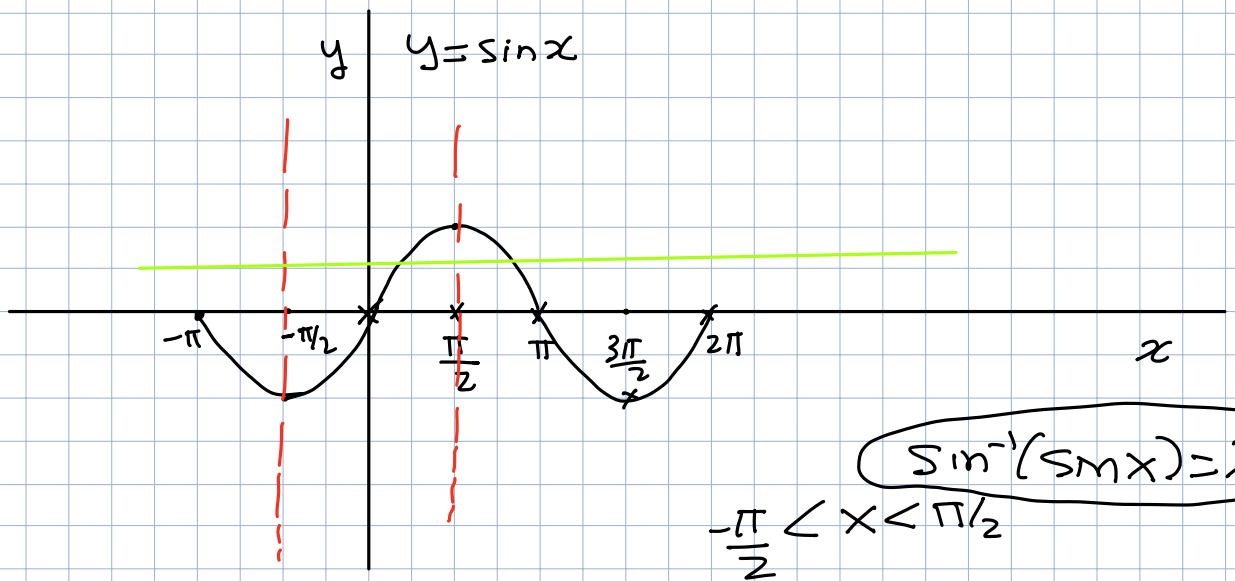
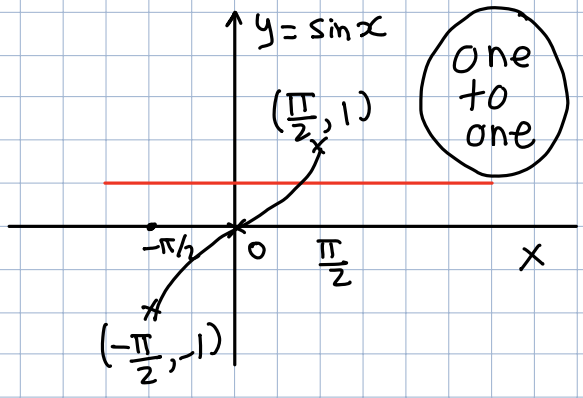
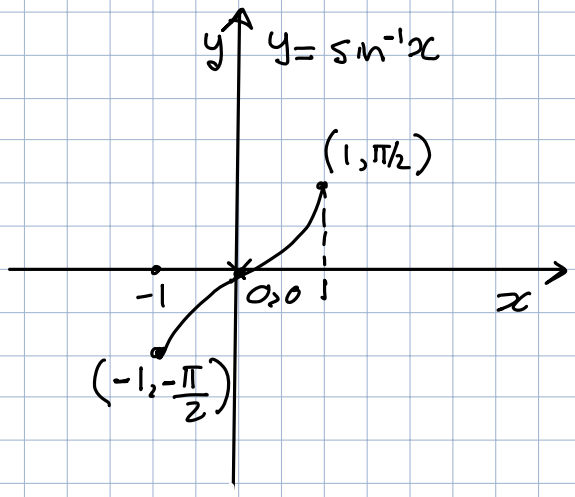
$f(x) = \sin x$

