Partial Fractions I

Background:

1] Every Polynomial can be factored as a product of linear and/or quadratic factors

Example: $x^3 + x = \underbrace{x}_{\text{Linear}}\,\underbrace{(x^2+1)}_{\text{quadratic}}$

2] Every rational function $f(x) = \dfrac{P(x)}{Q(x)}$

where degree of $P(x)$ < degree of $Q(x)$

can be decomposed into partial fractions

2] $f(x) = \dfrac{3x-1}{x^2-1}$ $P(x) = 3x-1$ deg. $P(x) = 1$

$Q(x) = x^2-1$ deg. $Q(x) = 2$

deg. $P(x)$ < deg $Q(x)$ $\therefore$ apply Partial fractions

let's start with $f(x) = \dfrac{1}{x-1} + \dfrac{2}{x+1}$

$$f(x) = \frac{(x+1)+2(x-1)}{(x-1)(x+1)} = \frac{3x-1}{(x-1)(x+1)} = \frac{3x-1}{x^2-1}$$

Goal of Partial Fractions is to reverse this process: $\dfrac{1}{x-1} + \dfrac{2}{x+1} = \dfrac{3x-1}{x^2-1}$

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

How to find A and B ?

Rational function $\qquad$ Partial Frac. Decomp.

$$\frac{3x-1}{x^2-1} \Longrightarrow \frac{A}{x-1} + \frac{B}{x+1}$$

Hard to Integrate $\qquad$ Easy to Integrate

$$\int \frac{3x-1}{x^2-1}\,dx \qquad \int \left(\frac{A}{x-1} + \frac{B}{x+1}\right) dx$$

How to Find A and B ?

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$\frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

Mult. both sides by $(x-1)(x+1)$

$$\frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

Clear out the fractions multiply both sides by $(x-1)(x+1)$

$$3x-1 = A(x+1) + B(x-1)$$

Collect equal powers of x

$$3x-1 = (A+B)x + (A-B)$$

Equate coefficients of x on both sides

$$\boxed{3}x + \boxed{-1} = \boxed{(A+B)}x + \boxed{A-B}$$

x^1: $3 = A+B$

x^0: $-1 = A-B$

$A = 3-B$ substitute into $-1 = A-B$

$$-1 = 3-B-B \Rightarrow -4 = -2B \Rightarrow B = 2$$

$$B = 2 \Rightarrow A = 3-B \Rightarrow A = 3-2 = 1$$

$$\frac{3x-1}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{2}{x+1}$$

Partial Fraction decomposition

Now we can integrate!

$$\int \frac{3x-1}{(x-1)(x+1)}\,dx = \int\left(\frac{1}{x-1} + \frac{2}{x+1}\right)dx$$

$$= \ln|x-1| + 2\ln|x+1| + C$$

$$\int \frac{3x-1}{x^2-1}\,dx = \ln|x-1| + 2\ln|x+1| + C$$

Integration Review + U-Substitution

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$$

$$\int \frac{1}{x+1}\,dx = \ln|x+1| + C$$

$$\int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a} + C$$

$$u = ax+b \qquad du = a\,dx \qquad dx = \frac{du}{a}$$

$$\int \frac{1}{2x+1}\,dx = \frac{\ln|2x+1|}{2} + C$$

Partial Fractions 2 Case I Distinct linear factors in Denominator

$f(x) = \dfrac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are Polynomials with no common factors and Degree of $P(x) <$ Degree of $Q(x)$

Assume $Q(x)$ can be factored as a product of distinct linear factors.

Step 1 Factor $Q(x)$ in the form:

$$Q(x) = (a_1x + b_1)(a_2x + b_2)\ldots(a_nx + b_n)$$

Step 2 | Apply Partial Fraction Decomposition

$$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_n}{a_nx+b_n}$$

Step 3 | Clear out the denominators by multiplying both sides of the above equation by $Q(x) = (a_1x+b_1)(a_2x+b_2)\dots(a_nx+b_n)$

Step 4 | Solve for the coefficients $A_1, A_2, \dots, A_n$ by equating powers of x or substituting special values of x that solves the equation for $A_1, A_2, \dots, A_n$.

Step 5 Integrate $\int \frac{P(x)}{Q(x)}\,dx$

$$\int \frac{P(x)}{Q(x)}\,dx = \int\left(\frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots \frac{A_n}{a_nx+b_n}\right)dx$$

$$= \frac{A_1 \ln|a_1x+b_1|}{a_1} + \frac{A_2 \ln|a_2x+b_2|}{a_2} + \cdots + \frac{A_n \ln|a_nx+b_n|}{a_n}$$

$$+ C$$

Recall $\int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a} + C$

$u = ax+b \quad du = a\,dx \quad dx = \frac{du}{a}$

Evaluate ∫ (3x^2+13x-2)/((2x-1)(x+1)(x+3)) dx Partial fractions ∫ example

Ex Evaluate $\int \frac{3x^2+13x-2}{(2x-1)(x+1)(x+3)}\,dx$

Step 1 Degree $P(x) = 2$; Degree $Q(x) = 3$; $2 < 3$

Step 2 Apply Partial Fraction Decomposition

$$\frac{3x^2+13x-2}{(2x-1)(x+1)(x+3)} = \frac{A}{2x-1} + \frac{B}{x+1} + \frac{C}{x+3}$$

