Basic Integration I

$$\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int c\,dx = c\int dx = cx + D$$

$$\int 2\pi\,dx = 2\pi\int 1\,dx = 2\pi x + D$$

① $\int \left[\frac{x^2+x}{x} - e^x + 2\pi\right] dx$

$\int \frac{x^2}{x} + \frac{x}{x} - e^x + 2\pi \quad dx$

$\int x + 1 - e^x + 2\pi \; dx$

$\int x\,dx + \int 1\,dx - \int e^x\,dx + \boxed{\int 2\pi\,dx}$

$\frac{x^2}{2} + x - e^x + 2\pi x + 2\pi x + C$

② $\int (x^{3/2} + x^{-3/2})^2 \, dx$ Let's simplify the integrand

$$(x^{3/2} + x^{-3/2})^2 = (x^{3/2})^2 + 2x^{3/2}x^{-3/2} + (x^{-3/2})^2$$

$$= x^3 + 2x^0 + x^{-3} = \boxed{x^3 + 2 + x^{-3}}$$

$$\int x^3 + 2 + x^{-3} \, dx$$

$$\int x^3 dx + \int 2 \, dx + \int x^{-3} dx$$

$$\frac{x^4}{4} + 2x + \frac{x^{-2}}{-2} + C$$

$$\boxed{\frac{x^4}{4} + 2x - \frac{1}{2x^2} + C}$$

Recall

$x^a x^b = x^{a+b}$

$x^0 = 1$

$(x^{\frac{a}{b}})^c = x^{ac/b}$

$\frac{x^a}{x^b} = x^{a-b}$

③ $\displaystyle\int \frac{x^3 - \sqrt[3]{x}}{2\sqrt{x}}\,dx$

Recall

$\sqrt[n]{x^m} = x^{m/n}$

$\dfrac{x^a}{x^b} = x^{a-b}$

$\displaystyle\int \frac{x^3}{2\sqrt{x}}\,dx - \int \frac{\sqrt[3]{x}}{2\sqrt{x}}\,dx$

$\displaystyle\int \frac{x^3}{2x^{1/2}}\,dx - \int \frac{x^{1/3}}{2x^{1/2}}\,dx$

$\dfrac{1}{3} - \dfrac{1}{2} = \dfrac{-1}{6}$

$\displaystyle\frac{1}{2}\int x^{5/2}\,dx - \frac{1}{2}\int x^{-1/6}\,dx$

$\dfrac{1}{2}\,\dfrac{x^{7/2}}{7/2} - \dfrac{1}{2}\,\dfrac{x^{5/6}}{5/6} + C = \dfrac{1}{\cancel{2}}\cdot\dfrac{\cancel{2}}{7}x^{7/2} - \dfrac{1}{\cancel{2}}\cdot\dfrac{\cancel{6}^{\,3}}{5}x^{5/6} + C$

$\dfrac{1}{\cancel{2}}\cdot\dfrac{\cancel{2}}{7}x^{7/2} - \dfrac{1}{\cancel{2}}\cdot\dfrac{\cancel{6}^{\,3}}{5}x^{5/6} + C = \dfrac{1}{7}x^{7/2} - \dfrac{3}{5}x^{5/6} + C$

④ $$\int \frac{5}{3t^7} + \frac{1}{t^9} - \frac{3}{\sqrt[3]{t^7}}\,dt$$

$$\frac{1}{t^7} = t^{-7}$$

$$\frac{3}{\sqrt[3]{t^7}} = \frac{3}{t^{7/3}} = 3t^{-7/3}$$

$$\int \frac{5}{3}t^{-7} + t^{-9} - 3t^{-7/3}\,dt$$

$$\int \frac{5}{3}t^{-7}\,dt + \int t^{-9}\,dt - \int 3t^{-7/3}\,dt$$

$$\frac{5}{3}\,\frac{t^{-6}}{-6} + \frac{t^{-8}}{-8} - 3\,\frac{t^{-4/3}}{-4/3} + C$$

$$\frac{-5}{18}\,\frac{1}{t^6} - \frac{1}{8t^8} - 3\cdot\frac{3}{-4}\,t^{-4/3} + C$$

$$\frac{-5}{18}\cdot\frac{1}{t^6} - \frac{1}{8t^8} + \frac{9}{4}\,t^{-4/3} + C$$

$$\frac{-5}{18t^6} - \frac{1}{8t^8} + \frac{9}{4t^{4/3}} + C$$

Basic Integration 2

⑤ $\int \sqrt{x}\left(x^3 - \frac{1}{2x}\right) dx$

Recall

$x^{1/2}x^3 = x^{\frac{1}{2}+3} = x^{7/2}$

$x^{1/2}x^{-1} = x^{-1/2}$

$\int x^{m/n} dx = \frac{x^{m/n+1}}{\frac{m}{n}+1} + C$

Solution:

$\int x^{1/2}\left(x^3 - \frac{1}{2}x^{-1}\right) dx$

$\int x^{1/2}x^3 - \frac{1}{2}x^{1/2}x^{-1} dx$

$\int x^{7/2} - \frac{1}{2}x^{-1/2} dx$

$\int x^{7/2} dx - \int \frac{1}{2}x^{-1/2} dx$

$\frac{x^{9/2}}{9/2} - \frac{1}{2}\frac{x^{1/2}}{1/2} + C \Rightarrow \boxed{\frac{2}{9}x^{9/2} - x^{1/2} + C}$

⑥ $\int 2e^t + e - \frac{1}{2t}\, dt$

Solution:

$\int 2e^t dt + \int e\, dt + \int \frac{-1}{2t}\, dt$

$2\int e^t dt + e\int 1\, dt - \frac{1}{2}\int \frac{1}{t}\, dt$

$2e^t + et - \frac{1}{2}\ln|t| + C$

Recall:

$\int e^t dt = e^t + C$

$\int c\, dt = ct + D$

$\int \frac{1}{t}\, dt = \ln|t| + D$

$\int c f(t)\, dt = c\int f(t)\, dt$

⑦ $\int \frac{1}{1+t^2} + \frac{2}{\sqrt{1-t^2}} + \sin t - \cos t \, dt$

SOLN: $\int \frac{1}{1+t^2} \, dt + \int \frac{2}{\sqrt{1-t^2}} \, dt + \int \sin t \, dt + \int -\cos t \, dt$

$\int \frac{1}{1+t^2} \, dt + 2\int \frac{1}{\sqrt{1-t^2}} \, dt + \int \sin t \, dt - \int \cos t \, dt$

$\underline{\tan^{-1} t + 2\sin^{-1} t - \cos t - \sin t + C}$

Recall: $\int \frac{1}{1+t^2} \, dt = \tan^{-1} t + C$

$\int \cos x \, dx = \sin x + C$

$\int \frac{1}{\sqrt{1-t^2}} \, dt = \sin^{-1} t + C$

(8) $\int \sec^2 x + \csc^2 x + \sec x \tan x \, dx$

Solution

$\int \sec^2 x \, dx + \int \csc^2 x \, dx + \int \sec x \tan x \, dx$

$\tan x + -\cot x + \sec x + C$

Recall: $\frac{d}{dx} \tan x = \sec^2 x$ $\quad \frac{d}{dx} \cot x = -\csc^2 x$

$\frac{d}{dx} \sec x = \sec x \tan x$

$\int \sec^2 x \, dx = \tan x + C$ $\quad \int \sec x \tan x \, dx = \sec x + C$

$\int \csc^2 x \, dx = -\cot x + C$

⑨ $\int \sec^2 x(\cos^2 x + \sin^2 x)\,dx$

Recall
$\cos^2 x + \sin^2 x = 1$

Solution: $\int \sec^2 x(1)\,dx$

$\int \sec^2 x\,dx = \tan x + C$

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⑩ $\int \frac{x^8 - 2x^3 + 3x^2 - 4}{x^3}\,dx$

Solution:

$\int \frac{x^8}{x^3} - \frac{2x^3}{x^3} + \frac{3x^2}{x^3} - \frac{4}{x^3}\,dx$

$\int x^5 - 2 + \frac{3}{x} - \frac{4}{x^3}\,dx$

(10) Continued

$$\int x^5\,dx - \int 2\,dx + \int \frac{3}{x}\,dx - \int 4x^{-3}\,dx$$

$$\int x^5\,dx - 2\int 1\,dx + 3\int \frac{1}{x}\,dx - 4\int x^{-3}\,dx$$

$$\frac{x^6}{6} - 2x + 3\ln|x| - 4\,\frac{x^{-2}}{-2} + C$$

$$\frac{x^6}{6} - 2x + 3\ln|x| + \frac{2}{x^2} + C$$

Basic Integration 3

(11) $\int x^3 - \left(\frac{e^{-x} - 2}{e^{-x}}\right) dx$

SOLN: $\int x^3 - \left(\frac{e^{-x}}{e^{-x}} - \frac{2}{e^{-x}}\right) dx$

$\int x^3 - (1 - 2e^x)\, dx$

$\int x^3 - 1 + 2e^x\, dx$

$\int x^3\, dx - \int 1\, dx + \int 2e^x\, dx$

$\frac{x^4}{4} - x + 2e^x + C$

Recall

$\int e^x dx = e^x + C$

$\int 1\, dx = x + C$

$\frac{1}{e^{-x}} = e^x$

$e^{-x} = \frac{1}{e^x}$

(12) Find $f(x)$ if $f'(x) = e^x + (1+x^2)^{-1} + \sin x$ $\boxed{f(0)=1}$

Solution: $f(x) = \int f'(x)\,dx$

$f(x) = \int e^x + (1+x^2)^{-1} + \sin x \; dx$

$f(x) = \int e^x\,dx + \int \frac{1}{1+x^2}\,dx + \int \sin x\,dx$

$f(x) = e^x + \tan^{-1}x + -\cos x + C$

We are given $f(0)=1$ $\Rightarrow$ subst. $x=0$ and $f(0)=1$ and solve for C

$1 = e^0 + \tan^{-1}0 - \cos 0 + C$

$1 = 1 + 0 - 1 + C \Rightarrow C = 1$

$$f(x) = e^x + \tan^{-1}x - \cos x + 1$$

Recall

$\tan^{-1}0 = 0$

$e^0 = 1$

$\cos 0 = 1$

Let's now verify our solution

$$f'(x) = e^x + \frac{1}{1+x^2} - -\sin x$$

$$f'(x) = e^x + \frac{1}{1+x^2} + \sin x \quad \checkmark$$

Let's also check $f(0) = 1$

$$f(x) = e^x + \tan^{-1}x - \cos x + 1$$

$$f(0) = e^0 + \tan^{-1}0 - \cos 0 + 1$$

$$f(0) = 1 + 0 - \cancel{1} + \cancel{1} = 1 \Rightarrow f(0) = 1 \quad \checkmark$$

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Basic Integration 4

13] A ball is thrown downward with a speed of $10 \frac{m}{sec}$ from the edge of a cliff 100 metres above the ground. Find the equation for the height of the ball t seconds later?

Solution : we choose the upwards direction to be positive and the downwards direction to be negative and since the force of gravity is downwards the acceleration of the ball must be negative and the initial velocity of the ball is also negative since the ball is thrown downwards.

$a(t) = -9.8 \frac{m}{s^2}$ $\quad V(0) = -10 \frac{m}{sec}$ $\quad S(0) = 100\,m$

$S(t)$ is the height of ball above the ground at time t

Recall: $V(t) = S'(t) \quad V'(t) = a(t) = S''(t)$

$a(t) = -9.8 \Rightarrow S''(t) = -9.8$

$S'(t) = V(t) = \int S''(t)\,dt = \int -9.8\,dt = -9.8t + C$

but $V(0) = -10\,m/sec \Rightarrow -10 = -9.8(0) + C$

$\therefore C = -10 \Rightarrow S'(t) = -9.8t - 10$

$S(t) = \int S'(t)\,dt = \int (-9.8t - 10)\,dt$

$S(t) = \int -9.8t\,dt - \int 10\,dt$

$S(t) = -9.8 \int t\,dt - 10 \int 1\,dt$

$S(t) = -\frac{9.8t^2}{2} - 10t + D$

$$S(t) = -\frac{9.8t^2}{2} - 10t + D$$

We are given $S(0) = 100$ m   initial height of ball

$t = 0 \quad S(t) = 100$ m

$$100 = -\frac{9.8(0)^2}{2} - 10(0) + D$$

$D = 100$

$$\boxed{S(t) = -4.9t^2 - 10t + 100}$$

height of ball t seconds after it was thrown from edge of cliff.

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