Power Series I

Definition: A Power Series has the form of an infinite polynomial: $\sum_{n=0}^{\infty} C_n X^n = C_0 + C_1 X + C_2 X^2 + \cdots + C_n X^n + \cdots$

This is called a Power Series in X, where C_n's are the coefficients and X is the variable.

If we let $C_n = 1$ for all n, the Power Series takes the form of a geometric series:

$\sum_{n=0}^{\infty} X^n = 1 + X + X^2 + X^3 + \cdots + X^n + \cdots$ which converges

if $|x| < 1$ or $-1 < X < 1$ and diverges if $|x| \geq 1$

; $x \geq 1$ or $x \leq -1$.

In general, a Power Series centered at $x=a$ takes the form of an infinite polynomial in powers of $x-a$.

$$\sum_{n=0}^{\infty} C_n(x-a)^n = C_0 + C_1(x-a) + C_2(x-a)^2 + \dots + C_n(x-a)^n + \dots$$

Theorem: Given a power series about $x=a$; $\sum_{n=0}^{\infty} C_n(x-a)^n$; there are only 3 cases to consider:

1] The series $\sum_{n=0}^{\infty} C_n(x-a)^n$ converges only at $x=a$

2] The series $\sum_{n=0}^{\infty} C_n(x-a)^n$ converges for all x in $(-\infty, \infty)$

3] The series $\sum_{n=0}^{\infty} C_n(x-a)^n$ converges for $|x-a|<R$ and Diverges for $|x-a|>R$

R is called the Radius of Convergence.

Definition: The interval of convergence of a power series is $a-R<x<a+R$; however the Ratio Test does not tell us what happens at the endpoints $x=a-R$ and $x=a+R$, so these endpoints have to be plugged in the power series $\sum_{n=0}^{\infty} C_n(x-a)^n$ and apply a test of convergence to find out if the Power Series converges at an endpoint.

Interval of Convergence Diagram

So there are 4 possibilities for the interval of convergence: $(a-R, a+R)$, $(a-R, a+R]$, $[a-R, a+R)$ and $[a-R, a+R]$.

Ex] Find the Radius and Interval of Convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$

$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$; Let's apply the Ratio Test with

$a_n = \frac{(x-2)^n}{n} \Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$= \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{n+1} \Big/ \frac{(x-2)^n}{n} \right| = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(x-2)^n} \right| \frac{n}{n+1}$

$= \lim_{n\to\infty} |x-2| \frac{n}{n+1} = \lim_{n\to\infty} |x-2| \frac{n/n}{\frac{n}{n}+\frac{1}{n}}$

$= \lim_{n\to\infty} |x-2| \frac{1}{1+\frac{1}{n}} = |x-2|$

By the Ratio Test, the given series converges if $|x-2| < 1$ or $-1 < x-2 < 1 \Rightarrow 1 < x < 3$, and diverges if $|x-2| > 1$, so the Radius of Convergence is 1.

Let's check endpoints $x=1$ and $x=3$

Plug $x=1$ into $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ we obtain:

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(1-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$ which is

convergent by the Alternating Series Test.

Since $a_n = \frac{1}{n}$ is decreasing and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$

Plug $x=3$ into $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ we obtain :

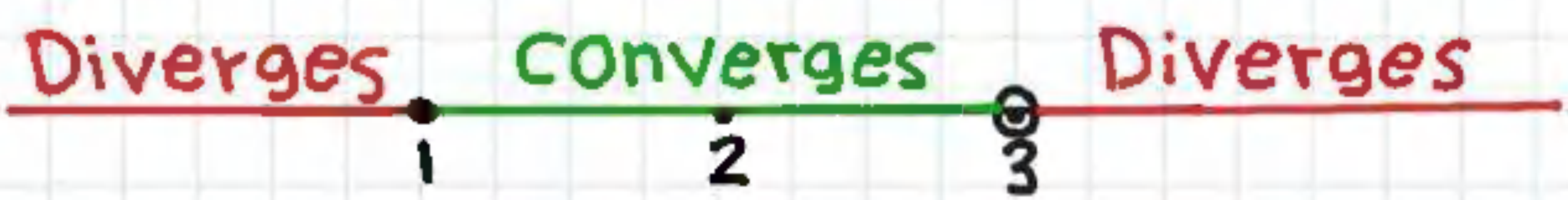
$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(3-2)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$$ which is a

Divergent P series with $P=1$ or a Divergent Harmonic series.

So Include $x=1$ and Exclude $x=3$ for the interval of convergence.

The Interval of Convergence for $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ is :

$1 \leq x < 3$ or $[1, 3)$



Power Series 2

Theorem: Given a Power Series about $x=0$; $\sum_{n=0}^{\infty} C_n x^n$

There are 3 cases to consider:

1] The series $\sum_{n=0}^{\infty} C_n x^n$ converges only at $x=0$

2] The series $\sum_{n=0}^{\infty} C_n x^n$ converges for all x in $(-\infty, \infty)$

3] The series $\sum_{n=0}^{\infty} C_n x^n$ converges for $|x| < R$ and Diverges for $|x| > R$. R is the Radius of convergence.

$-R < x < R$ is the Interval of Convergence.