Step 3 Multiply both sides by $(2x-1)(x+1)(x+3)$ to clear out the fractions

$$3x^2+13x-2 = A(x+1)(x+3) + B(2x-1)(x+3) + C(2x-1)(x+1)$$

Step 4 Substitute special values of x to solve for A, B and C

$$3x^2+13x-2=A(x+1)(x+3)+B(2x-1)(x+3)+C(2x-1)(x+1)$$

plug in $x=-1$

$$3(-1)^2+13(-1)-2=A(-1+1)(-1+3)+B(-3)(2)+C(-3)(0)$$

$$-12=A(0)(2)+-6B+C(-3)(0)$$

$$-12=-6B \Rightarrow B=2$$

Plug in $x=-3$

$$3(-3)^2+13(-3)-2=A(-2)(0)+B(-7)(0)+C(-7)(-2)$$

$$-14=0+0+14C \Rightarrow 14C=-14 \Rightarrow C=-1$$

Plug in $x = \frac{1}{2}$

$$3x^2+13x-2 = A(x+1)(x+3)+B(2x-1)(x+3)+C(2x-1)(x+1)$$

$$\frac{3}{4}+\frac{13}{2}-2 = A\left(\frac{3}{2}\right)(7/2)+B(0)(7/2)+C(0)(3/2)$$

$$\cancel{\frac{21}{4}} = A\left(\cancel{\frac{21}{4}}\right) \Rightarrow A=1$$

$\boxed{\text{Step 5}}$ Integrate!

$$\int \frac{3x^2+13x-2}{(2x-1)(x+1)(x+3)}\,dx = \int\left(\frac{1}{2x-1}+\frac{2}{x+1}+\frac{-1}{x+3}\right)dx$$

$$= \frac{\ln|2x-1|}{2} + 2\ln|x+1| - \ln|x+3| + C$$

Partial Fractions 3 Case II Repeated Linear Factors in Denominator

$f(x) = \frac{P(x)}{Q(x)}$; Assume $Q(x)$ is a product of Repeated linear factors

$$Q(x) = (a_1x+b_1)^{r_1}(a_2x+b_2)^{r_2}\dots(a_nx+b_n)^{r_n}$$

where $r_1, r_2, \dots, r_n \geq 1$

let's assume the first linear factor is repeated r_1 times then we repeat all the way up to the power r_1 times.

$$\frac{A_1}{a_1x+b_1} + \frac{A_2}{(a_1x+b_1)^2} + \dots + \frac{A_{r_1}}{(a_1x+b_1)^{r_1}}$$

Find the PFD (Partial Fraction Decomposition) of the following Rational functions.

1] $\frac{1}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

2] $\frac{2x+3}{(x-1)^2(x+2)^3(2x-1)(x+1)}$

$\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2} + \frac{E}{(x+2)^3} + \frac{F}{2x-1} + \frac{G}{x+1}$

KEY CONCEPT: Repeat all the way up to the highest power of repeated Factor.

Evaluate ∫ (x^2+5x+2)/(x^2+2x^3) dx ,Partial Fraction example solved

Ex| Evaluate $\int \frac{x^2+5x+2}{x^2+2x^3}\,dx$

deg. Num. = 2 ; deg. Denom = 3 ; Deg. Num < Deg. Denom

$$\frac{x^2+5x+2}{x^2+2x^3} = \frac{x^2+5x+2}{x^2(1+2x)}$$

Factor Denominator

$$\frac{x^2+5x+2}{x^2(1+2x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1+2x}$$

Clear out the fractions multiply both sides by $x^2(1+2x)$

$$x^2+5x+2 = Ax(1+2x) + B(1+2x) + Cx^2$$

$$x^2+5x+2 = 2Ax^2 + Cx^2 + Ax + 2Bx + B$$

$$x^2+5x+2 = (2A+C)x^2 + (A+2B)x + B$$

$$1x^2+5x+2=(2A+C)x^2+(A+2B)x+B$$

x^2: $\quad 1 = 2A+C$ equate equal powers of x

x^1: $\quad 5 = A+2B$

x^0: $\quad 2 = B$

$B=2$ plug in $5=A+2B \Rightarrow 5=A+4 \Rightarrow A=1$

$A=1$ plug in $1=2A+C \Rightarrow 1=2+C \Rightarrow C=-1$

$$\frac{x^2+5x+2}{x^2+2x^3}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x+1}=\frac{1}{x}+\frac{2}{x^2}+\frac{-1}{2x+1}$$

Partial Fraction Decomposition Completed!!

$$\int \frac{x^2+5x+2}{x^2+2x^3}\,dx = \int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{-1}{2x+1}\right) dx$$

$$= \int \left(\frac{1}{x} + 2x^{-2} + \frac{-1}{2x+1}\right) dx$$

$$= \ln|x| + \frac{2x^{-1}}{-1} - \frac{\ln|2x+1|}{2} + C$$

$$= \ln|x| - \frac{2}{x} - \frac{\ln|2x+1|}{2} + C$$

Partial Fractions with irreducible quadratic factors in denominator concise notes

Partial Fractions 4 Case III, $Q(x)$ contains the irreducible quadratic factor ax^2+bx+C ; $b^2-4ac<0$

$f(x)=\dfrac{P(x)}{Q(x)}$; then the PFD will have the pattern $\dfrac{Ax+B}{ax^2+bx+C}$; this term can be integrated by completing the square (if needed) and applying formula $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

$$\int \frac{Ax+B}{ax^2+bx+C}\,dx = \int \frac{Cu+D}{u^2+a^2}\,du = C\int \frac{u}{u^2+a^2}\,du + D\int \frac{1}{u^2+a^2}\,du$$

$$= \frac{C\ln|u^2+a^2|}{2} + \frac{D}{a}\tan^{-1}(u/a)+k$$

Find the PFD (Partial fraction Decomposition) of the following Rational functions?