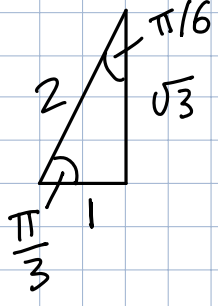


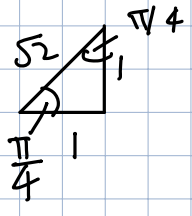
$\sin^{-1}(\sin x) = x$

$-\frac{\pi}{2} < x < \pi/2$









$\sin^{-1}(1/2) =$ what is the angle whose sine $= \frac{1}{2}$

$\sin^{-1}(1/2) = \pi/6$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi/3$

$\sin^{-1}(1/\sqrt{2}) = \pi/4$

$\tan^{-1}(\sqrt{3}) = \pi/3$

$\cos^{-1}(1/2) = \pi/3$

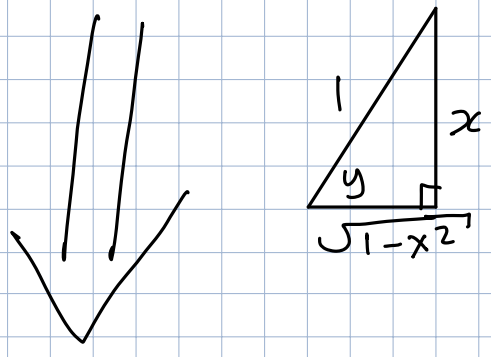
$\cos \pi/3 = 1/2$

$y = \sin^{-1} x \qquad \frac{dy}{dx} = ?$

$\sin y = \sin \sin^{-1} x$

$\sin y = x$

$\sin y = \frac{opp}{hyp} = \frac{x}{1}$



$\cos y \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \boxed{\frac{1}{\sqrt{1-x^2}}}$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad -1 < x < 1$

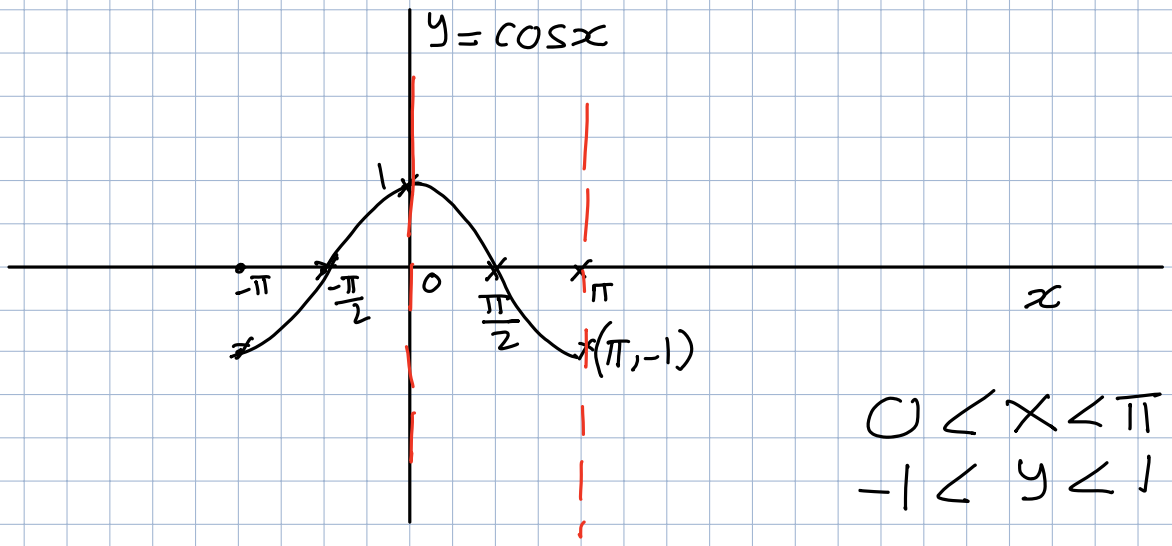
domain of $\sin x$ $\quad -\frac{\pi}{2} \leq x \leq \pi/2$

