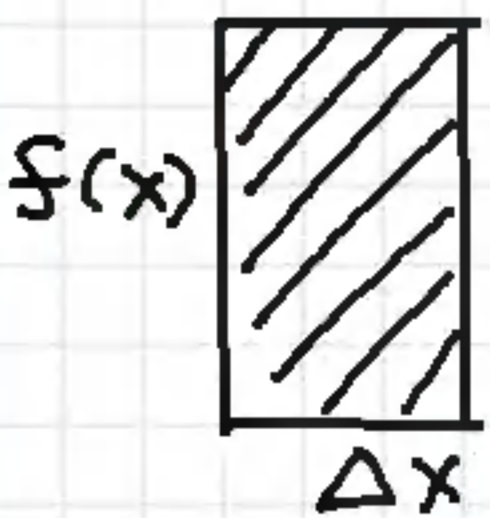
Adding up Rectangles 1

1] Apply right hand rule with $n=4$ rectangles to estimate the area under the parabola $y=x^2+1$ from 0 to 2 $\quad 0<x<2$

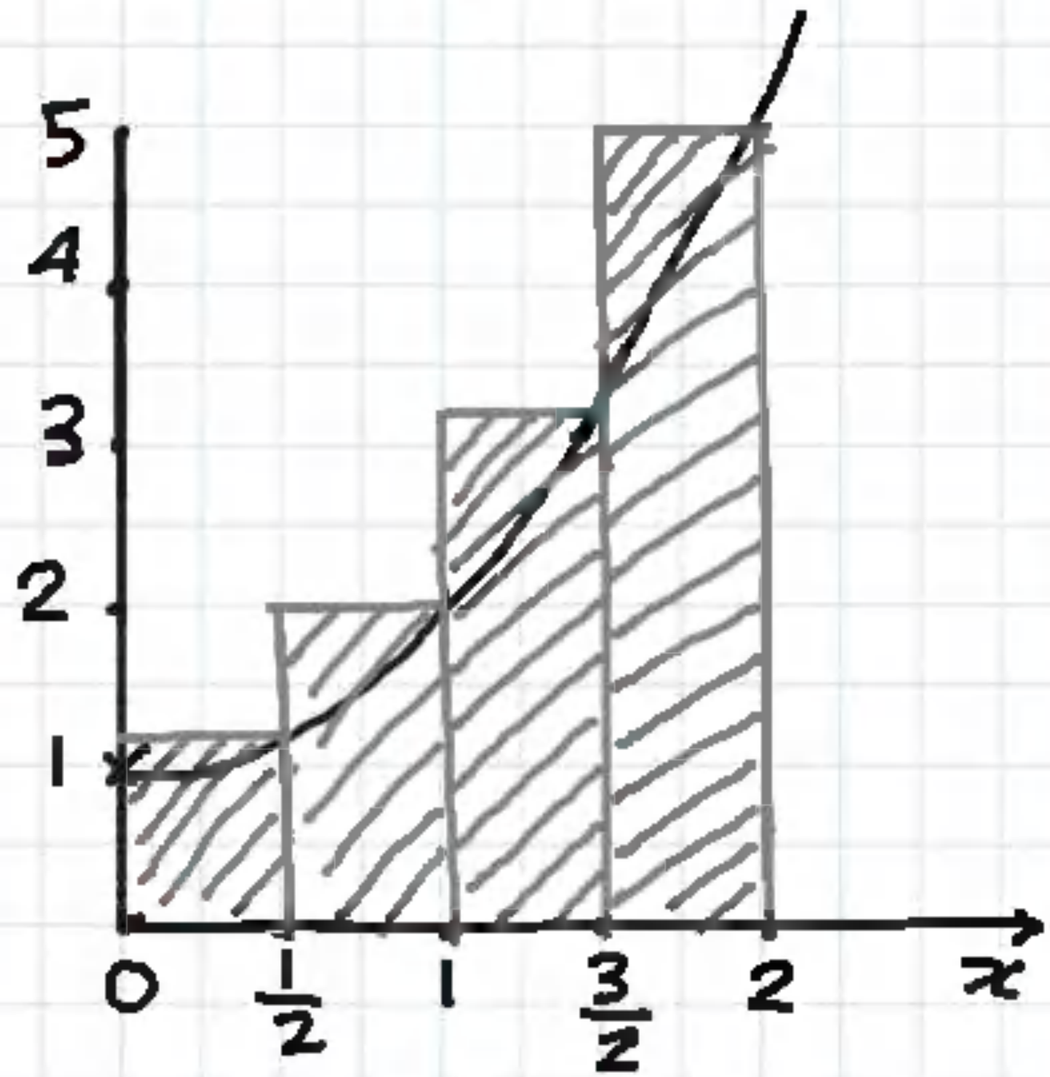
Solution:

Key concept

Area = Length × Width



$$dA = f(x)\Delta x$$



Since $n=4$ rectangles the width of every rectangle is $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

let's make a table of values

x	$f(x) = x^2+1$	Δx
$\frac{1}{2}$	$f(\frac{1}{2}) = (\frac{1}{2})^2+1 = \frac{5}{4} = 1.25$	$1/2$
1	$f(1) = 1^2+1 = 2$	$1/2$
$3/2$	$f(3/2) = (\frac{3}{2})^2+1 = \frac{13}{4} = 3.25$	$1/2$
2	$f(2) = (2)^2+1 = 5$	$1/2$

For the right hand rule evaluate the function at the right endpoint of every rectangle.

$$\text{Area} \cong f(1/2)\cdot\frac{1}{2} + f(1)\cdot\frac{1}{2} + f(3/2)\cdot\frac{1}{2} + f(2)\cdot\frac{1}{2}$$

$$\text{Area} \cong \frac{5}{4}\cdot\frac{1}{2} + 2\cdot\frac{1}{2} + \frac{13}{4}\cdot\frac{1}{2} + 5\cdot\frac{1}{2}$$

$$\text{Area} \cong \frac{5}{8} + 1 + \frac{13}{8} + \frac{5}{2} = \frac{46}{8} = 5.75$$

We have approximated the area under the curve to be 5.75 using 4 rectangles.

We notice since $f(x)$ is increasing Approximation is an overestimate of area under $f(x) = x^2+1$

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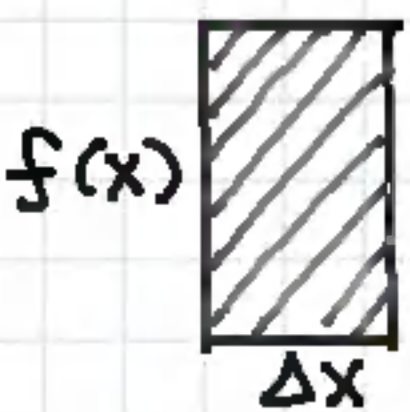
Adding up Rectangles 2

2| Apply left hand rule with $n = 4$ rectangles to estimate the area under the parabola $y = x^2 + 1$ from 0 to 2 $\quad 0 < x < 2$

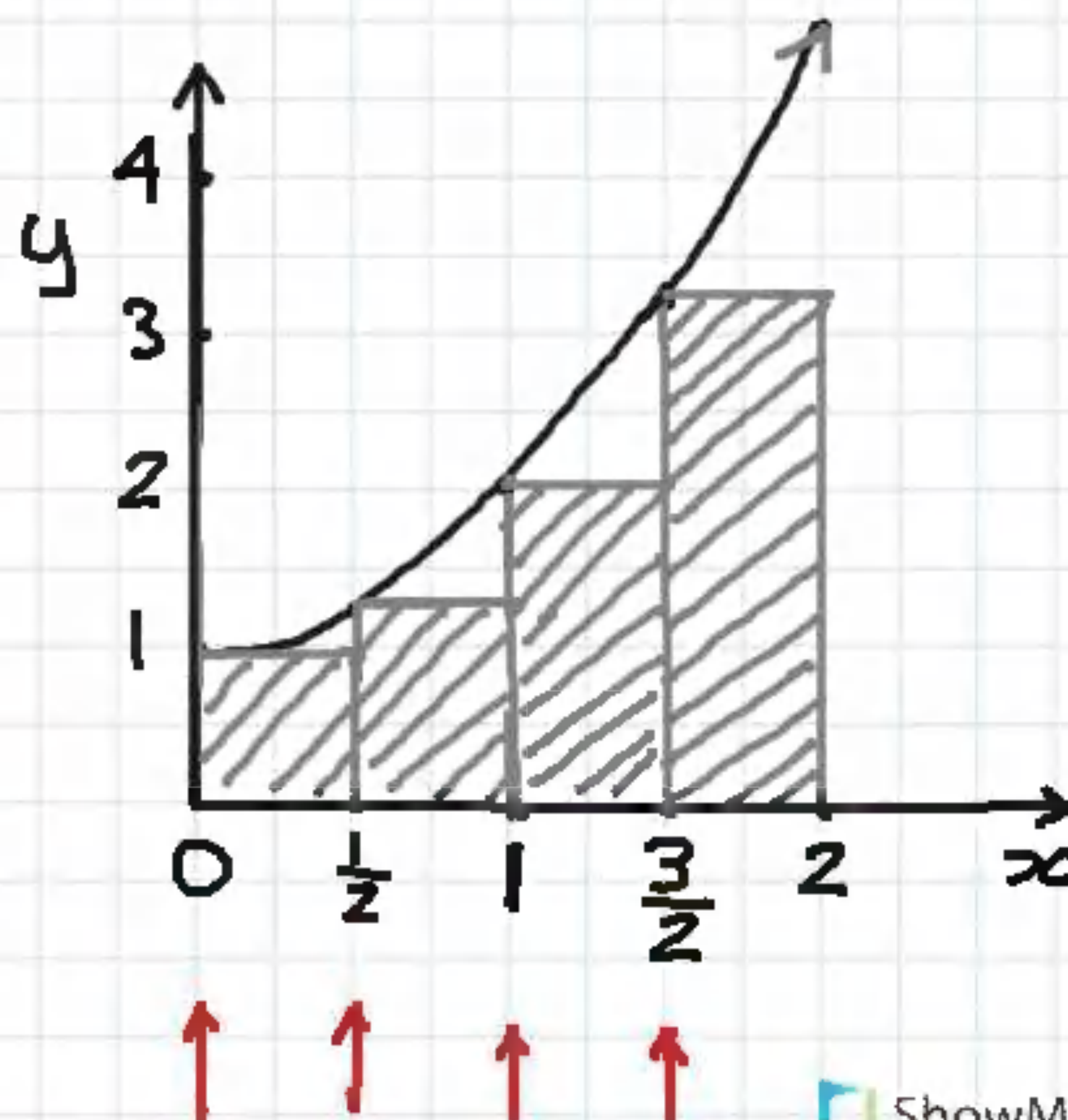
Solution:

Key Concept

Area = Length x Width



$$dA = f(x)\,dx$$



Since $n=4$ rectangles the width of every rectangle is $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

let's make a table of values

x	$f(x) = x^2+1$	Δx
0	$f(0) = 0^2+1 = 1$	$1/2$
$1/2$	$f(1/2) = (\frac{1}{2})^2+1 = 5/4$	$1/2$
1	$f(1) = 1^2+1 = 2$	$1/2$
$3/2$	$f(3/2) = (\frac{3}{2})^2+1 = \frac{13}{4} = 3.25$	$1/2$

For the Left hand rule evaluate the function at the left endpoint of every rectangle.

$$\text{Area} \cong f(0)\cdot\frac{1}{2} + f(1/2)\cdot\frac{1}{2} + f(1)\cdot\frac{1}{2} + f(3/2)\cdot\frac{1}{2}$$

$$\text{Area} \cong 1\cdot\frac{1}{2} + \frac{5}{4}\cdot\frac{1}{2} + 2\cdot\frac{1}{2} + \frac{13}{4}\,\frac{1}{2}$$

$$\text{Area} \cong \frac{1}{2} + \frac{5}{8} + 1 + \frac{13}{8} = \frac{30}{8} = \boxed{3.75}$$

We have approximated the area under the curve to be 3.75 using 4 rectangles. Since $f(x)$ is an increasing function and we are using the Left hand rule, Approximation is an underestimate.

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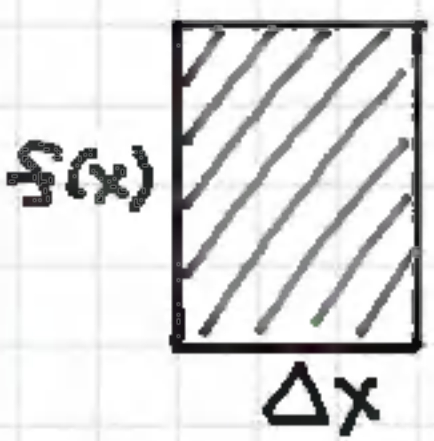
Adding up Rectangles 3

