Trignometric Integrals I

Strategy: Apply trig. Identity $\sin^2 x + \cos^2 x = 1$

and U-Substitution $u = \sin x$ or $u = \cos x$

$\int \sin^m x \cos^n x \, dx$ m, n integers

If either m or n is odd reduce odd power by 1, save a factor of sinx or cosx and apply $\sin^2 x + \cos^2 x = 1$

$\cos^0 x = (\cos x)^0 = 1$

Ex Integrate $\int \sin^3 x \, dx$ $m = 3$ $n = 0$

$\int \sin^2 x \sin x \, dx$ split off factor of sinx

Evaluate ∫ (sinx)^3 dx , Trigonometric integral solved example

$\int \sin^2 x \sin x \, dx$ reduce odd power by 1

$\int (1-\cos^2 x) \sin x \, dx$ $\sin^2 x = 1-\cos^2 x$

$u = \cos x \quad du = -\sin x \, dx$ U-Substitution

$-\int (1-u^2) \, du = -\left[u - \frac{u^3}{3}\right] + C$

$= -u + \frac{u^3}{3} + C$

$= \boxed{-\cos x + \frac{\cos^3 x}{3} + C}$ $u = \cos x$

Evaluate ∫ (sinx)^2 (cosx)^3 dx , Trigonometric Integral solved example

Ex) Evaluate $\int \sin^2 x \cos^3 x \, dx$

$\int \sin^2 x \cos^2 x \cos x \, dx$ reduce odd power by 1

$\int \sin^2 x (1 - \sin^2 x) \cos x \, dx$ $\cos^2 x = 1 - \sin^2 x$

$u = \sin x \quad du = \cos x \, dx$ U-Substitution

$\int u^2 (1 - u^2) \, du = \int (u^2 - u^4) du$

$u = \sin x$

$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

$\int \sin^2 x \cos^3 x \, dx = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

Evaluate ∫ (sinx)^3(cosx)^-2 dx , Trigonometric Integral solved example

Ex| Evaluate $\int \sin^3 x (\cos x)^{-2} dx$

$\int \sin^2 x (\cos x)^{-2} \sin x \, dx$ split off factor of sinx

$\int \frac{\sin^2 x}{\cos^2 x} \sin x \, dx$ re-arrange integrand

$\int \frac{1-\cos^2 x}{\cos^2 x} \sin x \, dx$ $\sin^2 x = 1-\cos^2 x$

$u = \cos x \quad du = -\sin x \, dx$ u-substitution

$$-\int \frac{1-u^2}{u^2} du = -\int (u^{-2} - 1) du = -\left[\frac{u^{-1}}{-1} - u\right] + C$$

$$-\int (u^{-2}-1)\,du = -\left[\frac{u^{-1}}{-1} - u\right] + C$$

$$= \frac{1}{u} + u + C$$

$$= \frac{1}{\cos x} + \cos x + C \qquad u = \cos x$$

$$\int \sin^3 x\,(\cos x)^{-2}\,dx = \boxed{\frac{1}{\cos x} + \cos x + C}$$

Trigonometric Integrals 2

$\int \sin^m x \cos^n x \, dx$ m, n integers

If both m and n are even use half-angle identities: $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

Ex Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

$\int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x) \, dx$ $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$\int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x)\,dx = \frac{1}{2}\int_0^{\pi/2}(1+\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]\Bigg|_0^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{\sin 2(\pi/2)}{2}\right] - \frac{1}{2}\left[0 + \frac{\overset{0}{\overset{\shortparallel}{\sin 0}}}{2}\right]$$

$$\int \cos 2x\,dx = \frac{\sin 2x}{2} + C$$

$t = 2x \quad dt = 2\,dx$

substitution

$$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{\overset{0}{\overset{\shortparallel}{\sin \pi}}}{2}\right] = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = \frac{\pi}{4}$$

Evaluate ∫ (cosx)^4 dx , Trigonometric integral solved example

Ex| Evaluate $\int \cos^4 x \, dx$

$$\int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx \qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\int (\cos^2 x)^2 \, dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1+\cos 2x)^2 \, dx = \frac{1}{4}\int (1+2\cos 2x+\cos^2 2x)\,dx$$

$$\cos^2 2x = \frac{1}{2}(1+\cos 4x)$$

$$= \frac{1}{4}\int \left[1+2\cos 2x+\frac{1}{2}(1+\cos 4x)\right]dx$$

$$= \frac{1}{4}\int \left[1+2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos 4x\right]dx$$

$$= \frac{1}{4}\int\left(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right)dx$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{\cancel{2}\sin 2x}{\cancel{2}} + \frac{1}{2}\,\frac{\sin 4x}{4}\right] + C$$

$$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

$$\int \cos^4 x\,dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

Note: $\int \cos(kx)\,dx = \frac{\sin(kx)}{k} + C$

Subst. method $\quad t = kx \quad dt = k\,dx \quad \frac{1}{k}dt = dx$

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Evaluate ∫ (cosx)^2 (sinx)^2 dx , Trigonometric Integral solved example