1] $$\frac{x+1}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$$

2] $$\frac{x}{(x+1)(x^2+9)(x^2+x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9} + \frac{Dx+E}{x^2+x+1}$$

KEY CONCEPT: Irreducible Quadratic factors x^2+4, x^2+9, x^2+x+1, x^2+a^2 have the PFD

$$\frac{Ax+B}{ax^2+bx+C}$$

Evaluate ∫(3x^2+x+4)/(x^3+4x) dx , Partial Fractions solved example

Ex] Evaluate $\int \frac{3x^2+x+4}{x^3+4x}\,dx$

Check: deg. Num < deg. Denom ; $2<3 \Rightarrow$ Apply PFD

$$\frac{3x^2+x+4}{x^3+4x} = \frac{3x^2+x+4}{x(x^2+4)}$$

Factor Denominator

$$\frac{3x^2+x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

Clear out the fractions by multiplying both sides by $x(x^2+4)$

$$3x^2+x+4 = A(x^2+4)+(Bx+C)x$$

$$3x^2+x+4 = Ax^2+Bx^2+Cx+4A$$

$$3x^2+x+4 = (A+B)x^2+Cx+4A$$

Rearrange

$$3x^2 + x + 4 = (A+B)x^2 + Cx + 4A$$

x^2 : $3 = A + B$ Equate equal powers of x

x^1 : $1 = C$

x^0 : $4 = 4A$

$4 = 4A \Rightarrow A = 1$ plug into $3 = A + B \Rightarrow 3 = 1 + B \Rightarrow B = 2$

$A = 1$, $B = 2$, $C = 1$

$$\frac{3x^2 + x + 4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4} = \frac{1}{x} + \frac{2x + 1}{x^2 + 4}$$

Partial Fraction Decomposition Completed!!

$$\int \frac{3x^2+x+4}{x(x^2+4)}\,dx = \int\left(\frac{A}{x}+\frac{Bx+C}{x^2+4}\right)dx = \int\left(\frac{1}{x}+\frac{2x+1}{x^2+4}\right)dx$$

Split up second term $\frac{2x+1}{x^2+4} = \frac{2x}{x^2+4} + \frac{1}{x^2+4}$

$$= \int \frac{1}{x}\,dx + \int \frac{2x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$$

$$= \ln|x| + \frac{\ln|x^2+4|}{2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$\int \frac{2x}{x^2+4}\,dx \qquad u = x^2+4 \qquad du = 2x\,dx$

U-Substitution was Applied

Find ∫1/(x^2+4) dx , U-Substitution review solved example

Integration Review U-Substitution

$$\int \frac{1}{x^2+4}\,dx = \int \frac{1/4}{\frac{x^2}{4}+1}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$$

let $u = \frac{x}{2}$ $\quad du = \frac{1}{2}\,dx \quad dx = 2\,du$

$$\frac{1}{4}\int \frac{1}{u^2+1}\,2\,du = \frac{2}{4}\int \frac{1}{u^2+1}\,du = \frac{1}{2}\tan^{-1}u + C$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}(x/a) + C$$

Easier to Apply this formula

Evaluate ∫(3x^2+7x+6)/((2x+1)(x^2+4x+5)) Partial Fractions solved example

Partial Fractions 5

Ex| Evaluate $\int \frac{3x^2+7x+6}{(2x+1)(x^2+4x+5)}\,dx$

check: deg. Num. = 2 ; deg. Denom. = 3 ; 2 < 3 ⇒ Apply PFD

$$\frac{3x^2+7x+6}{(2x+1)(x^2+4x+5)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4x+5}$$

multiply both sides by $(2x+1)(x^2+4x+5)$

$$3x^2+7x+6 = A(x^2+4x+5)+(Bx+C)(2x+1)$$

$$3x^2+7x+6 = Ax^2+4Ax+5A+2Bx^2+Bx+2Cx+C$$

$$3x^2+7x+6 = Ax^2+2Bx^2+4Ax+Bx+2Cx+5A+C$$

$$3x^2+7x+6 = (A+2B)x^2+(4A+B+2C)x+5A+C$$

$$3x^2+7x+6=(A+2B)x^2+(4A+B+2C)x+(5A+C)$$

x^2: $\quad 3=A+2B$ Equate equal powers of x

x^1: $\quad 7=4A+B+2C$

x^0: $\quad 6=5A+C$

Plug $A=3-2B$ into $7=4A+B+2C$ and $6=5A+C$

$$7=4(3-2B)+B+2C \Rightarrow 7=-7B+2C+12 \Rightarrow \boxed{-5=-7B+2C}$$

Plug $A=3-2B$ into $6=5A+C \Rightarrow 6=5(3-2B)+C$

$$6-15=-10B+C \Rightarrow \boxed{-9=-10B+C}$$

Plug $C=-9+10B$ into $-5=-7B+2C$

$$-5=-7B+2(-9+10B) \Rightarrow -5=13B-18 \Rightarrow 13=13B$$

$$\Rightarrow B=1$$

$B=1$ plug into $-9=-10B+C \Rightarrow -9=-10+C \Rightarrow C=1$

plug $C=1$ into $C=6-5A \Rightarrow C=6-5=1$

$A=1$; $B=1$; $C=1$

$$\frac{3x^2+7x+6}{(2x+1)(x^2+4x+5)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4x+5}$$

$$\frac{3x^2+7x+6}{(2x+1)(x^2+4x+5)} = \frac{1}{2x+1} + \frac{x+1}{x^2+4x+5}$$

Partial Fraction Decomposition Completed!

