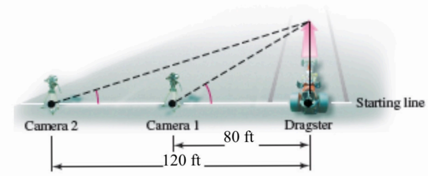
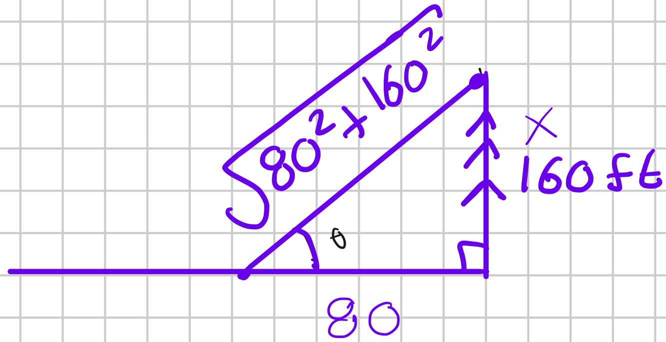
A camera is set up at the starting line of a drag race 80 ft from a dragster at the starting line (camera 1 in the figure). Two seconds after the start of the race, the dragster has traveled 160 ft and the camera is turning at 0.4 rad/s while filming the dragster. Answer parts **a.** and **b.** below.





2 sec

$$\frac{d\theta}{dt} = 0.4 \text{ rad/sec}$$

$$\tan\theta = \frac{x}{80}$$

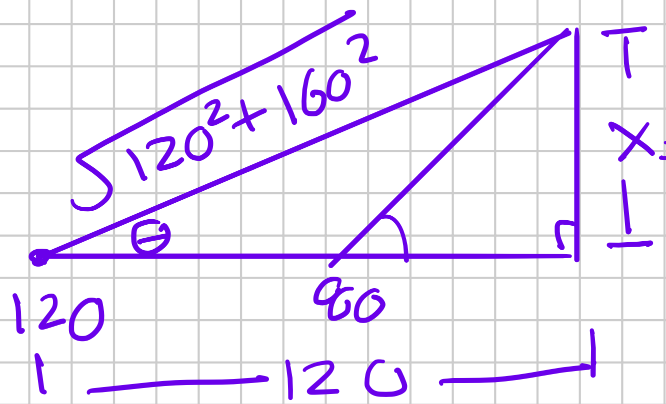
$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{80}\frac{dx}{dt}$$

$$\left(\frac{\sqrt{80^2+160^2}}{80}\right)^2 (0.4) = \frac{1}{80}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{\cancel{80}(0.4)(80^2+160^2)}{(80)^2}$$

$$\frac{dx}{dt} = \frac{0.4(80^2+160^2)}{80} = 160 \text{ ft/sec}$$

45 b)



$$\frac{dx}{dt} = 160 \frac{ft}{sec}$$

$$\tan\theta = \frac{x}{120}$$

$$\sec\theta = \frac{hyp}{adj}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{120}\frac{dx}{dt}$$

$$\left(\frac{\sqrt{120^2+160^2}}{120}\right)^2 \frac{d\theta}{dt} = \frac{1}{12\cancel{0}} \cdot 16\cancel{0}$$

$$\frac{120^2+160^2}{120^2}\frac{d\theta}{dt} = \frac{16}{12}$$

$$\frac{d\theta}{dt} = \frac{16}{12} \cdot \frac{120^2}{120^2+160^2}$$

$d\theta = 0.48$ rad

$\frac{d}{dt}$ = ... $\overline{sec}$

$y = \underline{\cos^4 x} + \cos(x^4)$ $\qquad \frac{dy}{dx} = \_\_\_\_\_\_$

$y' = -\sin^4 x - \sin(x^4) \cdot 4x$

$=$

$y = (\cos x)^4 + \underline{\cos(x^4)}$

$\boxed{y' = 4(\cos x)^3 \cdot (-\sin x) + -\sin(x^4) \cdot 4x^3}$ ✓✓

$\frac{d}{dx} \cos x = -\sin x$

$y' = -4\sin x(\cos x)^3 - 4x^3 \sin(x^4)$ ✓✓

b) $y = \ln[\ln(2x)]$

$y = [g(x)]^4 + g(x^4)$

$4[g(x)]^3 \cdot g'(x) + g'(x^4) \cdot 4x^3$

$y = f(g(h(x)))$

$$y' = f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \qquad \frac{d}{dx}\ln g(x) = \frac{1}{g(x)}\cdot g'(x)$$