Ex] Find the Radius and Interval of Convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(-4)^n}$

let's apply the Ratio Test with $a_n = \frac{x^{2n}}{(-4)^n}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{(-4)^{n+1}} \Big/ \frac{x^{2n}}{(-4)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^{2n+2}}{x^{2n}} \cdot \frac{(-4)^n}{(-4)^{n+1}} \right| = \lim_{n\to\infty} \left| x^2 \cdot \frac{1}{-4} \right| = \frac{x^2}{4}$$

By the Ratio Test, the given series converges Absolutely if $\frac{x^2}{4} < 1 \Rightarrow x^2 < 4 \Rightarrow -2 < x < 2$

and Diverges if $\frac{x^2}{4} > 1 \Rightarrow x^2 > 4 \Rightarrow x > 2$ or $x < -2$

So the Radius of Convergence $R = 2$.

Let's check endpoints $x = -2$ and $x = 2$

Plug $x = -2$ into $\sum_{n=1}^{\infty} \frac{x^{2n}}{(-4)^n}$ we obtain:

$$\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{(-4)^n} = \sum_{n=1}^{\infty} \frac{((-2)^2)^n}{(-4)^n} = \sum_{n=1}^{\infty} \frac{\cancel{4^n}}{(-1)^n \cancel{4^n}} = \sum_{n=1}^{\infty} (-1)^n$$

which Diverges by the Divergence Test since

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n = \begin{cases} 1 & \text{for } n \text{ even} \\ -1 & \text{for } n \text{ odd} \end{cases} \quad ; \text{Since}$$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n \neq 0 \Rightarrow \sum_{n=1}^{\infty} \frac{x^{2n}}{(-4)^n}$ Diverges at $x = -2$

Plug $x=2$ into $\sum_{n=1}^{\infty} \frac{x^{2n}}{(-4)^n}$ we obtain:

$$\sum_{n=1}^{\infty} \frac{2^{2n}}{(-4)^n} = \sum_{n=1}^{\infty} \frac{\cancel{(4)^n}}{(-1)^n \cancel{4^n}} = \sum_{n=1}^{\infty} \frac{1}{(-1)^n} = \sum_{n=1}^{\infty} (-1)^n$$

which also Diverges by the Divergence Test:

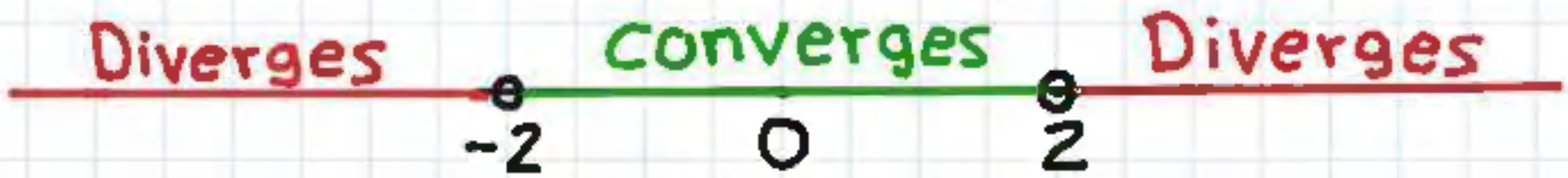
$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n \neq 0 \Rightarrow \sum_{n=0}^{\infty} \frac{x^{2n}}{(-4)^n} \text{ Diverges}$$

at $x=2$.

So Exclude both $x=-2$ and $x=2$ for the Interval of Convergence.

The Interval of Convergence for $\sum_{n=1}^{\infty} \frac{x^{2n}}{(-4)^n}$ is

$-2 < x < 2$ or $(-2, 2)$



Power Series 3

Ex] Find the Radius and Interval of Convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(2x-4)^n}{n}$

Let's apply the Ratio Test with $a_n = \dfrac{2^n(2x-4)^n}{n}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2^{n+1}(2x-4)^{n+1}}{n+1} \Big/ \frac{2^n(2x-4)^n}{n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2x-4)^{n+1}}{(2x-4)^n}\cdot\frac{2^{n+1}}{2^n}\cdot\frac{n}{n+1}\right| = \lim_{n\to\infty}|2x-4|\cdot 2\cdot\frac{n}{n+1}$$

$$= |2x-4| \lim_{n\to\infty} \frac{2n/n}{\frac{n}{n}+\frac{1}{n}} = |2x-4| \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = 2|2x-4|$$

By the Ratio Test, the given series converges Absolutely if $2|2x-4| < 1 \Rightarrow |2x-4| < \frac{1}{2}$

$$\Rightarrow \frac{-1}{2} < 2x-4 < \frac{1}{2} \Rightarrow \frac{-1}{2}+4 < 2x < \frac{1}{2}+4$$

$$\Rightarrow \frac{7}{2} < 2x < \frac{9}{2} \Rightarrow \frac{7}{4} < x < \frac{9}{4}$$ and Diverges