range of $\sin x$ $\quad -1 \leq y \leq 1$

domain of $\sin^{-1} x$ $\quad -1 \leq x \leq 1$

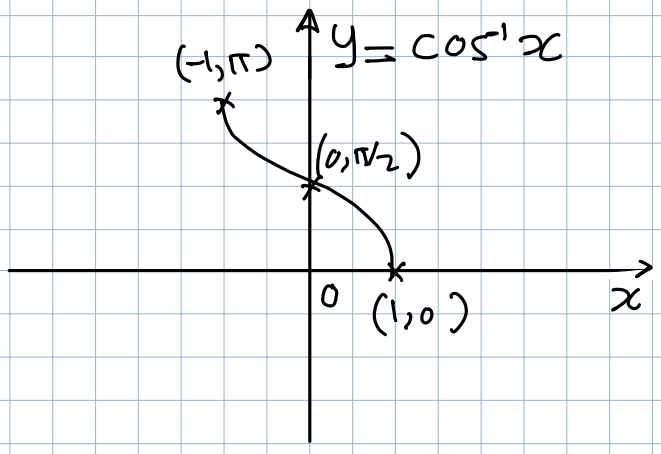
range of $\sin^{-1} x$ $\quad -\frac{\pi}{2} \leq y \leq \pi/2$

$y = \cos^{-1} x$



$0 < x < \pi$

$-1 < y < 1$



$-1 \leq x \leq 1$ Domain

$0 \leq y \leq \pi$ Range

$y = \cos^{-1} x$

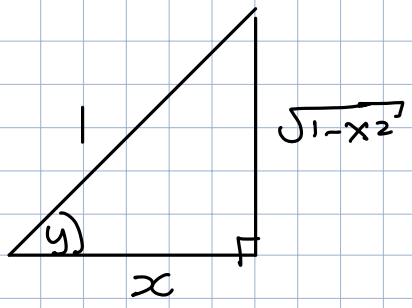
$\frac{dy}{dx} = ?$

$\cos y = \cos \cos^{-1} x$

$\frac{d}{dx}(\cos y = x)$

$\Downarrow$

$-\sin y \frac{dy}{dx} = 1$



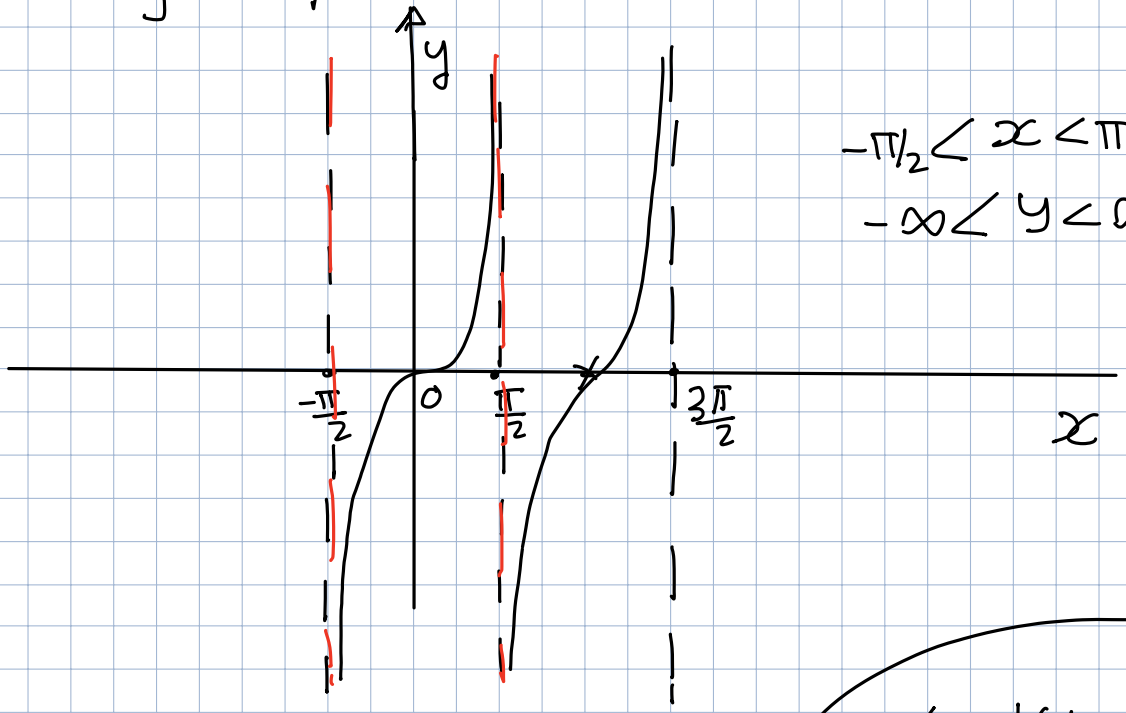
$\cos y = \frac{x}{1}$

$\frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\sqrt{1-x^2}}$ $\boxed{-1 < x < 1}$

