6]

$$f(x,y,z) = x^3\cos(y+z)$$

$$2 \le x \le 3 \qquad 0 \le y \le \pi \qquad 0 \le z \le \pi$$

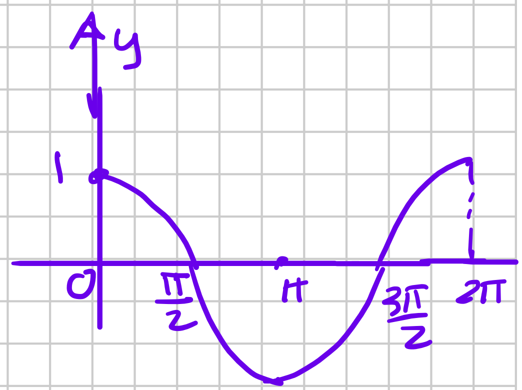
$$\int_2^3 \int_0^\pi \int_0^\pi \underline{x^3\cos(y+z)}\, dz\, dy\, dx$$

$$\int_3^2 \int_0^\pi x^3\sin(y+z)\Big|_0^\pi \, dy\, dx$$

$$\int_2^3 \int_0^\pi \left(x^3\sin(y+\pi) - x^3\sin(y) \right) dy\, dx$$

$$\int_2^3 -x^3\cos(y+\pi) + x^3\cos(y)\Big|_0^\pi \, dx$$

$$\int_2^3 \left[-x^3\overset{=1}{\cos(2\pi)} + x^3\overset{=-1}{\cos(\pi)} \right] - \left[-x^3\overset{=-1}{\cos(\pi)} + x^3\overset{=1}{\cos(0)} \right] dx$$



$$\int_2^3 (-x^3 - x^3 - x^3 - x^3)\, dx$$

$$\int_2^3 -4x^3\, dx = -\cancel{4}\frac{x^4}{\cancel{4}}\Big|_2^3$$

$$-[3^4 - 2^4] = \underline{\underline{2^4 - 3^4}}$$

9)

$$\int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx$$

$$\boxed{0 < z < y < x < 1} \quad **$$

$$0 < y < x$$

$$0 < x < 1$$

$$\int_0^1 \int_z^1 \int_y^1 f \; dx\,dy\,dz$$

$$\int_0^1 \int_0^x \int_z^x f\,dy\,\underline{dz\;dx}$$

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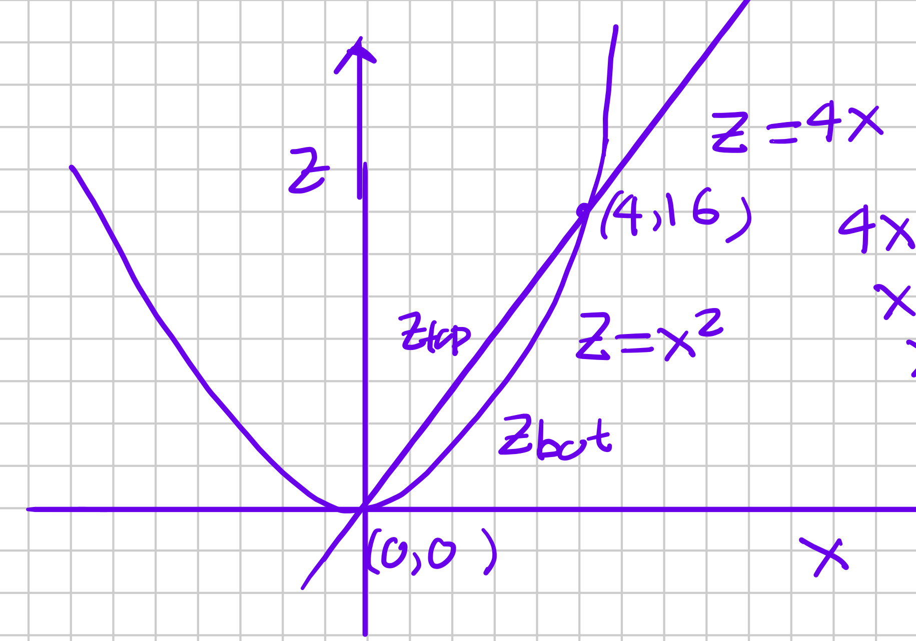
10)

