P. 478 A20, A21 Barron's

A20/a

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{\ln(n+1)}$$

apply Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{\ln(n+2)} \cdot \frac{\ln(n+1)}{x^n}\right|$$

$$= \lim_{n\to\infty} |x| \frac{\ln(n+1)}{\ln(n+2)} = |x| < 1$$

$$-1 < x < 1$$

Check endpts

$x=-1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^n}{\ln(n+1)} = \sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{\ln(n+1)}$$

$$= -\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$$

diverges by Basic Comp. test

$$\frac{1}{\ln(n+1)} > \frac{1}{n+1}$$

$x=1$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}$$ Converges by AST

$-1 < x \leq 1$ Interval of CONVG.

$0 < x \leq 1$ Since Question stated $x > 0$

b) $$f(0.5) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0.5)^n}{\ln(n+1)}$$

$$|S - S_n| \leq |b_{n+1}| \leq 0.01$$

$$b_{n+1} = \frac{0.5^{n+1}}{\ln(n+2)}$$

$n=2$ $$\frac{(0.5)^3}{\ln 4} = 0.09$$

$n=3 \quad \frac{(0.5)^4}{\ln 5} = 0.039$

$n=4 \quad \frac{(0.5)^5}{\ln 6} = 0.017$

$n=5 \quad \frac{(0.5)^6}{\ln 7} = 0.008$ ✓

Thus 5 terms are needed so that $|E| \leq 0.01$

c) $f(-0.5) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-0.5)^n}{\ln(n+1)}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{2n+1}(0.5)^n}{\ln(n+1)} = \sum_{n=1}^{\infty} \frac{-(0.5)^n}{\ln(n+1)}$$

all terms are negative, AST does not apply, we expect f(0.5) which is alternating series to converge faster and hence more accurate than f(-0.5) using a non alternating neg. term series

A21)

$\frac{dF}{dm} = 0.0002F(600-F)$

a) $F'' = 0.0002(600-F) + 0.0002F(-1)$

$F'' = 0.12 - 0.0002F - 0.0002F$

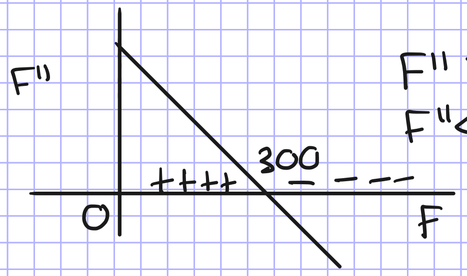
$F'' = 0.12 - 0.0004F = 0$

$0.12 = 0.0004F$

$F = \frac{0.12}{0.0004} * \frac{10\,000}{10\,000} = \frac{1200}{4} = 300$

$F''' = -0.0004 < 0$ $\therefore$ F' is maximum when $F = 300$

or sign test on F''



$F'' > 0$ when $0 < F < 300$

$F'' < 0$ 〃 $F > 300$

$\therefore$ F' is maximised at $F = 300$

A21) b Solve d.e

A21) c $F = 300$ solve for t

B1 Open ended Barron's

Sketch $f(x)$

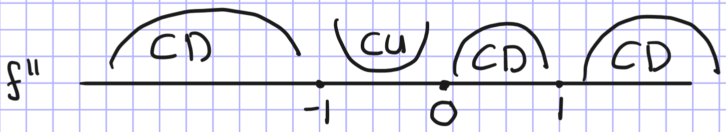
a) $f'(-1) = f'(1) = 0$

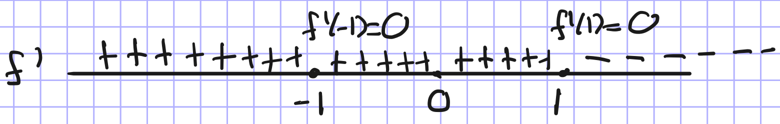
b) $x < -1$, $f'(x) > 0$ but $f'' < 0$

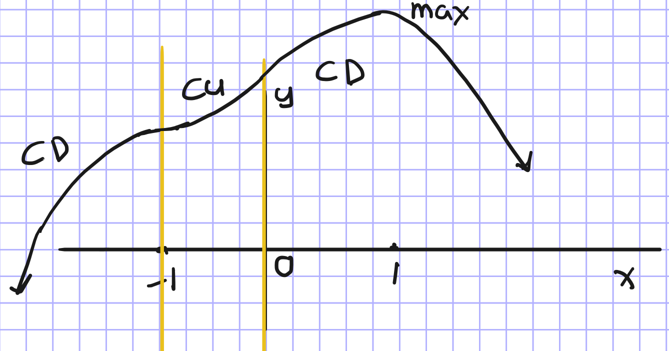
c) $-1 < x < 0$ $f'(x) > 0$ and $f'' > 0$