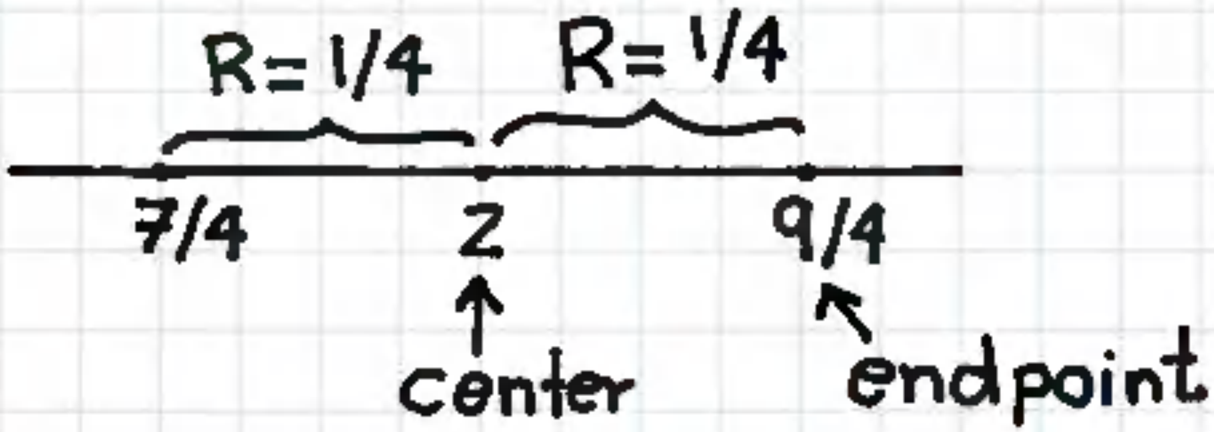
if $|2x-4| > \frac{1}{2} \Rightarrow 2x-4 > \frac{1}{2}$ or $2x-4 < \frac{-1}{2}$

$$\Rightarrow x > \frac{9}{4} \text{ or } x < \frac{7}{4}$$

To find the Radius of Convergence set $(2x-4)^n=0$

$\Rightarrow \sqrt[n]{(2x-4)^n} = \sqrt[n]{0} \Rightarrow 2x-4=0 \Rightarrow x=2$

So Radius of convergence $R=\frac{9}{4}-2=\frac{1}{4}$; Radius of Convergence is the distance from the endpoint to the center of the Power Series.



To find center look at $\sum_{n=1}^{\infty} \frac{2^n(2x-4)^n}{n}$ and set $(2x-4)^n=0$

$x=2$ is the center

let's check endpoints $x = 7/4$ and $x = 9/4$

Plug $x = 7/4$ into $\sum_{n=1}^{\infty} \frac{2^n(2x-4)^n}{n}$ we obtain:

$$\sum_{n=1}^{\infty} \frac{2^n(2(7/4)-4)^n}{n} = \sum_{n=1}^{\infty} \frac{2^n\left(\frac{-1}{2}\right)^n}{n} = \sum_{n=1}^{\infty} \frac{\cancel{2^n}(-1)^n}{\cancel{2^n}\,n}$$

$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ which is Convergent by the Alternating

Series Test since $a_n = \frac{1}{n}$ is decreasing and

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$

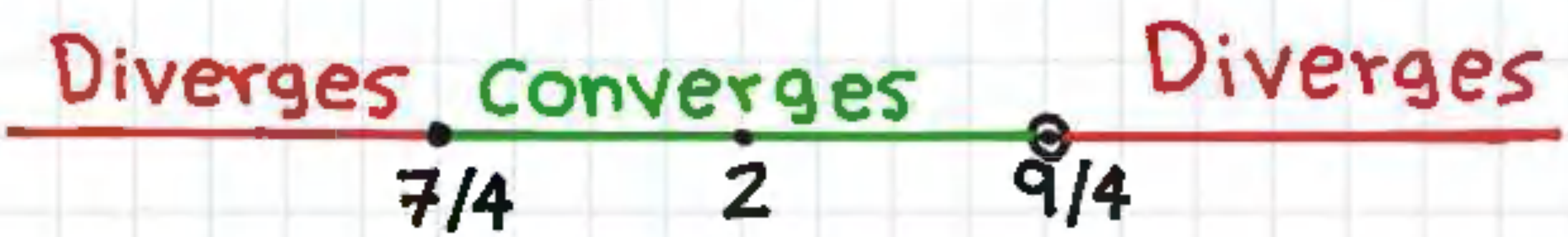
Plug $x = 9/4$ into $\sum_{n=1}^{\infty} \frac{2^n(2x-4)^n}{n}$ we obtain:

$x = 9/4$; $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(2x-4)^n}{n} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n(2(9/4)-4)^n}{n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{2^n(1/2)^n}{n} = \sum_{n=1}^{\infty} \frac{\cancel{2^n}}{\cancel{2^n}\cdot n} = \sum_{n=1}^{\infty} \frac{1}{n}$ which is

a Divergent P series with $P=1$ or a Divergent Harmonic Series. So <u>Include</u> $x = 7/4$ and <u>Exclude</u> $x = 9/4$ for the Interval of Convergence.

The Interval of Convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(2x-4)^n}{n}$ is $\frac{7}{4} \le x < \frac{9}{4}$ or $[7/4, 9/4)$



Power Series 4

Ex] Find the Radius and Interval of Convergence of the following Power Series.