$$z = 4x \qquad z = x^2$$

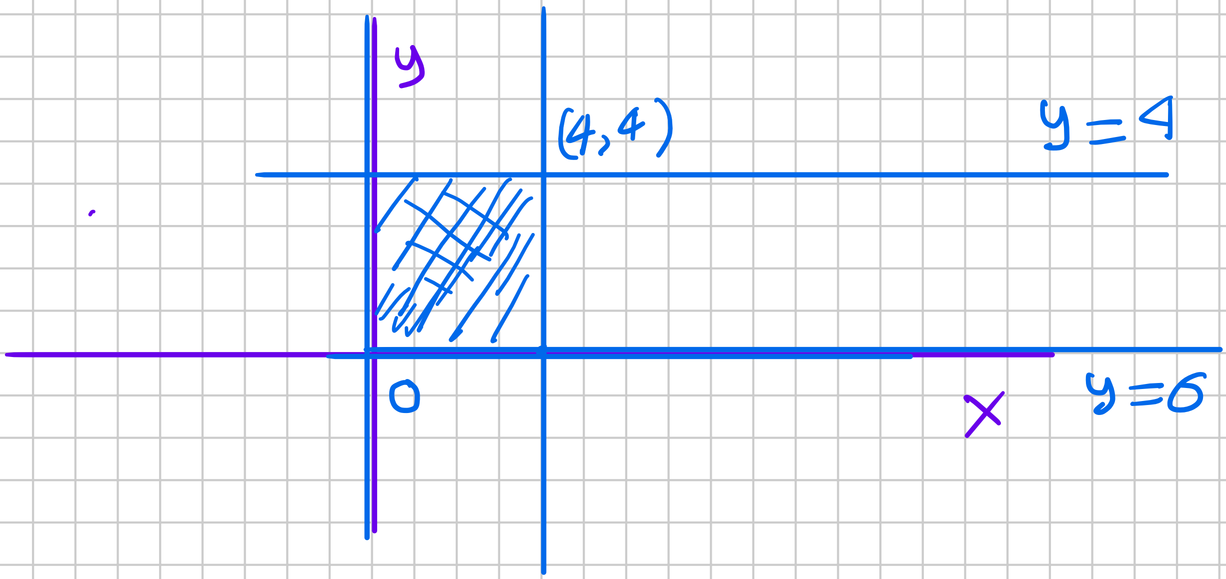
$$y = 0 \qquad y = 4$$



$4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$



$z = z$

$4x = x^2$

$x^2 - 4x = 0$

$x = 0 \quad x = 4$

$$\int_0^4 \int_0^4 \int_{x^2}^{4x} f \; dz\,dy\,dx$$

11)

