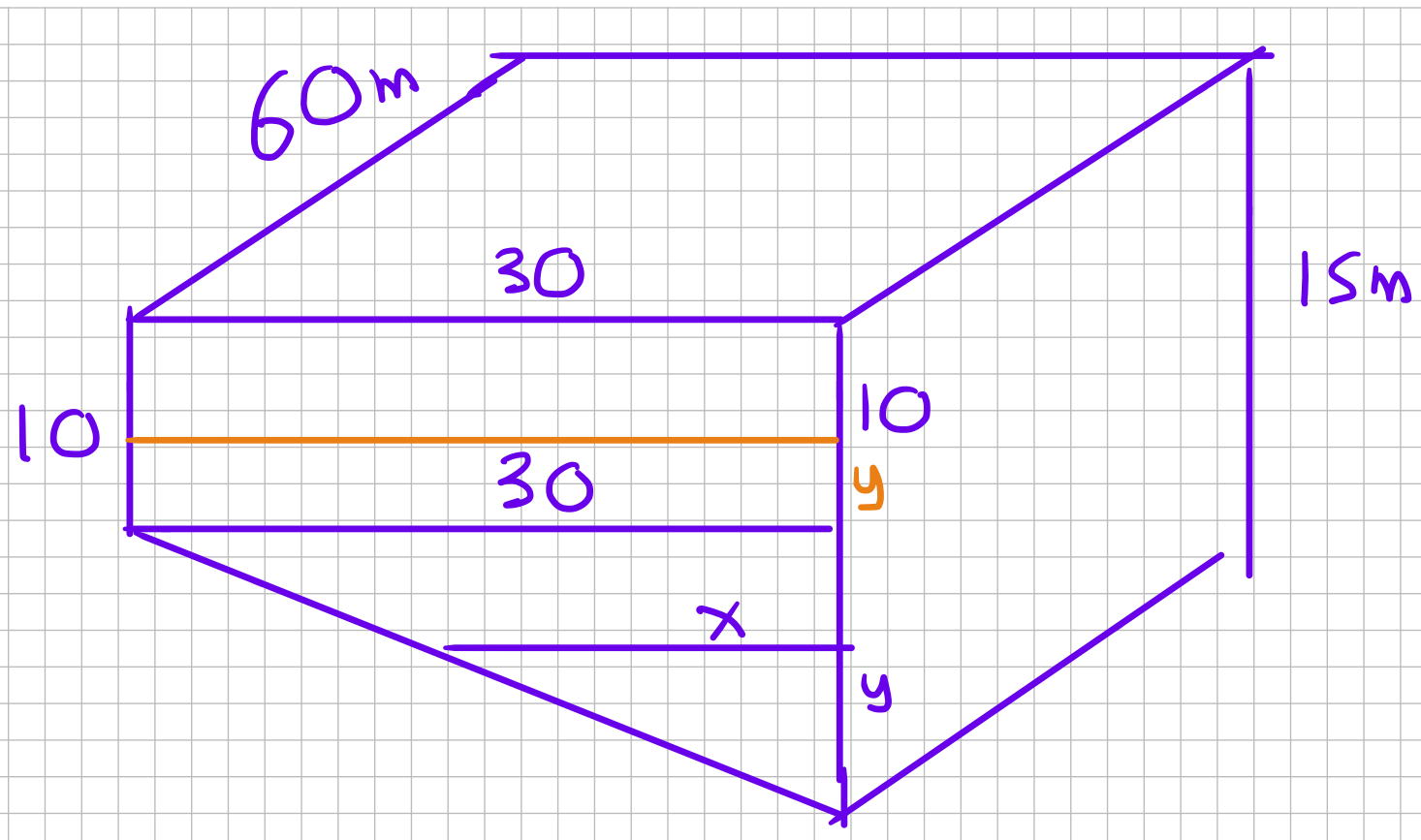
$$\frac{dV}{dt} = 16\pi \frac{dh}{dt}$$

$$4\pi = 16\pi \frac{dh}{dt}$$

$$\frac{4\pi}{16\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{4}$$ cm/min

7)



$y = 3$

$\frac{dV}{dt} = 5\ m^3/min$

$V = \frac{1}{2} xy \cdot 60$

$\frac{x}{y} = \frac{30}{5}$

$\frac{x}{y} = 6$

$x = 6y$

$V = \frac{1}{2} 6y \cdot y \cdot 60$

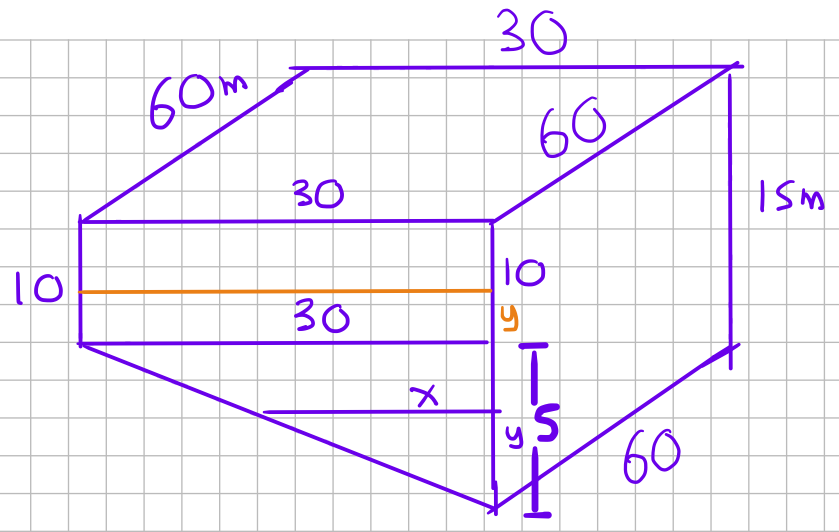
$V = 180y^2$

$\frac{dV}{dt} = 360y \frac{dy}{dt}$

$5 = 360(3) \frac{dy}{dt}$

$\frac{5}{3(360)} = \frac{1}{216}$ ft/min

b) When water level is $y = 12$



$y = 2$

$$V = \frac{1}{2}(30)(5)(60) + 30y(60)$$

$$\frac{dV}{dt} = 1800\,\frac{dy}{dt}$$

$$5 = 1800\,\frac{dy}{dt}$$

$$\frac{5}{1800} = \frac{dy}{dt}$$

$$\frac{1}{360}\ \frac{m}{min} = \frac{dy}{dt}$$