A] $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{2n+1}(n+1)!\,n!}$; Also Known as Bessel Function of Order 1

B] $\sum_{n=1}^{\infty} n!\,(3x-9)^n$

A] $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{2n+1}(n+1)!\,n!}$

Let's apply the Ratio Test with $a_n = \dfrac{(-1)^n x^{2n+1}}{2^{2n+1}(n+1)!\,n!}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2(n+1)+1}}{2^{2(n+1)+1}(n+2)!(n+1)!} \Big/ \frac{(-1)^n x^{2n+1}}{2^{2n+1}(n+1)!\,n!}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^{2n+3}}{x^{2n+1}}\cdot\frac{(-1)^{n+1}}{(-1)^n}\cdot\frac{2^{2n+1}}{2^{2n+3}}\cdot\frac{\cancel{(n+1)!}\;\cancel{n!}}{(n+2)\cancel{(n+1)!}\;(n+1)\cancel{n!}}\right|$$

$$= \lim_{n\to\infty}\left| x^2\cdot(-1)\cdot\frac{1}{2^2}\cdot\frac{1}{(n+2)(n+1)}\right| = \lim_{n\to\infty}\left|\frac{-x^2}{4(n+2)(n+1)}\right|$$

$$= \lim_{n\to\infty} \frac{x^2}{4(n+2)(n+1)} = 0 < 1$$

By the Ratio Test the given series converges for all real x values since limit of the Ratio $\left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$ regardless of the values of x. The Interval of Convergence is $(-\infty, \infty)$ and the Radius of Convergence is infinite. $(R = \infty)$

B] $\sum_{n=1}^{\infty} n!(3x-9)^n$

Let's apply the Ratio Test with $a_n = n!(3x-9)^n$

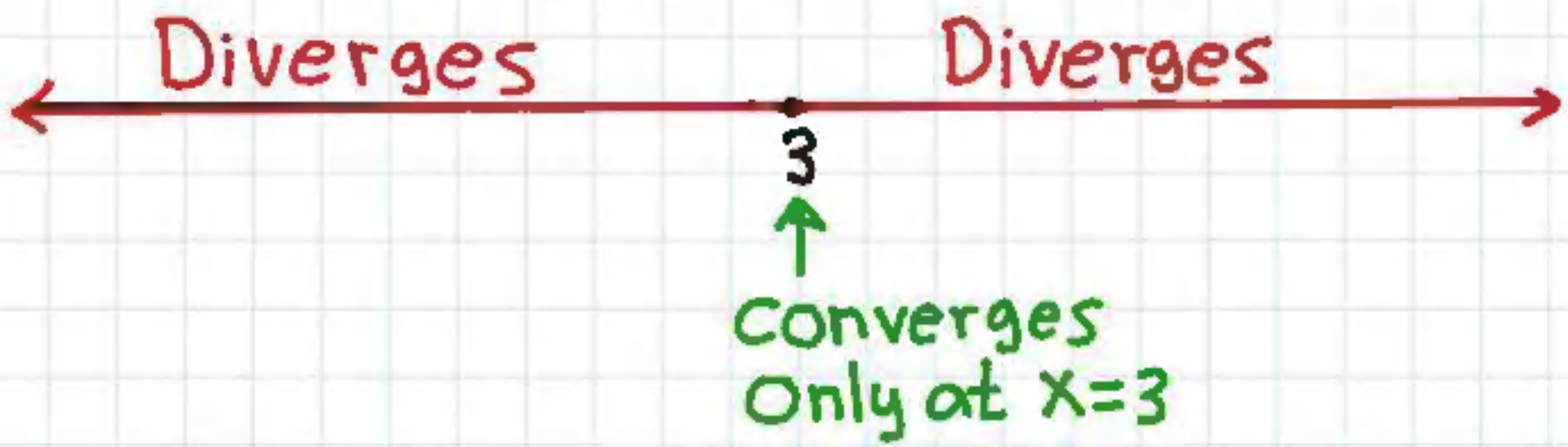
$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!(3x-9)^{n+1}}{n!(3x-9)^n}\right|$$

$$= \lim_{n\to\infty}|3x-9|\frac{(n+1)!}{n!} = \lim_{n\to\infty}|3x-9|\frac{(n+1)\cdot\cancel{n!}}{\cancel{n!}}$$

$= |3x-9| \cdot \lim_{n\to\infty}(n+1) = \infty$ for $x \neq 3$; the given series converges only if $|3x-9| = 0 \Rightarrow 3x-9=0 \Rightarrow x=3$

So the Radius of Convergence is $R=0$, the Interval of convergence is $\{3\}$; Only $x=3$

Conclusion: This series $\sum_{n=1}^{\infty} n!\,(3x-9)^n$ only converges at $x=3$ and Diverges otherwise.