$$\int \frac{3x^2+7x+6}{(2x+1)(x^2+4x+5)}\,dx = \int \left(\frac{1}{2x+1} + \frac{x+1}{x^2+4x+5} \right) dx$$

$$= \int \frac{1}{2x+1}\,dx + \int \frac{x+1}{x^2+4x+5}\,dx$$

split up integrals

$$= \frac{\ln|2x+1|}{2} + \int \frac{x+1}{(x+2)^2+1}\,dx$$

Complete the square

$$= \frac{\ln|2x+1|}{2} + \int \frac{(x+2)-1}{(x+2)^2+1}\,dx$$

re-write $x+1$ as
$x+1 = (x+2)-1$

$$= \frac{\ln|2x+1|}{2} + \int \frac{x+2}{(x+2)^2+1}\,dx - \int \frac{1}{(x+2)^2+1}\,dx$$

$$= \frac{\ln|2x+1|}{2} + \int \frac{x+2}{(x+2)^2+1}\,dx - \int \frac{1}{(x+2)^2+1}\,dx$$

$u = x+2 \quad du = dx \qquad u = x+2 \quad du = dx$

$$= \frac{\ln|2x+1|}{2} + \int \frac{u}{u^2+1}\,du - \int \frac{1}{u^2+1}\,du$$

$$= \frac{\ln|2x+1|}{2} + \frac{\ln|u^2+1|}{2} - \tan^{-1}(u) + C$$

$$= \frac{\ln|2x+1|}{2} + \frac{\ln|(x+2)^2+1|}{2} - \tan^{-1}(x+2) + C$$

Completing the square review y=x^2+4x+5 solved example

Completing the square + U-Substitution Review

$y = x^2 + 4x + 5$

Key Concept

$y = \underbrace{x^2 + 4x + 4}\ - 4 + 5$ $\qquad \left(\frac{4}{2}\right)^2 = 4$ add and subtract

$y = (x+2)^2 + 1$

$$\int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a} + C$$

$u = ax+b \qquad du = a\,dx \qquad dx = \frac{du}{a}$

$$\int \frac{x}{x^2+1}\,dx = \frac{\ln|x^2+1|}{2} + C$$

$u = x^2+1 \qquad du = 2x\,dx \qquad \frac{du}{2} = x\,dx$

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Partial Fraction theory when deg. of numerator is >= deg. of numerator

Partial Fractions 6

Given a rational function $f(x) = \frac{P(x)}{Q(x)}$ where the degree of the Numerator $P(x)$ is equal or greater than the degree of the denominator $Q(x)$, the first step is to apply Long Division so that the degree of the Numerator becomes Less than the degree of the Denominator. Afterwards apply PFD (Partial Fraction Decomposition) as usual.

Ex $f(x) = \frac{x^3}{x^2-1}$ deg. $P(x) = 3 >$ deg. $Q(x) = 2$

Apply Long Division First

Evaluate ∫(x^3+3x^2+4)/(x^2-1) dx , Partial Fraction with long division

Ex Evaluate $\int \frac{x^3+3x^2+4}{x^2-1}\,dx$

deg. $P(x)=3$, deg $Q(x)=2$; $3>2$; Apply Long Division

$$
\begin{array}{r}
x+3 \\
x^2-1 \,\big)\, x^3+3x^2+0x+4 \\
-(x^3+0x^2-x+0) \\
\hline
3x^2+x+4 \\
-(3x^2+0x+-3) \\
\hline
x+7
\end{array}
$$

$$\frac{x^3+3x^2+4}{x^2-1} = \underbrace{x+3}_{\text{Easy to Integrate Polynomial}} + \underbrace{\frac{x+7}{x^2-1}}_{\text{Apply PFD (Partial fractions)}}$$

$$\int \frac{x^3+3x^2+4}{x^2-1}\,dx = \int\left[(x+3) + \frac{x+7}{x^2-1}\right]dx$$

$$= \int (x+3)\,dx + \int \frac{x+7}{x^2-1}\,dx$$ split up integrals

$$= \frac{x^2}{2} + 3x + \int \frac{x+7}{x^2-1}\,dx$$ Integrate easy part Polynomial

$$g(x) = \frac{x+7}{x^2-1}$$ deg. Num $=1$; deg. Denom. $=2$

$1 < 2 \Rightarrow$ Apply PFD

$$\frac{x+7}{x^2-1} = \frac{x+7}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$ Multiply both sides by $(x-1)(x+1)$

$$x+7 = A(x+1) + B(x-1)$$ Clear out the fractions

$$x+7 = Ax + A + Bx - B$$

$$x+7 = Ax + A + Bx - B$$

$$x+7 = Ax + Bx + A - B$$ Re-arrange

$$x+7 = (A+B)x + A - B$$

x^1: $\quad 1 = A+B$ Equate powers of x

x^0: $\quad 7 = A-B$

Plug $A = 1-B$ into $7 = A-B \Rightarrow 7 = 1-B-B$

$\Rightarrow 7 = 1-2B \Rightarrow 6 = -2B \Rightarrow B = -3$

Plug $B = -3$ into $A = 1-B \Rightarrow A = 1--3 \Rightarrow A = 4$

$$\frac{x+7}{x^2-1} = \frac{x+7}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{4}{x-1} + \frac{-3}{x+1}$$

Partial Fraction Decomposition Completed!