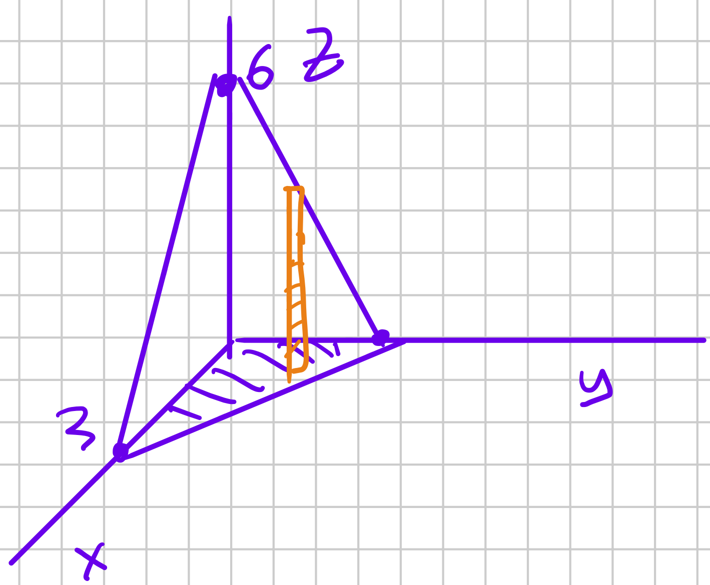
xy   yz   xz

$$\frac{x}{3} + \frac{y}{2} + \frac{z}{6} = 1 \qquad f(x,y) = x+y$$

$x=0 \quad y=0 \quad z=6$

$x=3 \quad y=0 \quad z=0$

$x=0 \quad y=2 \quad z=0$



xy plane

$z=0$

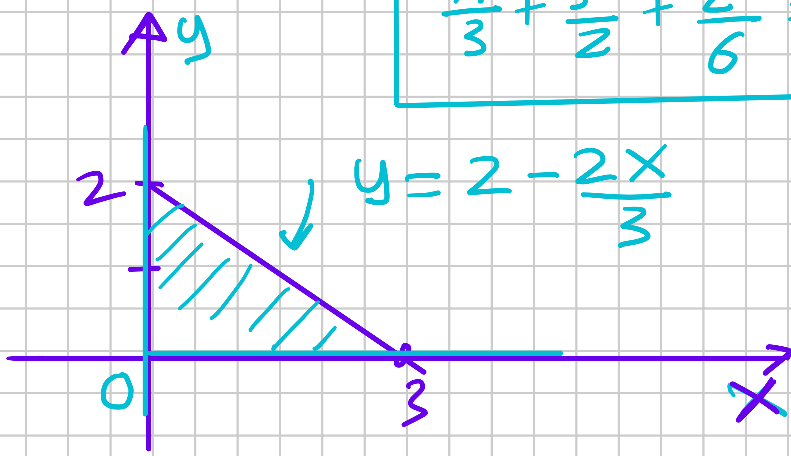
$x=0$
yz

$y=0$
xz

$$z=0 \qquad \frac{x}{3} + \frac{y}{2} = 1$$

$$\frac{y}{2} = 1 - \frac{x}{3} \qquad y = 2 - \frac{2x}{3}$$

$$\boxed{\frac{x}{3} + \frac{y}{2} + \frac{z}{6} = 1}$$



$$y = 2 - \frac{2x}{3}$$

$$\int_0^3 \int_0^{2-\frac{2x}{3}} \int_0^{6-2x-3y} (x+y)\,dz\,dy\,dx$$

$$\frac{z}{\cancel{6}} = \left(1 - \frac{x}{3} - \frac{y}{2}\right) 6$$

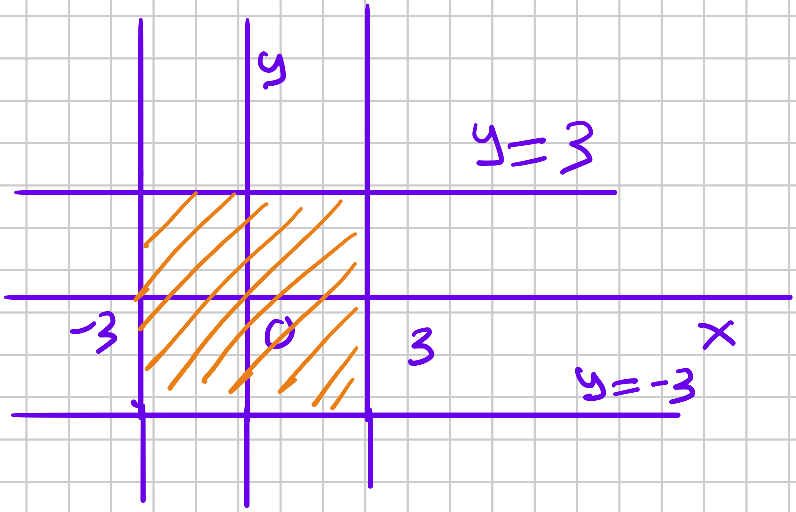
$$z = 6 - \frac{6x}{3} - \frac{6y}{2}$$

$$z = 6 - 2x - 3y$$

12)