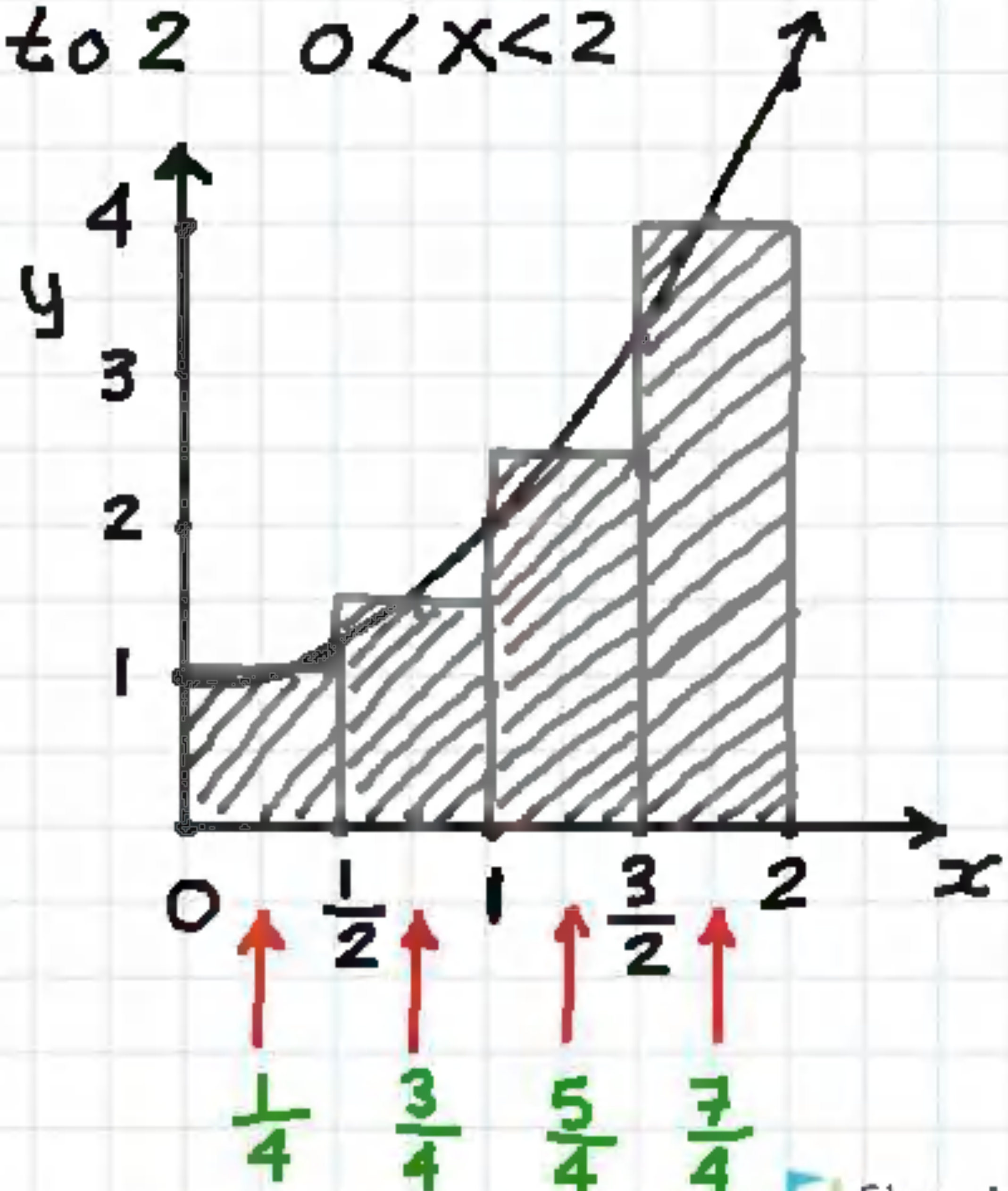
3] Apply the midpoint rule with $n=4$ rectangles to estimate the area under $f(x)=x^2+1$ and above the x axis from 0 to 2 $0<x<2$

Solution

Key concept
Area = Length × Width



$$\Delta A = f(x)\Delta x$$



Since $n=4$ rectangles the width of every rectangle is $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

Important : we need to find the midpoints of every interval. For example for the first interval $x = \frac{0+1/2}{2} = \frac{1}{4}$ and for the Last interval $x = \frac{\frac{3}{2}+2}{2} = \frac{7}{4}$

Let's make a table of values

x	$f(x) = x^2+1$	Δx
1/4	$f(1/4) = \left(\frac{1}{4}\right)^2 + 1 = \frac{17}{16} = 1.0625$	1/2
3/4	$f(3/4) = \left(\frac{3}{4}\right)^2 + 1 = \frac{25}{16} = 1.5625$	1/2
5/4	$f(5/4) = (5/4)^2 + 1 = 41/16 = 2.5625$	1/2
7/4	$f(7/4) = (7/4)^2 + 1 = 65/16 = 4.0625$	1/2

For the midpoint rule we evaluate the function at the midpoint of every rectangle.

$$\text{Area} \cong f(1/4)\cdot\frac{1}{2} + f(3/4)\cdot\frac{1}{2} + f(5/4)\cdot\frac{1}{2} + f(7/4)\cdot\frac{1}{2}$$

$$\text{Area} \cong \frac{1}{2}\left[f(1/4 + f(3/4) + f(5/4) + f(7/4) \right]$$

$$\text{Area} \cong \frac{1}{2}\left[1.0625 + 1.5625 + 2.5625 + 4.0625 \right]$$

$$\text{Area} \cong \frac{1}{2}\left[9.25 \right] = 4.625$$

We have approximated the area below the function $f(x) = x^2 + 1$ and above the x axis to be 4.625 applying the midpoint rule with $n = 4$ rectangles.

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