$$\int \frac{x^3+3x^2+4}{x^2-1}\,dx = \int \left[(x+3) + \frac{x+7}{x^2-1}\right] dx$$

$$= \frac{x^2}{2} + 3x + \int \frac{x+7}{(x-1)(x+1)}\,dx$$

Split up and integrate easy part x+3

$$= \frac{x^2}{2} + 3x + \int \left[\frac{4}{x-1} + \frac{-3}{x+1}\right] dx$$

Apply PFD

$$= \frac{x^2}{2} + 3x + \int \frac{4}{x-1}\,dx + \int \frac{-3}{x+1}\,dx$$

Integrate term by term

$$= \frac{x^2}{2} + 3x + 4\int \frac{1}{x-1}\,dx - 3\int \frac{1}{x+1}\,dx$$

$$= \frac{x^2}{2} + 3x + 4\ln|x-1| - 3\ln|x+1| + C$$

Integration Review + U-Substitution

$$\int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a} + C$$

Let $u = ax+b$ $\quad du = a\,dx \Rightarrow dx = \frac{du}{a}$

$$\int \frac{1}{x-1}\,dx = \ln|x-1| + C$$

$u = x-1 \quad du = dx$

$$\int \frac{1}{x+1}\,dx = \ln|x+1| + C$$

$u = x+1 \quad du = dx$

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Evaluate ∫ 1/(x^2-a^2) dx , Partial fraction decomposition solved example

Partial Fractions 7

Ex] Evaluate $\int \frac{dx}{x^2-a^2}$ $\quad a \neq 0$

$$\frac{1}{x^2-a^2} = \frac{1}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$$

Clear out the fractions by multiplying both sides by $(x-a)(x+a)$

$$1 = A(x+a) + B(x-a)$$

Plug in $x = -a \Rightarrow 1 = A(0) + B(-2a) \Rightarrow 1 = -2aB$

$\Rightarrow B = \frac{-1}{2a}$; Plug in $x = a \Rightarrow 1 = A(2a) + B(0)$

$\Rightarrow A = \frac{1}{2a}$

$$\frac{1}{x^2-a^2} = \frac{A}{x-a} + \frac{B}{x+a} = \frac{1/2a}{x-a} + \frac{-1/2a}{x+a}$$

$$\int \frac{dx}{x^2-a^2} = \int \left(\frac{1/2a}{x-a} + \frac{-1/2a}{x+a} \right) dx$$

$$= \frac{1}{2a} \int \left(\frac{1}{x-a} - \frac{1}{x+a} \right) dx$$

$$= \frac{1}{2a} \left(\ln|x-a| - \ln|x+a| \right) + C$$

$$= \frac{1}{2a} \left(\ln \left| \frac{x-a}{x+a} \right| \right) + C \qquad \ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

Let's take a look at a few more cases of integrating rational functions with a Quadratic Denominators $\int \frac{x}{x^2-a^2} dx$; $\int \frac{x^2}{x^2-a^2} dx$; $\int \frac{1}{x^2+a^2} dx$

Evaluate ∫ x^2/(x^2-a^2) dx , Partial fractions integration solved example

Ex| Evaluate $\int \frac{x^2}{x^2-a^2}\,dx \qquad a\neq 0$

Deg. Num = 2 ; Deg. Denom = 2 $\Rightarrow$ Apply Long Division First

$$\begin{array}{r l} & 1 \\ x^2-a^2 \,\big) & x^2+0 \\ & -(x^2-a^2) \\ \hline & a^2 \end{array}$$

Recall:
If Deg P(x) $\geq$ Deg Q(x) apply Long Division first.

$$\frac{x^2}{x^2-a^2} = 1 + \frac{a^2}{x^2-a^2} = \text{Quotient} + \frac{\text{Remainder}}{\text{Denominator}}$$

$$\int \frac{x^2}{x^2-a^2}\,dx = \int\left(1+\frac{a^2}{x^2-a^2}\right)dx = \int 1\,dx + \int \frac{a^2}{x^2-a^2}\,dx$$

$$\int \frac{x^2}{x^2-a^2}\,dx = \int 1\,dx + a^2 \int \frac{1}{x^2-a^2}\,dx$$

$$= x + a^2 \underbrace{\int \frac{1}{x^2-a^2}\,dx}_{\text{Solved already!}}$$

$$= x + a^2 \cdot \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$= x + \frac{a}{2} \ln \left| \frac{x-a}{x+a} \right| + C$$

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Compute ∫ x/(x^2-a^2) dx , partial fraction integral solved example

Ex Compute $\int \frac{x}{x^2-a^2}\,dx$

We can apply PFD or U-Substitution

$$\frac{x}{x^2-a^2} = \frac{x}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a} \qquad \text{P.F.D}$$

Let's Do U-Substitution (it is easier!)

$$\int \frac{x}{x^2-a^2}\,dx$$

$u = x^2-a^2 \qquad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{1/2}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2-a^2| + C$$

Find ∫ 1/(x^2+a^2) dx , Partial Fraction Integral solved by U-Substitution

Ex Find $\int \frac{1}{x^2+a^2}\,dx$

Let's apply formula derived by applying algebra and U-Substitution.