$$y = \ln(\ln(\boxed{2x}))$$

$$y' = \frac{1}{\ln(2x)}\cdot\frac{1}{2x}\cdot 2$$

$$y = \ln[\underbrace{\ln[g(x)]}]$$

$$y' = \frac{1}{\ln[g(x)]}\cdot\frac{1}{g(x)}\cdot g'(x)$$

$$\frac{1}{\ln(2x)} \qquad \frac{1}{2x} \qquad 2$$

12)

$$\frac{d}{dx}(\ln(2x^8))$$

$$= \frac{1}{2x^8}\cdot 16x^7 = \frac{16x^7}{2x^8} = \frac{8}{x}$$

20)

$$\frac{d}{dx}\ln(\cos^2 x) = \ln([\cos(x)]^2)$$

$$\frac{d}{dx} = \frac{1}{\cos^{\cancel{2}} x}\cdot 2(\cancel{\cos} x)\cdot -\sin x$$

$$= \frac{-2\sin x}{\cos x}$$

22)

$$\frac{d}{dx} \ln(e^x + e^{-x})$$

$$= \frac{1}{e^x + e^{-x}} \cdot (e^x - e^{-x})$$

$$= \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

45)

$$y = x^{\cos x} \qquad a = \pi/2$$

$$\frac{dy}{dx} = x^{\cos x} \cdot (-\sin x)$$

$$y = f(x)^{g(x)} \qquad \underline{\text{LOG-Diff}}$$

$$\ln y = \ln f(x)^{g(x)} = \underline{g(x)}\, \underline{\ln f(x)}$$

$$\cancel{y} \cdot \frac{1}{\cancel{y}} \frac{dy}{dx} = \left(g'(x) \ln f(x) + g(x) \cdot \frac{1}{f(x)} f'(x)\right) y$$

$$\frac{dy}{dx} = \left(g'(x) \ln f(x) + \frac{g(x) f'(x)}{f(x)} \right) f(x)^{g(x)}$$

$$y = x^{\sin x} \qquad a = \pi/2$$

$$\boxed{\ln x^r = r \ln x}$$

$$\ln y = \ln x^{\sin x}$$

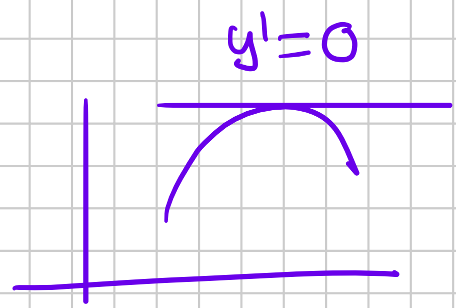
$$\ln y = \sin x \ln x$$

$$\cancel{y}\cdot\frac{1}{\cancel{y}}\frac{dy}{dx} = \left(\cos x \ln x + \sin x \cdot \frac{1}{x}\right) x^{\sin x}$$

$$\frac{dy}{dx} = \left(\cos x \ln x + \frac{\sin x}{x}\right) x^{\sin x}$$

Ex. 6 $\qquad y = x^x \qquad x > 0$

horiz. tangent line



$$y = x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = x \cdot \frac{1}{x} + (1)\cdot \ln x$$

$$\frac{dy}{dx} = x^x (1 + \ln x)$$

$$x^x > 0$$

$y' = 0$ $\quad 1 + \ln x = 0$

$$\ln x = -1$$

$$e^{\ln x} = e^{-1}$$

$$\boxed{x = 1/e}$$

$\ln 1 = 0 \qquad \log_e 1 = 0$

$\ln e = 1 \qquad \log_e e = 1$

$$\ln x^r = r \ln x$$

$$\ln(xy) = \ln x + \ln y$$

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\log_b x = \frac{\ln x}{\ln b}$$

$$\frac{d}{dx} \underbrace{\log_4(x^2+x)} =$$

STU $\quad \log_4(x^2+x) = \dfrac{\ln(x^2+x)}{\ln 4}$

Change of base

$$y = \frac{1}{\ln 4} \underbrace{\ln(x^2+x)}$$

$$\frac{dy}{dx} = \frac{1}{\ln 4} \cdot \frac{1}{x^2+x} \cdot (2x+1)$$

Ex. 5)