$y = \tan^{-1} x$

$\tan y = x$

$y = \tan x$



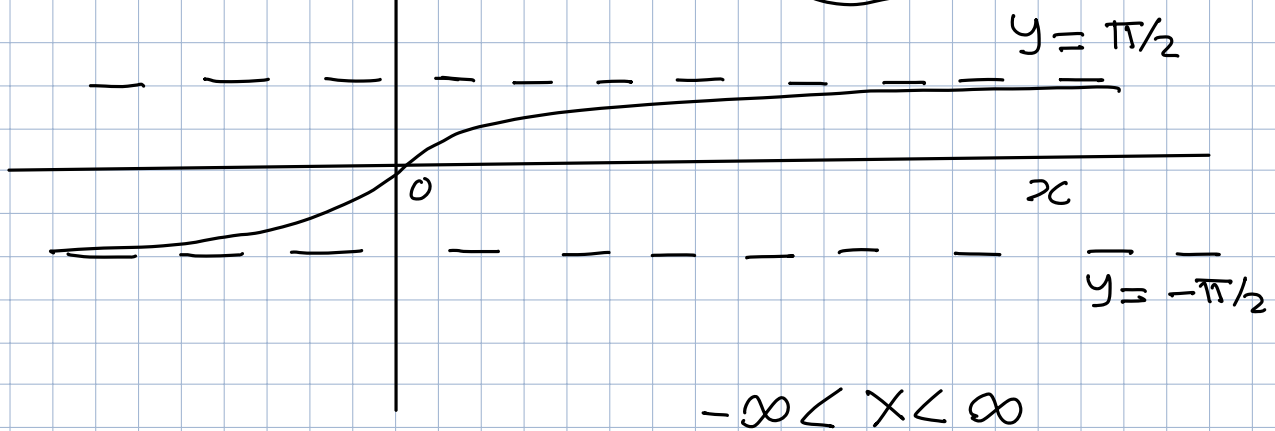
$-\pi/2 < x < \pi/2$

$-\infty < y < \infty$

$\tan^{-1}(\tan x) = x$

$-\frac{\pi}{2} \leq x \leq \pi/2$

$y = \tan^{-1} x$



$-\infty < x < \infty$

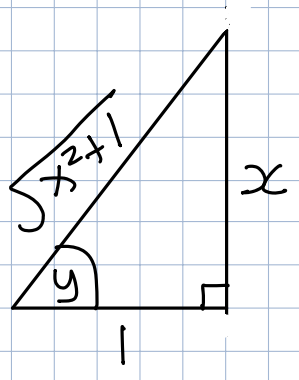
$-\frac{\pi}{2} < y < \pi/2$

$y = \tan^{-1} x$

$\frac{dy}{dx} =$

$\tan y = \tan \tan^{-1} x = x$

$\tan y = \frac{x}{1}$



$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \left(\frac{1}{\sqrt{x^2+1}}\right)^2 = \frac{1}{x^2+1}$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

---

② $y = \ln(x+5)$

$e^y = e^{\ln(x+5)}$

$e^y = x+5$

$x = e^y - 5$

$y = e^x - 5$

$$\boxed{f^{-1}(x) = e^x - 5}$$

---

(9) $\frac{d}{dx}\left(x^2y^2 + xy = 2\right)$ $\qquad m = -1$

$$2xy^2 + x^2 \cdot 2y\frac{dy}{dx} + 1y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(2yx^2 + x\right) = -2xy^2 - y$$

$$\frac{dy}{dx} = \frac{-2xy^2 - y}{2yx^2 + x} = \frac{-1}{1}$$

$$-2xy^2 - y = -2yx^2 - x$$

$$-2xy^2 + 2yx^2 - y + x = 0$$

$$-2xy[y - x] - 1[y - x] = 0$$

$$(y - x)(-2xy - 1) = 0$$

$y - x = 0$ $\qquad \boxed{y = x}$

$$x^2y^2 + xy = 2$$

$$x^4 + x^2 - 2 = 0$$

$$x^2 = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$$

$$x^2 = \frac{-1 \pm 3}{2} = 1, -2$$

$x^2 = 1$ $\qquad \boxed{x = 1 \quad x = -1}$

$x^2 = -2$ $\qquad$ REJECT !!!