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1/a^2}{\left(\frac{x}{a}\right)^2+1}\,dx \qquad u=\frac{x}{a}\,;\ du=\frac{1}{a}dx\,;\ dx=a\,du$$

$$\frac{1}{a^2}\int \frac{1}{u^2+1}\,a\,du = \frac{1}{a}\int\frac{1}{u^2+1}\,du = \frac{1}{a}\tan^{-1}u+C$$

$$=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

Summary: 4 different Partial fraction integrals with Quadratic denominators

Summary: Integrals with Quadratic Denominators

1. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{x^2}{x^2-a^2}\,dx = x + \frac{a}{2} \ln\left|\frac{x-a}{x+a}\right| + C$

3. $\int \frac{x}{x^2-a^2}\,dx = \frac{1}{2} \ln\left|x^2-a^2\right| + C$

4. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a} \tan^{-1}(x/a) + C$

Partial Fractions Case IV, repeated irreducible Quadratic factors in denominator

Partial Fractions 8 Case IV, $Q(x)$ contains repeated irreducible Quadratic factors $(ax^2+bx+c)^m$; $b^2-4ac<0$; $f(x)=\frac{P(x)}{Q(x)}$; then the PFD will be in the form of a sum:

$$f(x)=\frac{P(x)}{Q(x)}=\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\frac{A_3x+B_3}{(ax^2+bx+c)^3}+\cdots+\frac{A_mx+B_m}{(ax^2+bx+c)^m}$$

Some examples of repeated irreducible quadratic factor

$(x^2+x+1)^2$, $(2x+5)(x^2+4)^2$, $x(x-1)(x^2+2)^3$

Find the PFD (Partial Fraction Decomposition) of the following Rational functions?

1) $$\frac{1}{x^2(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$$

2) $$\frac{t}{(t+1)(t^2+4)^3} = \frac{A}{t+1} + \frac{Bt+C}{t^2+4} + \frac{Dt+E}{(t^2+4)^2} + \frac{Ft+G}{(t^2+4)^3}$$

Key Concept: Repeat all the way up to the highest power of repeated irreducible Quadratic factor $(ax^2+bx+c)^m$

Evaluate ∫ 1/(x(x^2+4)^2) dx , Partial Fraction Integral Solved example

Ex Evaluate $\int \frac{1}{x(x^2+4)^2}\,dx$

Check: deg. Num < deg. Denom. ; Deg. Num = 0, Deg. Denom = 5

$0 < 5 \Rightarrow$ Apply PFD (Partial Fraction Decomposition)

$$\frac{1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

Clear out the fractions by multiplying both sides by $x(x^2+4)^2$

$$1 = A(x^2+4)^2 + (Bx+C)(x)(x^2+4) + (Dx+E)x$$

$$1 = A(x^4+8x^2+16) + (Bx+C)(x^3+4x) + Dx^2 + Ex$$

$$1 = (A+B)x^4 + Cx^3 + (8A+4B+D)x^2 + (4C+E)x + 16A$$

$$1 = (A+B)x^4 + Cx^3 + (8A+4B+D)x^2 + (4C+E)x + 16A$$

x^4: $0 = A + B$ Equate equal powers of x

x^3: $0 = C$

x^2: $0 = 8A + 4B + D$

x^1: $0 = 4C + E$

x^0: $1 = 16A$

$A = \frac{1}{16}$, $C = 0$, $B = -A \Rightarrow B = -\frac{1}{16}$; $E = -4C \Rightarrow E = 0$

plug $A = \frac{1}{16}$ and $B = \frac{-1}{16}$ into $0 = 8A + 4B + D$

$$0 = 8\left(\frac{1}{16}\right) + 4\left(\frac{-1}{16}\right) + D \Rightarrow D = \frac{-8}{16} + \frac{4}{16} = \frac{-4}{16} = \frac{-1}{4}$$

$A = 1/16$; $B = -1/16$; $C = 0$; $D = -1/4$; $E = 0$

$A = 1/16$; $B = -1/16$; $C = 0$; $D = -1/4$; $E = 0$

$$\frac{1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

$$\frac{1}{x(x^2+4)^2} = \frac{1/16}{x} + \frac{(-1/16)x}{x^2+4} + \frac{-1/4\, x}{(x^2+4)^2}$$

Partial Fraction Decomposition Completed !!

$$\int \frac{1}{x(x^2+4)^2}\,dx = \int \left(\frac{1/16}{x} - \frac{(1/16)x}{x^2+4} - \frac{(1/4)x}{(x^2+4)^2} \right) dx$$

$$= \frac{1}{16}\int \frac{1}{x}\,dx - \frac{1}{16}\int \frac{x}{x^2+4}\,dx - \frac{1}{4}\int \frac{x}{(x^2+4)^2}\,dx$$

$$= \frac{1}{16} \ln|x| - \frac{1}{16}\int \frac{x}{x^2+4}\,dx - \frac{1}{4}\int \frac{x}{(x^2+4)^2}\,dx$$

$$= \frac{1}{16}\ln|x| - \frac{1}{16}\int \frac{x}{x^2+4}\,dx - \frac{1}{4}\int \frac{x}{(x^2+4)^2}\,dx$$

$u = x^2+4$ $\qquad$ $u = x^2+4$ $\qquad$ Apply U-Subst.