Power Series 5

EX] Find the Radius and Interval of Convergence of the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (x+1)^n 3^n}{n^2-1}$

Let's apply the Ratio Test with $a_n = \dfrac{(-1)^n (x+1)^n 3^n}{n^2-1}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x+1)^{n+1} 3^{n+1}}{(n+1)^2-1} \Big/ \frac{(-1)^n (x+1)^n 3^n}{n^2-1} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{(x+1)^n} \cdot \frac{3^{n+1}}{3^n} \cdot \frac{(-1)^{n+1}}{(-1)^n} \cdot \frac{n^2-1}{(n+1)^2-1} \right|$$

$$= \lim_{n\to\infty} \left| (x+1).3(-1).\frac{(n^2-1)}{n^2+2n} \right| = \lim_{n\to\infty} \left| x+1 \right| .3\,\frac{(n^2-1)}{n^2+2n}$$

$$= \left| x+1 \right| \lim_{n\to\infty} \frac{3(n^2-1)}{n^2+2n} = \left| x+1 \right| \lim_{n\to\infty} \frac{3(n^2-1)/n^2}{\frac{n^2}{n^2}+\frac{2n}{n^2}}$$

$$= \left| x+1 \right| \lim_{n\to\infty} \frac{3(n^2/n^2-1/n^2)}{n^2/n^2+2n/n^2} = \left| x+1 \right| \lim_{n\to\infty} \frac{3(1-1/n^2)}{1+2/n}$$

$$= \left| x+1 \right| . 3$$

By the Ratio Test, the given series converges Absolutely if $3|x+1| < 1 \Rightarrow |x+1| < 1/3$

$|x+1| < \frac{1}{3} \Rightarrow \frac{-1}{3} < x+1 < \frac{1}{3} \Rightarrow -\frac{1}{3} - 1 < x < \frac{1}{3} - 1$

$\frac{-4}{3} < x < \frac{-2}{3}$ and Diverges if $|x+1| > \frac{1}{3}$

$\Rightarrow x+1 > \frac{1}{3}$ or $x+1 < \frac{-1}{3} \Rightarrow x > \frac{-2}{3}$ or $x < \frac{-4}{3}$

To find the Radius of Convergence set $(x+1)^n = 0$

$\Rightarrow \sqrt[n]{(x+1)^n} = \sqrt[n]{0} \Rightarrow x+1=0 \Rightarrow x=-1$; Radius of Convergence is the distance from the endpoint to the center of the series. So $R = \frac{-2}{3} - -1 = \frac{-2}{3} + 1 = \frac{1}{3}$

Or simply look at $|x+1| < \frac{1}{3}$ from Ratio Test.

let's check endpoints $x=-4/3$ and $x=-2/3$

Plug $x=-4/3$ into $\sum_{n=2}^{\infty} \frac{(-1)^n (x+1)^n 3^n}{n^2-1}$ we obtain:

$$\sum_{n=2}^{\infty} \frac{(-1)^n \left(\frac{-4}{3}+1\right)^n 3^n}{n^2-1} = \sum_{n=2}^{\infty} \frac{(-1)^n \left(\frac{-1}{3}\right)^n 3^n}{n^2-1}$$

$$= \sum_{n=2}^{\infty} \frac{(-1)^n (-1)^n \cancel{3^n}}{\cancel{3^n}(n^2-1)} = \sum_{n=2}^{\infty} \frac{(-1)^{2n}}{n^2-1} = \sum_{n=1}^{\infty} \frac{1}{n^2-1}$$

Since $\frac{1}{n^2-1} \approx \frac{1}{n^2}$ for Large n

Apply Limit Comparison Test with $b_n = \frac{1}{n^2}$ and

$a_n = \frac{1}{n^2-1}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{n^2-1} \Big/ \frac{1}{n^2} = \lim_{n\to\infty} \frac{n^2}{n^2-1}$$

$$= \lim_{n\to\infty} \frac{n^2/n^2}{\frac{n^2}{n^2}-\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1}{1-1/n^2} = 1 > 0$$

Since $\lim\limits_{n\to\infty} \frac{a_n}{b_n}$ exists and $\sum\limits_{n=2}^{\infty} \frac{1}{n^2}$ is a Convergent

P-series with $P=2>1$, the given series $\sum\limits_{n=2}^{\infty} \frac{1}{n^2-1}$

also converges by the Limit Comparison Test.

so Power Series $\sum\limits_{n=2}^{\infty} \frac{(-1)^n (x+1)^n 3^n}{n^2-1}$ converges

at $x = -4/3$

Plug $x = -2/3$ into $\sum_{n=2}^{\infty} \frac{(-1)^n (x+1)^n 3^n}{n^2-1}$ we obtain:

$$\sum_{n=2}^{\infty} \frac{(-1)^n \left(\frac{-2}{3}+1\right)^n 3^n}{n^2-1} = \sum_{n=2}^{\infty} \frac{(-1)^n \left(\frac{1}{3}\right)^n 3^n}{n^2-1}$$

$$= \sum_{n=2}^{\infty} \frac{(-1)^n \cancel{3^n}}{\cancel{3^n}(n^2-1)} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2-1}$$; By applying the

Absolute Convergence Test:

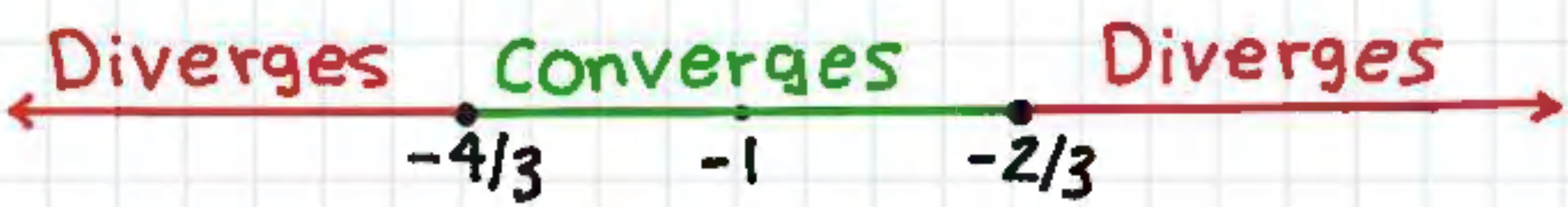
$\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{n^2-1} \right| = \sum_{n=2}^{\infty} \frac{1}{n^2-1}$ which was proven to be convergent earlier. (Limit Comparison Test with $b_n = \frac{1}{n^2}$)

So $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2-1}$ also converges.

So Include both endpoints $x=-4/3$ and $x=-2/3$ for the Interval of Convergence.

The Interval of Convergence for $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (x+1)^n 3^n}{n^2-1}$

is $\dfrac{-4}{3} \le x \le \dfrac{-2}{3}$ or $\left[\dfrac{-4}{3}, \dfrac{-2}{3}\right]$



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Power Series 6

Find the Radius and Interval of Convergence of the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{2^n \ln n}$

Let's apply the Ratio Test with $a_n = \dfrac{(-1)^n x^n}{2^n \ln n}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2^{n+1} \ln(n+1)} \Big/ \frac{(-1)^n x^n}{2^n \ln n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{(-1)^n} \cdot \frac{x^{n+1}}{x^n} \cdot \frac{\ln n}{\ln(n+1)} \cdot \frac{2^n}{2^{n+1}} \right|$$

$$= \lim_{n\to\infty} \left| -x \cdot \frac{Ln\, n}{Ln(n+1)} \cdot \frac{1}{2} \right| = |x| \cdot \lim_{n\to\infty} \frac{Ln\, n}{2Ln(n+1)}$$

Since both Numerator $Ln\, n$ and Denominator $2Ln(n+1)$ approach ∞ as $n \to \infty \Rightarrow$ Apply L'Hopital's Rule

$$|x| \lim_{n\to\infty} \frac{(Ln\, n)'}{(2Ln(n+1))'} = |x| \lim_{n\to\infty} \frac{1/n}{2/(n+1)}$$

$$= |x| \lim_{n\to\infty} \frac{n+1}{2n} = |x| \lim_{n\to\infty} \frac{(n+1)/n}{2n/n} = |x| \lim_{n\to\infty} \frac{1+1/n}{2}$$

$= \frac{|x|}{2}$; By the Ratio Test, the given series converges

Absolutely if $\frac{|x|}{2} < 1 \Rightarrow |x| < 2 \Rightarrow -2 < x < 2$

and Diverges if $\frac{|x|}{2} > 1 \Rightarrow |x| > 2 \Rightarrow x > 2$ or $x < -2$

So the Radius of Convergence is $R = 2$.

Let's check endpoints $x = -2$ and $x = 2$

Plug $x = -2$ into $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{2^n \ln n}$ we obtain:

$$\sum_{n=2}^{\infty} \frac{(-1)^n(-2)^n}{2^n \ln n} = \sum_{n=2}^{\infty} \frac{(-1)^n(-1)^n \cancel{2^n}}{\cancel{2^n} \ln n} = \sum_{n=2}^{\infty} \frac{(-1)^{2n}}{\ln n} = \sum_{n=2}^{\infty} \frac{1}{\ln n}$$

since $\ln n < n$ For $n \geq 2 \Rightarrow \frac{1}{\ln n} > \frac{1}{n}$ for $n \geq 2$

Apply Basic comparison Test with $\sum_{n=2}^{\infty} b_n = \sum_{n=2}^{\infty} \frac{1}{n}$

Since $\sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum_{n=2}^{\infty} \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{n}$ is a Divergent P-series with $P=1$ or Divergent Harmonic Series, the given series $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ Diverges by the Basic Comparison Test.

Plug $x=2$ into $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{2^n \ln n}$ we obtain:

$\sum_{n=2}^{\infty} \frac{(-1)^n \cancel{2^n}}{\cancel{2^n} \ln n} = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ which is Convergent

by the Alternating Series Test since $a_n = \frac{1}{\ln n}$ is decreasing and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\ln n} = 0$

So Exclude $x=-2$ and Include $x=2$ for the Interval of Convergence.

The Interval of Convergence for $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{2^n \ln n}$

is $-2 < x \leq 2$ or $(-2, 2]$

Power Series 7

EX] Suppose that $\sum_{n=0}^{\infty} C_n x^n$ converges when $x=-5$ and Diverges when $x=7$. What can you conclude about the Convergence or Divergence of the following Series?