d) $0 < x < 1$, $f'(x) > 0$ but $f'' < 0$

e) If $x > 1$, $f'(x) < 0$ and $f'' < 0$







B2] a) f(x) is cont. since f'(x) is continuous which means f(x) is differentiable and diff. ⇒ continuity

B2/b

Since f'(x) changes from negative to positive at $x = -2$ ⇒ f(x) has a local minimum at $x = -2$

B2/C] Since f'(x) changes sign from positive to negative at $x = 7$ ⇒ f(x) has a local max. at $x = 7$

$A(7) > A(-3)$

$A(x) = \int_{-3}^{x} f(t)\,dt$ $\quad A(7) - A(-3) = \int_{-3}^{7} f(t)\,dt > 0$

B2/d) At $x = 7$ f(x) achieves a global max. since $f' \geq 0$ on $(-2, 7)$ and hence accumulated area $\int_{-3}^{7} f'(x)\,dx$ is maximum and exceeds the area below the x axis from $(-3, -2)$ and $(7, 9)$

B2/e] f has a pt of inflection when f'' changes sign from pos. to neg or neg. to positive.

$x < 2 \quad f'' > 0$

$2 < x < 4 \quad f'' < 0$

$\therefore x = 2$ P.O.I of f(x)

$2 < x < 4 \quad f'' < 0$

$4 < x < 6 \quad f'' > 0$

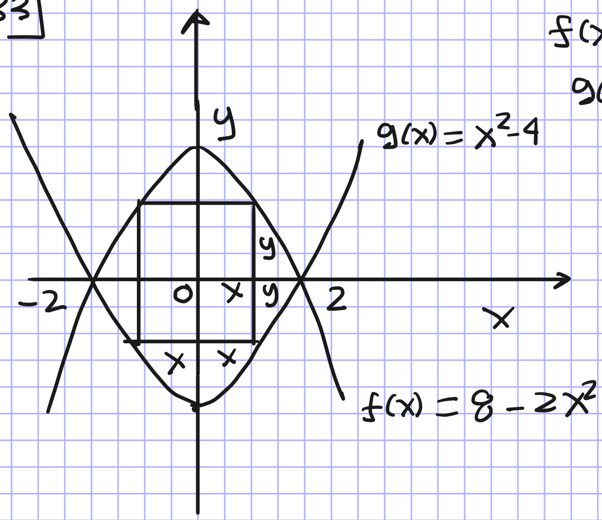
$\therefore x = 4$ P.O.I of f(x)

$4 < x < 6 \quad f'' > 0$

$6 < x < 9 \quad f'' < 0$

$\therefore x = 6$ P.O.I of f(x)

33



$f(x) = 8 - 2x^2$

$g(x) = x^2 - 4$

$f(x) = g(x)$

$8 - 2x^2 = x^2 - 4$

$12 = 3x^2$

$x^2 = 4$

$x = \pm 2$

$A = 2x\,[y_{top} - y_{bot}]$

$A = 2x\,[8 - 2x^2 - (x^2 - 4)]$

$A = 2x\,[12 - 3x^2]$

$A = 24x - 6x^3$ $\qquad 0 < x < 2$

$A' = 24 - 18x^2 = 0$

$24 = 18x^2 \Rightarrow x^2 = \frac{24}{18} = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