$du = 2x\,dx$ $\qquad$ $du = 2x\,dx$

$du/2 = x\,dx$ $\qquad$ $du/2 = x\,dx$

$$= \frac{1}{16}\ln|x| - \frac{1}{16}\int \frac{1/2}{u}\,du - \frac{1}{4}\int \frac{1/2}{u^2}\,du$$

$$= \frac{1}{16}\ln|x| - \frac{1}{32}\int \frac{1}{u}\,du - \frac{1}{8}\int u^{-2}\,du$$

$$= \frac{1}{16}\ln|x| - \frac{1}{32}\ln|u| - \frac{1}{8}\frac{u^{-1}}{-1} + C$$

Subst. $u = x^2+4$

$$= \frac{1}{16}\ln|x| - \frac{1}{32}\ln|x^2+4| + \frac{1}{8(x^2+4)} + C$$

Evaluate ∫ x^3/((x-2)(x-3)) dx , Partial Fractions integral solved example

Partial Fractions 9

Ex| Evaluate $\int \frac{x^3}{(x-2)(x-3)}\,dx$

Check: Deg. Num $= 3$; Deg. Denom $= 2$; $3 > 2$

Apply Long Division first

$$\frac{x^3}{(x-2)(x-3)} = \frac{x^3}{x^2-5x+6}$$

$$\begin{array}{r l}
 & x+5 \\
x^2-5x+6 \,\big) & x^3+0x^2+0x \\
 & -(x^3-5x^2+6x\ ) \\
\hline
 & 5x^2-6x+0 \\
 & -(5x^2-25x+30) \\
\hline
 & 19x-30
\end{array}$$

$$\frac{x^3}{x^2-5x+6} = \underbrace{x+5}_{\text{Easy to integrate Polynomial}} + \underbrace{\frac{19x-30}{x^2-5x+6}}_{\text{Apply PFD (Partial Fractions)}}$$

$$\int \frac{x^3}{x^2-5x+6}\,dx = \int \left[(x+5) + \frac{19x-30}{x^2-5x+6} \right] dx$$

$$= \int (x+5)\,dx + \int \frac{19x-30}{x^2-5x+6}\,dx$$ split up integrals

$$= \frac{x^2}{2} + 5x + \int \frac{19x-30}{x^2-5x+6}\,dx$$ Integrate easy part

$$\frac{19x-30}{x^2-5x+6} = \frac{19x-30}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$\frac{19x-30}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

Multiply both sides by $(x-3)(x-2)$ to clear out the fractions

$$19x-30 = A(x-2) + B(x-3)$$

plug $x=2$; $8 = A(0) + B(-1) \Rightarrow -B = 8 \Rightarrow B = -8$

plug $x=3$; $27 = A(1) + B(0) \Rightarrow A = 27$

$$\frac{19x-30}{x^2-5x+6} = \frac{19x-30}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2} = \frac{27}{x-3} - \frac{8}{x-2}$$

$$\int \frac{x^3}{x^2-5x+6}\,dx = \int \left[(x+5) + \frac{19x-30}{x^2-5x+6} \right] dx$$

$$= \frac{x^2}{2} + 5x + \int \frac{19x-30}{(x-3)(x-2)}\,dx$$

$$= \frac{x^2}{2} + 5x + \int \frac{19x-30}{(x-3)(x-2)}\,dx$$

split up and integrate easy part x+5

$$= \frac{x^2}{2} + 5x + \int \left(\frac{27}{x-3} - \frac{8}{x-2}\right) dx$$

Apply PFD

$$= \frac{x^2}{2} + 5x + \int \frac{27}{x-3}\,dx - \int \frac{8}{x-2}\,dx$$

Integrate term by term

$$= \frac{x^2}{2} + 5x + 27\int \frac{1}{x-3}\,dx - 8\int \frac{1}{x-2}\,dx$$

$u = x-3$ $\quad$ $u = x-2$

$du = dx$ $\quad$ $du = dx$

U-Substitution

$$= \frac{x^2}{2} + 5x + 27\ln|x-3| - 8\ln|x-2| + C$$

Partial Fractions 10

Ex Evaluate $\int \frac{x^{1/3}+1}{x^{1/3}-1}\,dx$

Strategy : Apply U-Substitution + PFD (Partial Fractions)

$u = x^{1/3} \Rightarrow u^3 = x$ and $3u^2\,du = dx$

$$\int \frac{x^{1/3}+1}{x^{1/3}-1}\,dx = \int \frac{u+1}{u-1}\,3u^2\,du = \int \frac{3u^3+3u^2}{u-1}\,du$$

Since deg. Num $= 3$; deg. Denom $= 1$; $3 > 1$

Apply Long Division First and then apply PFD

$$\int \frac{x^{1/3}+1}{x^{1/3}-1}\,dx = \int \frac{3u^3+3u^2}{u-1}\,du$$

$$\begin{array}{r|l}
 & 3u^2+6u+6 \\
\hline
u-1 & 3u^3+3u^2 \\
 & -(3u^3-3u^2) \\
\hline
 & 6u^2+0u \\
 & -(6u^2-6u) \\
\hline
 & 6u+0 \\
 & -(6u-6) \\
\hline
 & 6
\end{array}$$

$$\frac{3u^3+3u^2}{u-1} = 3u^2+6u+6+\frac{6}{u-1}$$

$$\int \frac{3u^3+3u^2}{u-1}\,du = \int \left[3u^2+6u+6+\frac{6}{u-1}\right]du$$

$$= \int 3u^2du + \int 6u\,du + \int 6\,du + \int \frac{6}{u-1}\,du$$

$$= 3\int u^2du + 6\int u\,du + 6\int 1\,du + 6\int \frac{1}{u-1}\,du$$

$$= \frac{3u^3}{3} + \frac{6u^2}{2} + 6u + 6\ln|u-1| + C$$

$$= u^3 + 3u^2 + 6u + 6\ln|u-1| + C$$ plug $u = x^{1/3}$

$$= (x^{1/3})^3 + 3(x^{1/3})^2 + 6x^{1/3} + 6\ln|x^{1/3}-1| + C$$

$$= x + 3x^{2/3} + 6x^{1/3} + 6\ln|x^{1/3}-1| + C$$

Evaluate ∫ 1/(x^(1/2)-x^(1/3)) dx , Partial Fractions + U-Substitution example

Partial Fractions II

Ex| Evaluate $\int \frac{1}{\sqrt[2]{x} - \sqrt[3]{x}}\, dx$

Strategy : Apply U-Substitution + PFD (Partial Fractions)