$f(x) = x^{\sin x}$ $\quad x \geq 0$

$\boxed{y = g(x)^{h(x)}}$ LOG-DIFF

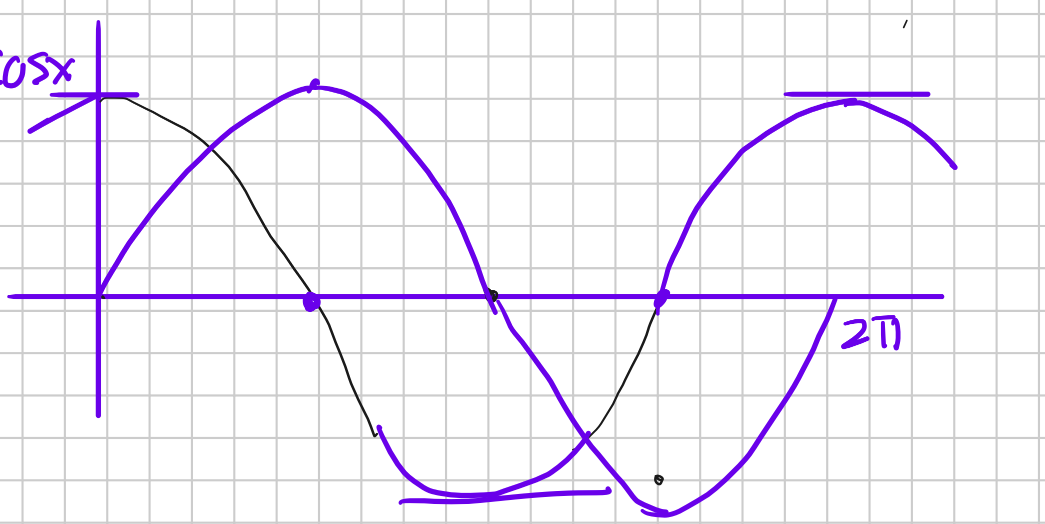
$\ln f(x) = \sin x \cdot \ln x$

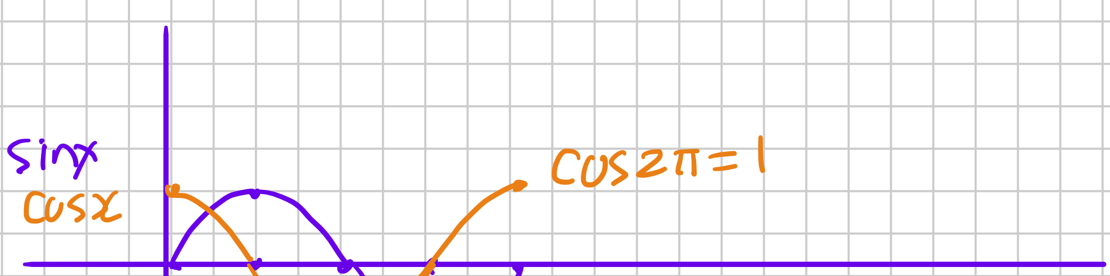
$\frac{1}{f(x)} \cdot f'(x) = \left(\sin x \cdot \frac{1}{x} + \cos x \cdot \ln x\right)$

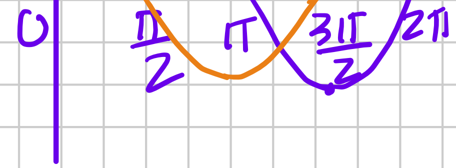
$f'(x) = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \ln x\right)$

b) $f'(\pi/2)$

$\sin \pi/2 = 1$
$\cos \pi/2 = 0$







$$f'(x) = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \ln x\right)$$

$$f'\left(\tfrac{\pi}{2}\right) = \left(\tfrac{\pi}{2}\right)^{\sin\frac{\pi}{2}}\left(\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)} + \cos\left(\tfrac{\pi}{2}\right)\cdot \ln\left(\tfrac{\pi}{2}\right)\right)$$

$$= \frac{\pi}{2}\left(\frac{1}{\frac{\pi}{2}} + 0\right)$$

$$= \frac{\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{1}{1} = 1$$

$$\frac{d}{dx}\log_b x = \frac{1}{x\ln b} \qquad x > 0$$

$$\log_b x = \frac{\ln x}{\ln b}$$

$$\frac{d}{dx}\log_b x = \frac{d}{dx}\left(\frac{\ln x}{\ln b}\right)$$

$$\frac{1}{\ln b}\,\frac{d}{dx}\ln x$$

$$\frac{1}{\ln b}\cdot\frac{1}{x}$$

7/a] Exam

$y = \log_5(2x+1)$

$$y = \frac{\ln(2x+1)}{\ln 5} = \frac{1}{\ln 5} \boxed{\ln(2x+1)}$$

$$y' = \frac{1}{\ln 5} \cdot \frac{1}{2x+1} \cdot 2 = \frac{2}{\ln 5} \cdot \frac{1}{2x+1}$$

$$= \boxed{\frac{2}{\ln 5(2x+1)}}$$