$x = 0 \qquad A = 0$

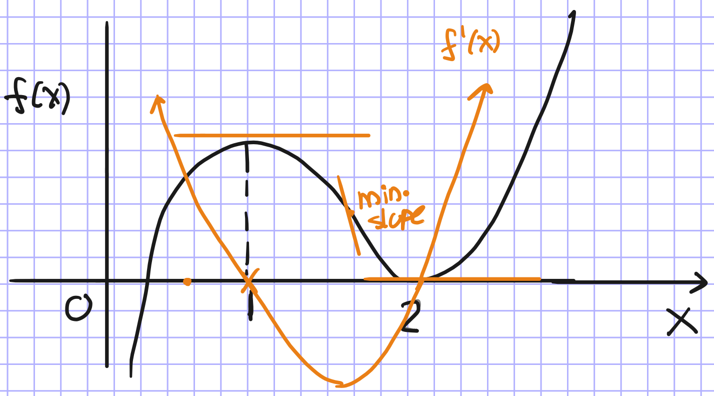
$x = 2/\sqrt{3} \qquad A = 24x - 6x^3 = \frac{24(2)}{\sqrt{3}} - 6\left(\frac{8}{3\sqrt{3}}\right)$

$x = 2 \qquad A = 0$

max. Area $\quad A_{max} = \frac{48 \cdot 3}{\sqrt{3} \cdot 3} - \frac{48}{3\sqrt{3}}$

$A_{max} = \frac{96}{3\sqrt{3}} = \frac{32}{\sqrt{3}}$

B4



B5

$S = 6x^2$

$\frac{dS}{dt} = 72 \text{ in}^2/\text{sec}$ $\qquad S = 54 \text{ ft}$

$1 \text{ in}^2 = \left(\frac{1}{12} \text{ ft}\right)^2$

$\frac{dS}{dt} = 72 \cdot \left(\frac{1}{12}\right)^2 = \frac{1}{2} \text{ ft}^2/\text{sec}$

$\frac{dS}{dt} = 12x \frac{dx}{dt}$ $\qquad S = 6x^2 \Rightarrow 54 = 6x^2$

$x^2 = 9$

$x = 3 \text{ ft}$

$\frac{1}{2} = 12(3)\frac{dx}{dt}$

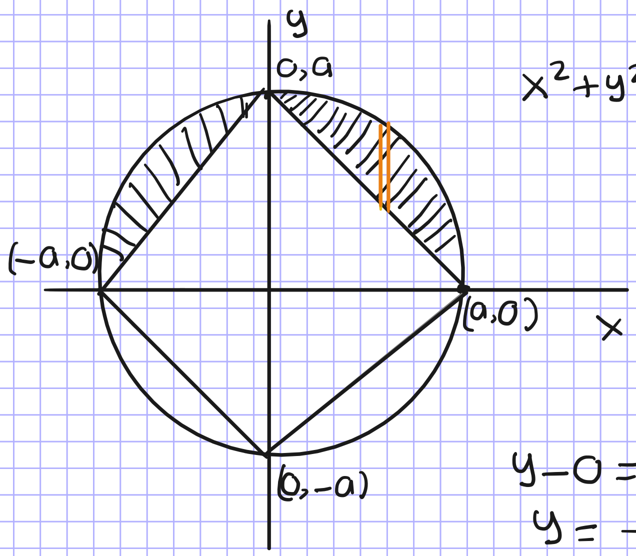
$\frac{dx}{dt} = \frac{1}{72} \text{ ft/sec}$

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$\frac{dV}{dt} = 3(3)^2\left(\frac{1}{72}\right) = \frac{9(3)}{72} = \frac{3}{8} \text{ ft}^3/\text{sec}$

B6



$$x^2+y^2=a^2$$

$$y-0=-1(x-a)$$

$$y=-x+a$$

$$V=2\pi\int_0^a \left(r_{out}^2-r_{in}^2\right)\,dx$$

$$V=2\pi\int_0^a \left[a^2-x^2-(-x+a)^2\right]dx$$

$$V=2\pi\int_0^a a^2-x^2-(x^2-2ax+a^2)\,dx$$

$$V=2\pi\int_0^a (\cancel{a^2}-x^2-x^2+2ax-\cancel{a^2})\,dx$$

$$V=2\pi\int_0^a (2ax-2x^2)\,dx$$

$$V=2\pi\left[\frac{\cancel{2}ax^2}{\cancel{2}}-\frac{2x^3}{3}\right]_0^a$$

$$V=2\pi\left[a^3-\frac{2a^3}{3}\right]=\frac{2\pi a^3}{3}$$