$$\iiint x^4 e^y \, dV$$

$$z = 9 - y^2 \qquad z = 0 \qquad x = 3 \qquad x = -3$$



$$z = 0$$

$$9 - y^2 = 0$$

$$y^2 = 9$$

$$y = \pm 3$$

$$0 \le z \le 9-y^2$$

$$-3 \le y \le 3$$

$$-3 \le x \le 3$$

$$\int_{-3}^{3}\int_{-3}^{3}\int_{0}^{9-y^2} x^4 e^y \, dz \, dy \, dx$$

$$x^4 e^y z \Big|_0^{9-y^2}$$

$$\int_{-3}^{3}\int_{-3}^{3} x^4 e^y (9-y^2) \, dy \quad dx$$

$$\int_{-3}^{3} \boxed{\int_{-3}^{3} e^y (9-y^2) \, dy} \; x^4 \, dx$$

$u = 9-y^2 \qquad du = -2y \, dy$

$dv = e^y \, dy \qquad v = e^y$

$$e^y(9-y^2) - \int e^y \cdot (-2y) \, dy$$

$$e^y(9-y^2) + 2 \boxed{\int y e^y \, dy}$$

$u = y \quad du = dy$

$dv = e^y dy \quad v = e^y$

$$\downarrow\; ye^y - \int e^y\,dy$$

$$e^y(9-y^2) + 2(ye^y - e^y)$$

$$e^y(9-y^2) + 2ye^y - 2e^y \Big|_{-3}^{3}$$

$$e^3(0) + \underbrace{6e^3} - \underbrace{2e^3} - [0 - 6e^{-3} - 2e^{-3}]$$

$$\int_{-3}^{3} (4e^3 + 8e^{-3})\; x^4\,dx$$

$$(4e^3 + 8e^{-3}) \boxed{\int_{-3}^{3} x^4\,dx}$$

$$\frac{x^5}{5}\Big|_{-3}^{3}$$

$$\frac{3^5}{5} - \frac{(-3)^5}{5}$$

$$\left(\frac{3^5}{5} + \frac{3^5}{5}\right)(4e^3 + 8e^{-3})$$

Web 12)

1) Cartesian

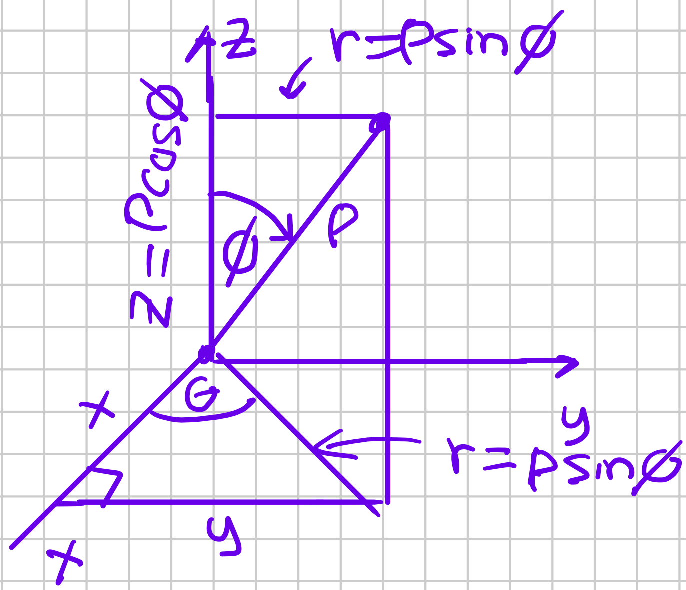
2) $\iiint z\, dV \qquad 1 \le x \le 2$

Cartesian

3) Spherical

4) Polar

Spherical



$z = \rho \cos \phi$

$\cos \theta = \dfrac{x}{\rho \sin \phi} \qquad x = \rho \sin \phi \cos \theta$

$\sin \theta = \dfrac{y}{\rho \sin \phi} \qquad y = \rho \sin \phi \sin \theta$

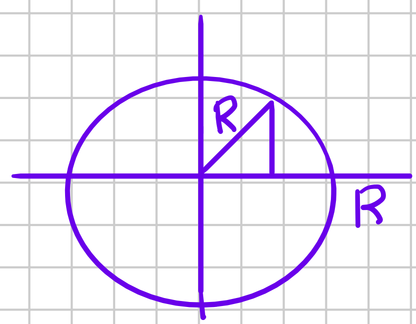
$x^2 + y^2 + z^2 = \rho^2 \qquad x^2 + y^2 = r^2$

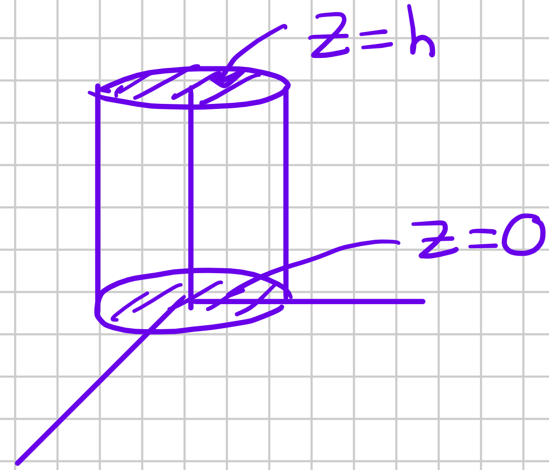
Cylindrical   Polar + z

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

Find Volume of Cylinder using cylindrical coordinates

radius R height H

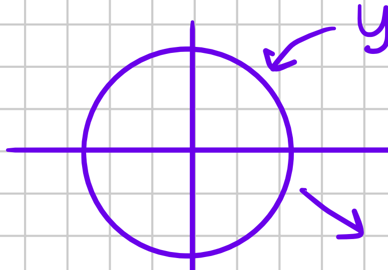




$$V = \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{0}^{H} 1 \, dz\, dy\, dx$$