$x = 1 \qquad x = -1$

$y = x$

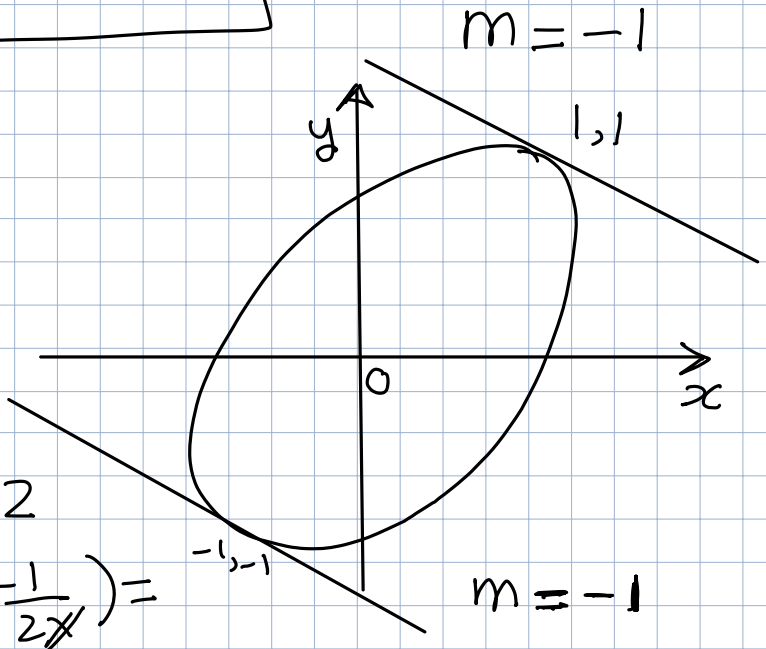
$y = 1 \qquad y = -1$

$\boxed{(1,1)} \quad \boxed{(-1,-1)}$

$-2xy - 1 = 0$

$-2xy = 1$

$y = \frac{-1}{2x}$



$x^2 y^2 + xy = 2$

$x^2 \cdot \frac{1}{4x^2} + x\left(\frac{-1}{2x}\right) =$

$\frac{1}{4} - \frac{1}{2} = 2$ NO!!!

LOG Diff.

$y = f(x)^{g(x)}$

$\ln y = \ln f(x)^{g(x)} = g(x) \ln f(x)$

$y \cdot \frac{1}{y} \frac{dy}{dx} = \left(g'(x) \ln f(x) + g(x) \cdot \frac{1}{f(x)} \cdot f'(x)\right) f(x)^{g(x)}$

$y = x^{x}$

$\ln y = \ln x^x = x \ln x$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = (1 \ln x + x \cdot \frac{1}{x}) x^x$$

$$\frac{dy}{dx} = (\ln x + 1) x^x$$

$$y = x^{\sqrt{x}} \qquad \frac{d}{dx} \ln y = \frac{1}{y} \frac{dy}{dx}$$

$$\ln y = \ln x^{\sqrt{x}} = \sqrt{x} \ln x$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left(\frac{1}{2\sqrt{x}} \ln x + \sqrt{x} \cdot \frac{1}{x}\right) x^{\sqrt{x}}$$

$$\frac{dy}{dx} = \left(\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}\right) x^{\sqrt{x}}$$

$$y = \sin x^{\ln(\cos x)}$$

$$\ln y = \ln \sin x^{\cos x} = \cos x \ln \sin x$$

$$y \cdot \frac{1}{y} \frac{dy}{dx} = \left[-\sin x \ln(\sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x\right] \sin x^{\cos x}$$

Paul's Notes

(3) $y = h(t) = \dfrac{\sqrt{5t+8}\ \sqrt[3]{1-9\cos(4t)}}{\sqrt[4]{t^2+10t}}$

$$\ln y = \ln(5t+8)^{1/2} + \ln(1-9\cos(4t))^{1/3} - \ln(t^2+10t)^{1/4}$$

$$\ln y = \frac{1}{2}\ln(5t+8) + \frac{1}{3}\ln(1-9\cos(4t)) - \frac{1}{4}\ln(t^2+10t)$$

$$\frac{1}{y}\frac{dy}{dt} = \frac{1}{2} \cdot \frac{1}{5t+8} \cdot 5 + \frac{1}{3} \cdot \frac{1}{1-9\cos(4t)} \cdot [36\sin(4t)]$$

$$-\frac{1}{4}\cdot\frac{1}{t^2+10t}\cdot(2t+10)$$

$$\frac{dy}{dt}=\left[\frac{5}{2(5t+8)}+\frac{12\sin(4t)}{1-9\cos(4t)}-\frac{(2t+10)}{4(t^2+10t)}\right]y$$