A] $\sum_{n=0}^{\infty} C_n$

B] $\sum_{n=0}^{\infty} C_n (5)^n$

C] $\sum_{n=0}^{\infty} (-1)^n C_n 9^n$

D] $\sum_{n=0}^{\infty} C_n (-4.9)^n$

A] $\sum_{n=0}^{\infty} C_n$

The center of the power Series $\sum_{n=0}^{\infty} C_n X^n$ is 0

The Interval of convergence has the form $(-a, a)$

Since we know that the series $\sum_{n=0}^{\infty} C_n X^n$ converges at $X = -5$; the Radius of convergence is at least as large as $R = 5$, so the given Series $\sum_{n=0}^{\infty} C_n X^n$ must converge for all X in $(-5, 5)$,

Plug $X = 1$ into $\sum_{n=0}^{\infty} C_n X^n$ to obtain $\sum_{n=0}^{\infty} C_n$

And Since $X = 1$ is inside the interval $(-5, 5)$

the given series $\sum_{n=0}^{\infty} C_n$ Converges.

B] $\sum_{n=0}^{\infty} C_n 5^n$; Since we know that $\sum_{n=0}^{\infty} C_n X^n$ converges at $X=-5$, then $\sum_{n=0}^{\infty} C_n X^n$ converges for all X in $(-5,5)$, since the Radius of Convergence R is at least as large as $R=5$, however we dont know anything about convergence of a Power Series at an endpoint, so $\sum_{n=0}^{\infty} C_n 5^n$ may Converge or Diverge. (we dont know!)

So Interval of Convergence has 2 possibilities

$[-5,5)$ or $[-5,5]$; We cannot conclude anything about Convergence of $\sum_{n=0}^{\infty} C_n 5^n$

$\underline{C]}$ $\sum_{n=0}^{\infty} (-1)^n C_n 9^n$; The center of $\sum_{n=0}^{\infty} C_n X^n$ is 0

Since we know that $\sum_{n=0}^{\infty} C_n X^n$ converges at $X=-5$ and Diverges at $X=7$, the Radius of Convergence R must satisfy $5 \leq R \leq 7$, so Interval of Convergence must be smaller than $(-7,7)$ since we are given that $\sum_{n=0}^{\infty} C_n X^n$ Diverges at $X=7$.

let's rewrite $\sum_{n=0}^{\infty} (-1)^n C_n 9^n$ as $\sum_{n=0}^{\infty} C_n (-9)^n$

$$\sum_{n=0}^{\infty} (-1)^n C_n 9^n = \sum_{n=0}^{\infty} C_n (-9)^n$$

Plug $x=-9$ into $\sum_{n=0}^{\infty} C_n x^n$ to obtain $\sum_{n=0}^{\infty} C_n (-9)^n$

Since $x=-9$ is outside the Interval $(-7,7)$ the given series $\sum_{n=0}^{\infty} (-1)^n C_n 9^n$ Diverges.

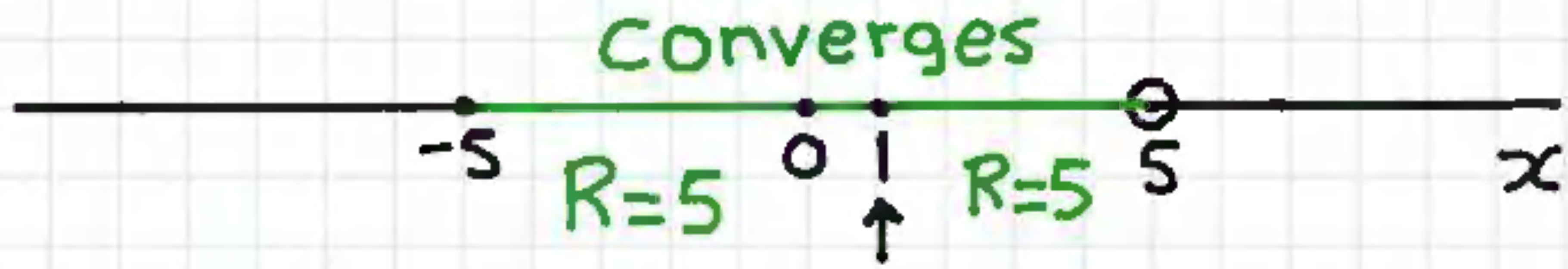
D] $\sum_{n=0}^{\infty} C_n (-4.9)^n$; The center of the series $\sum_{n=0}^{\infty} C_n x^n$ is 0. Since we know that the series $\sum_{n=0}^{\infty} C_n x^n$ converges at $x=-5$; the Radius of Convergence is at least as large as $R=5$, so the power series $\sum_{n=0}^{\infty} C_n x^n$ must converge for all x in $(-5,5)$, Plug $x=-4.9$ into $\sum_{n=0}^{\infty} C_n x^n$ to obtain $\sum_{n=0}^{\infty} C_n(-4.9)^n$, and since $x=-4.9$ is

inside the interval $(-5,5)$ the given series $\sum_{n=0}^{\infty} C_n(-4.9)^n$ Converges.

Check out the Interval of Convergence Diagrams on the next 2 pages!