Let $u = x^{1/6}$ $\qquad u^2 = (x^{1/6})^2 = x^{2/6} = x^{1/3} = \sqrt[3]{x}$

$u^6 = x$ $\qquad u^3 = (x^{1/6})^3 = x^{3/6} = x^{1/2} = \sqrt{x}$

$6u^5\, du = dx$

$$\int \frac{1}{\sqrt[2]{x} - \sqrt[3]{x}}\, dx = \int \frac{1}{u^3 - u^2} \cdot 6u^5\, du$$

$$\int \frac{1}{\sqrt[2]{x} - \sqrt[3]{x}}\,dx = \int \frac{6u^5}{u^3 - u^2}\,du$$

$$= \int \frac{6u^5}{u^2(u-1)}\,du = \int \frac{6u^3}{u-1}\,du$$

Since deg. Num $= 3$; Deg. Denom $= 1$; $3 > 1$

Apply Long Division First and then apply PFD

$$\begin{array}{r l}
 & 6u^2 + 6u + 6 \\
u - 1 \,\big) & 6u^3 + 0u^2 \\
 & -(6u^3 - 6u^2) \\
\hline
 & 6u^2 + 0u \\
 & -(6u^2 - 6u) \\
\hline
 & 6u + 0 \\
 & -(6u - 6) \\
\hline
 & 6
\end{array}$$

$$\int \frac{6u^3}{u-1}\,du = \int \left(6u^2+6u+6+\frac{6}{u-1}\right) du$$

$$= \int 6u^2\,du + \int 6u\,du + \int 6\,du + \int \frac{6}{u-1}\,du$$

$$= 6\int u^2\,du + 6\int u\,du + 6\int du + 6\int \frac{1}{u-1}\,du$$

$$= \frac{6u^3}{3} + \frac{6u^2}{2} + 6u + 6\ln|u-1| + C$$

$$= 2u^3 + 3u^2 + 6u + 6\ln|u-1| + C$$

$$= 2x^{1/2} + 3x^{1/3} + 6x^{1/6} + 6\ln|x^{1/6}-1| + C$$

$$= 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x}-1| + C$$

Subst. $u = x^{1/6}$

$u^3 = x^{1/2}$

$u^2 = x^{1/3}$

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Evaluate ∫ √(1+e^(x)) dx , Partial Fractions+U-Substitution,Let U=√(1+e^x)

Partial Fractions 12

Ex Evaluate $\int \sqrt{1+e^x}\, dx$

Strategy: Apply U-Substitution + PFD (Partial Fractions)

$u = \sqrt{1+e^x} \Rightarrow u^2 = 1+e^x$ and $e^x = u^2-1$

$2u\,du = e^x dx \Rightarrow dx = \dfrac{2u}{e^x} du = \dfrac{2u}{u^2-1} du$

$$\int \sqrt{1+e^x}\, dx = \int \frac{u \cdot 2u}{u^2-1} du$$

$$= 2\int \frac{u^2}{u^2-1} du$$

deg. Num = deg. Denom = 2

Apply long Division or "shortcut"

$$\int \sqrt{1+e^x}\,dx = 2\int \frac{u^2}{u^2-1}\,du = 2\int \frac{(u^2-1)+1}{u^2-1}\,du$$

$$= 2\int \left(\frac{u^2-1}{u^2-1} + \frac{1}{u^2-1}\right) du = 2\int 1\,du + 2\int \frac{1}{u^2-1}\,du$$

$$= 2u + \int \frac{2}{u^2-1}\,du$$

Apply PFD (Partial Fractions)

$$\frac{2}{u^2-1} = \frac{2}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1} = \frac{A(u+1)+B(u-1)}{(u-1)(u+1)}$$

$$\frac{2}{(u-1)(u+1)} = \frac{A(u+1)+B(u-1)}{(u-1)(u+1)}$$

Clear out the fractions by multiplying both sides by $(u-1)(u+1)$

$$\frac{2}{(u-1)(u+1)} = \frac{A(u+1)+B(u-1)}{(u-1)(u+1)}$$

$$2 = A(u+1) + B(u-1)$$

plug $u=-1 \Rightarrow 2 = A(0) + B(-2) \Rightarrow B=-1$

plug $u=1 \Rightarrow 2 = A(2) + B(0) \Rightarrow A=1$

$$\frac{2}{(u-1)(u+1)} = \frac{1}{u-1} - \frac{1}{u+1}$$

PFD completed

Putting it all together!

$$\int \sqrt{1+e^x}\, dx = 2u + \int \frac{2}{u^2-1}\, du = 2u + \int \left(\frac{1}{u-1} - \frac{1}{u+1}\right) du$$

$$\int \sqrt{1+e^x}\,dx = 2u + \int \left(\frac{1}{u-1} - \frac{1}{u+1}\right) du$$

$$= 2u + \int \frac{1}{u-1}\,du - \int \frac{1}{u+1}\,du$$

split off integrals and integrate term by term.

$$= 2u + \ln|u-1| - \ln|u+1| + C$$

Substitute $u = \sqrt{1+e^x}$

$$= 2\sqrt{1+e^x} + \ln\left|\sqrt{1+e^x} - 1\right| - \ln\left|\sqrt{1+e^x} + 1\right| + C$$

Recall: $\int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a} + C$ U-Subst.

let $t = ax+b$ $dt = a\,dx$

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