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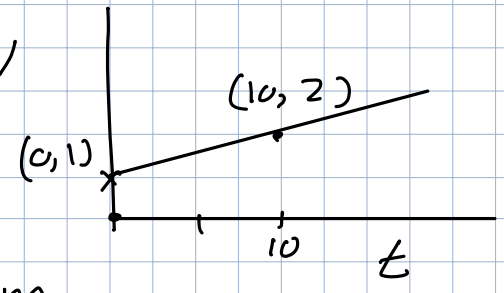
(2) C-L-P

$$1\ \frac{m}{sec}\rightarrow\frac{2\ m}{sec^2}$$



$$a=\frac{2-1}{10}=\frac{1}{10}\ \frac{m}{sec^2}$$

$$\frac{2m}{sec}\rightarrow\frac{3\ m}{sec^2}$$

$$a=\frac{1}{10}\ m/sec^2$$

$$\frac{13-3}{\Delta t}=\frac{1}{10}\ m/sec^2$$

$$3\frac{m}{sec}\rightarrow 13\frac{m}{sec^2}$$

$$\Delta t=100\ sec$$

(3)

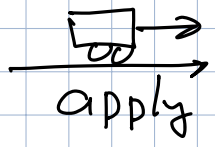
Object speeding UP ?

$$a>0$$
$$V>0$$

Object speeding up

$$a<0\quad V<0$$

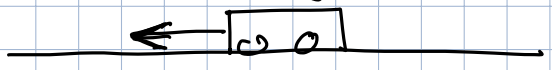
Slowing down



$a < 0 \quad v > 0$

$a > 0 \quad v < 0$

going in reverse



$v = -10 \frac{m}{sec}$

$a = \frac{5m}{sec^2}$

$t = 0 \quad v = -10$

$t = 1 \quad v = -5$

$t = 2 \quad v = 0$

$a > 0 \quad v < 0$

slow down

$s'(a) > 0 \quad s''(a) > 0$

speeding up

---

(9) $v(0) = 120 \frac{km}{hr}$

$a = \frac{\Delta v}{\Delta t} = \frac{120 \times 1000m}{3600 sec}$

$$\frac{\qquad\qquad}{\Delta t}$$

$a = C$

$V(t) = Ct + D$

$V(0) = \frac{120\,000}{3600}\ \frac{m}{sec} = \frac{1200}{36}\ \frac{m}{sec} = D$

$V(t) = Ct + \frac{1200}{36}$

$X(t) = \frac{Ct^2}{2} + \frac{1200}{36}t + E$

$X(0) = 0$

$0 = 0 + 0 + E$

$X(t) = \frac{Ct^2}{2} + \frac{1200}{36}t$

$V(t) = Ct + \frac{1200}{36}$

$V(t) = 0$

$0 = Ct + \frac{1200}{36}$

$Ct = -\frac{1200}{36}$

$$\boxed{t^* = \frac{-1200}{36C}}$$

$$X(t^*) = 100\ m$$

$$X(t) = \frac{Ct^2}{2} + \frac{1200}{36}t$$

$$100 = \frac{C}{2}\left(\frac{-1200}{36C}\right)^2 + \frac{1200}{36}\cdot\frac{(-1200)}{36C}$$

$$100 = \frac{C}{C^2}\ \frac{(1200)^2}{(36)^2 \cdot 2}\ -\frac{(1200)^2}{(36)^2 C}$$

$$100(36)^2 C = \frac{(1200)^2}{2} - (1200)^2$$

$$C = \frac{(1200)^2\left[-\frac{1}{2}\right]}{100(36)^2}$$

$$C = -\frac{(1200)^2}{100(36)^2}\cdot\frac{1}{2}$$

$$C = -\frac{555.55}{100}\ \frac{m}{sec^2} = -5.555\ \frac{m}{sec^2}$$

$$72000\ \frac{1000\ m}{(3600)(3600)}$$

$$72000\ \frac{km}{hr^2} \rightarrow$$

$$72000\ \frac{1000\ m}{(3600)^2} = 5.55\ \frac{m}{sec^2}$$