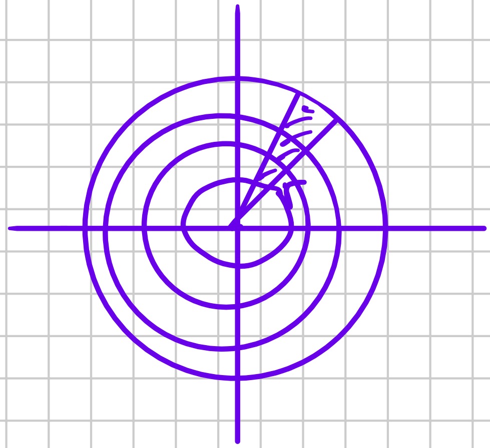
$x^2 + y^2 = R^2$

$y = \sqrt{R^2 - x^2}$



$y = -\sqrt{R^2 - x^2}$

$$V = \int_0^{2\pi} \int_0^{R} \int_0^{H} 1 \cdot dz \, \overbrace{r \, dr \, d\theta}^{dA}$$



$$= \int_0^{2\pi} \int_0^{R} zr \Big|_0^H dr \, d\theta$$

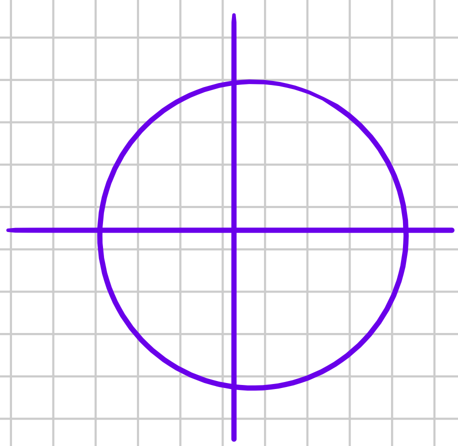
$$= \int_0^{2\pi} \int_0^{R} Hr \; dr \, d\theta$$

$$= \int_0^{2\pi} \frac{Hr^2}{2} \Big|_0^R d\theta = \int_0^{2\pi} \frac{HR^2}{2} d\theta$$

$$= \frac{HR^2}{2} \theta \Big|_0^{2\pi} = \frac{HR^2}{2} (\cancel{2}\pi) = HR^2 \pi$$

$$= \underline{\pi R^2 H}$$

Volume of sphere



$$x^2 + y^2 + z^2 = R^2$$

$$z^2 = R^2 - x^2 - y^2$$

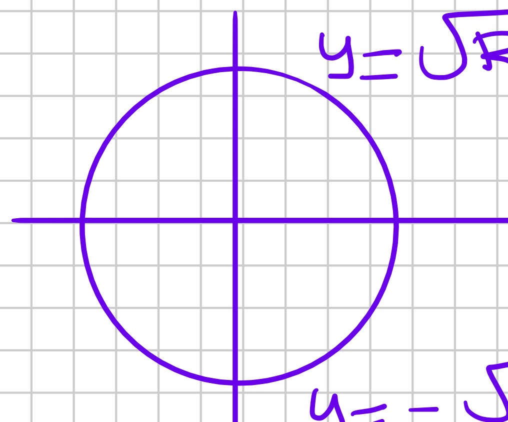
$$z = \pm\sqrt{R^2 - x^2 - y^2}$$

$$V = \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_{-\sqrt{R^2-x^2-y^2}}^{\sqrt{R^2-x^2-y^2}} 1 \cdot dz\, dy\, dx$$