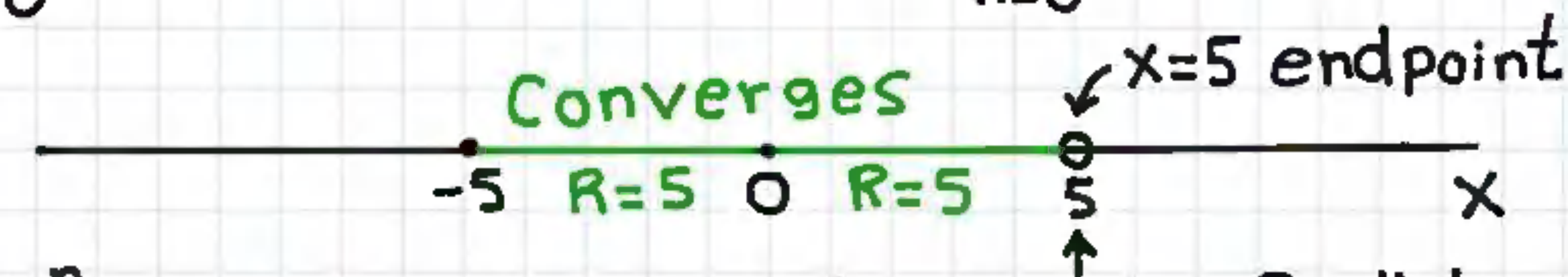
Interval of Convergence Diagrams

A) $\sum_{n=0}^{\infty} C_n$ Plug $x=1$ into $\sum_{n=0}^{\infty} C_n x^n$



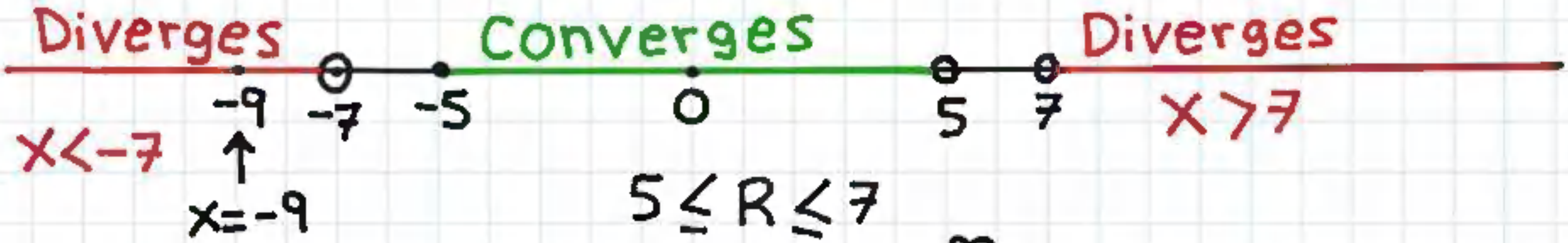
$x=1$ is included in $(-5,5) \Rightarrow \sum_{n=0}^{\infty} C_n$ converges

B) $\sum_{n=0}^{\infty} C_n 5^n$ Plug $x=5$ into $\sum_{n=0}^{\infty} C_n x^n$



$\sum_{n=0}^{\infty} C_n 5^n$ may converge or Diverge, We Don't know ?

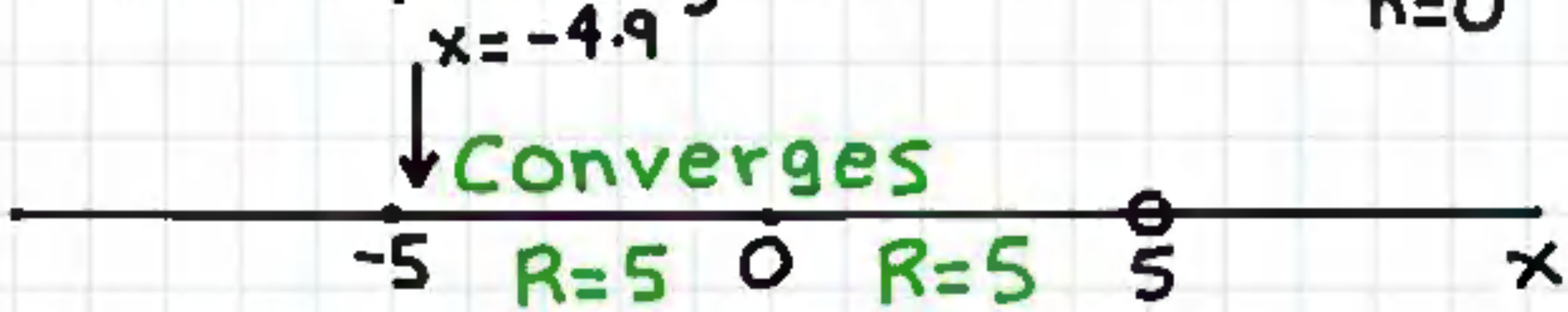
C] $\sum_{n=0}^{\infty}(-1)^n C_n 9^n = \sum_{n=0}^{\infty} C_n(-9)^n$; Plug $x=-9$ in $\sum_{n=0}^{\infty} C_n x^n$



$5 \leq R \leq 7$

Since $x=-9$ is outside $(-7,7) \Rightarrow \sum_{n=0}^{\infty}(-1)^n C_n 9^n$ Diverges

D] $\sum_{n=0}^{\infty} C_n(-4.9)^n$ Plug $x=-4.9$ into $\sum_{n=0}^{\infty} C_n x^n$



Since $x=-4.9$ is inside $(-5,5) \Rightarrow \sum_{n=0}^{\infty} C_n(-4.9)^n$ converges

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