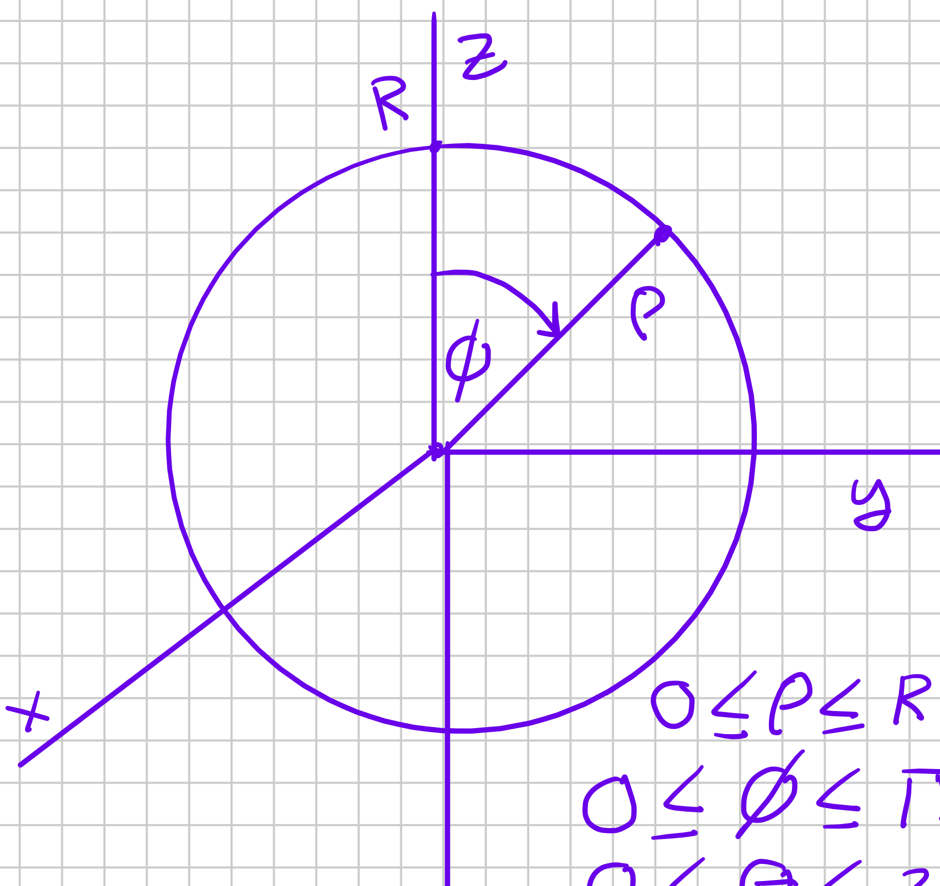
$$x^2+y^2+z^2=R^2$$

$$z=0 \qquad x^2+y^2=R^2$$

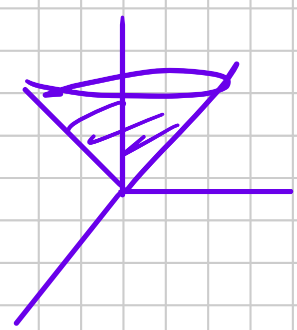
$y=\sqrt{R^2-x^2}$



$y=-\sqrt{R^2-x^2}$



$\phi=\pi/3$



$0 \le \rho \le R$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^{R} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{\rho^3}{3} \sin\phi \Big|_0^R \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{R^3}{3} \sin\phi \, d\phi \, d\theta = \int_0^{2\pi} -\frac{R^3}{3} \cos\phi \Big|_0^{\pi} \, d\theta$$

$$= \int_0^{2\pi} -\frac{R^3}{3} \overbrace{\cos(\pi)}^{=-1} + \frac{R^3}{3} \overbrace{\cos(0)}^{=1} \, d\theta = \int_0^{2\pi} \frac{R^3}{3} + \frac{R^3}{3} \, d\theta$$

$$= \int_0^{2\pi} \frac{2R^3}{3} \, d\theta = \frac{2R^3}{3} \theta \Big|_0^{2\pi} = \frac{2R^3}{3}(2\pi) = \frac{4R^3\pi}{3}$$

$$= \frac{4\pi R^3}{3}$$

2)

$$\iiint \underbrace{(x^2 + y^2 + z^2)}^{\rho^2} \, dV$$

$$x^2 + y^2 + z^2 \leq 1$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{1} \rho^2 \cdot \rho^2 \sin\phi \cdot d\rho \, d\phi \, d\theta$$

$$\frac{\rho^5}{5}\Big|_0^1$$

$$\int_0^{2\pi}\int_0^{\pi}\frac{1}{5}\sin\phi\,d\phi\,d\theta$$

$$\int_0^{2\pi}-\frac{1}{5}\cos\phi\Big|_0^{\pi}\,d\theta$$

$$\int_0^{2\pi}-\frac{1}{5}[\underbrace{\cos\pi}_{-1}-\underbrace{\cos 0}_{1}]\,d\theta$$

$$-\frac{1}{5}(-2)\int_0^{2\pi}1\,d\theta$$

$$\frac{2}{5}\,\theta\Big|_0^{2\pi}$$

$$\boxed{\frac{4\pi}{5}}$$