Trigonometric Integrals 3

Ex| Integrate $\int \cos^2 x \sin^2 x \, dx$

$$= \int \underbrace{\frac{1}{2}(1+\cos 2x)}_{\cos^2 x} \cdot \underbrace{\frac{1}{2}(1-\cos 2x)}_{\sin^2 x} \, dx$$

$$= \frac{1}{4} \int \left[1 - \cancel{\cos 2x} + \cancel{\cos 2x} - \cos^2 2x\right] dx$$

$$= \frac{1}{4} \int (1 - \cos^2 2x) \, dx = \frac{1}{4} \int \sin^2 2x \, dx$$

$$= \frac{1}{4} \int \underbrace{\frac{1}{2}(1 - \cos 4x)}_{\sin^2 2x} \, dx$$

$$= \frac{1}{8} \int (1 - \cos 4x) \, dX$$

$$= \frac{1}{8}\int (1-\cos 4x)\,dx = \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right] + C$$

$$\int \cos^2 x \sin^2 x \, dx = \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right] + C$$

Ex| Evaluate $\int \sqrt[3]{\sin x}\,\cos^3 x\,dx$

$$= \int \sqrt[3]{\sin x}\,\cos^2 x \cos x\,dx \qquad \cos^2 x = 1 - \sin^2 x$$

$$\int \overbrace{\sqrt[3]{\sin x}}^{\sqrt[3]{u}}\ \overbrace{(1-\sin^2 x)}^{1-u^2}\ \overbrace{\cos x\,dx}^{du}$$

Let $u = \sin x \quad du = \cos x\,dx$

$$\int u^{1/3}(1-u^2)\,du$$

$$= \int u^{1/3}(1-u^2)\,du$$

$$= \int \left(u^{1/3} - u^{2+\frac{1}{3}}\right) du = \int \left(u^{1/3} - u^{7/3}\right) du$$

$$= \frac{u^{4/3}}{4/3} - \frac{u^{10/3}}{10/3} + C = \frac{3}{4}u^{4/3} - \frac{3}{10}u^{10/3} + C$$

$u = \sin x$

$$= \frac{3}{4}(\sin x)^{4/3} - \frac{3}{10}(\sin x)^{10/3} + C$$

$$\int \sqrt[3]{\sin x}\,\cos^3 x\,dx = \text{ABOVE}$$

Evaluate ∫ (cosx+sinx)/sin(2x) dx , Trigonometric Integral example solved

Ex Evaluate $\int \frac{\cos x + \sin x}{\sin 2x}\,dx$

$= \int \frac{\cos x + \sin x}{2\sin x \cos x}\,dx$ $\quad \sin 2x = 2\sin x \cos x$

$= \int \left[\frac{\cancel{\cos x}}{2\sin x \cancel{\cos x}} + \frac{\cancel{\sin x}}{2\cancel{\sin x}\cos x}\right] dx$

$= \int \left(\frac{1}{2}\csc x + \frac{1}{2}\sec x\right) dx$

$= \frac{1}{2}\int (\csc x + \sec x)\,dx$

$$= \frac{1}{2}\int (\csc x + \sec x)\, dx$$

$$= \frac{1}{2}\int \csc x\, dx + \frac{1}{2}\int \sec x\, dx$$

$$= \frac{1}{2}\ln|\csc x - \cot x| + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \frac{\cos x + \sin x}{\sin 2x}\, dx = \text{ABOVE}$$

Trigonometric identities to memorize to solve ∫(trig. function) dx

Important stuff to memorize

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x + \sin^2 x = 1$$

$$\sin 2x = 2 \sin x \cos x$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\sin^2(2x) = \frac{1}{2}(1 - \cos 4x)$$

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Trigonometric Integrals 4

$\int \tan^m x \sec^n x \, dx$ m, n pos. integers

Strategy:

If power of secant is even factor out $\sec^2 x$ and replace remaining factors of $\sec^2 x$ in terms of $\tan x$ using $1 + \tan^2 x = \sec^2 x$; Then substitute $u = \tan x$ $du = \sec^2 x \, dx$

$$\int \tan^m x \sec^n x \, dx$$

If the power of $\tan x$ is odd, factor out $\sec x \tan x$ and apply $\tan^2 x = \sec^2 x - 1$ to replace remaining factors of $\tan^2 x$ in terms of $\sec x$; then substitute

$u = \sec x \quad du = \sec x \tan x \, dx$

Evaluate ∫ (tanx)^4(secx)^6 dx , Trigonometric integral solved example

Ex) Evaluate $\int \tan^4 x \sec^6 x \, dx$

Power of Secx EVEN

$\int \tan^4 x \sec^4 x \sec^2 x \, dx$ Factor out $\sec^2 x$

$\int \tan^4 x (\sec^2 x)^2 \sec^2 x \, dx$ $1+\tan^2 x = \sec^2 x$

$\int \overbrace{\tan^4 x}^{u^4} (\overbrace{1+\tan^2 x}^{(1+u^2)^2})^2 \overbrace{\sec^2 x \, dx}^{du}$

$u = \tan x \quad du = \sec^2 x \, dx$ Substitution

$\int u^4 (1+u^2)^2 \, du$

$\int u^4 (1+2u^2+u^4) \, du$

$$\int u^4(1+2u^2+u^4)\,du$$

$$\int (u^4+2u^6+u^8)\,du = \frac{u^5}{5}+\frac{2u^7}{7}+\frac{u^9}{9}+C$$

$u = \tan x$

$$= \frac{\tan^5 x}{5}+\frac{2}{7}\tan^7 x+\frac{1}{9}\tan^9 x + C$$

$$\int \tan^4 x \sec^6 x \, dx = \text{ABOVE}$$

Evaluate ∫ (tanx)^3(secx)^5 dx, odd power of tanx solved integral

Ex Evaluate $\int \tan^3 x \sec^5 x \, dx$

Power of tanx ODD

$$\int \tan^2 x \sec^4 x \tan x \sec x \, dx$$

Factor tanx secx

$$\int \overbrace{(\sec^2 x - 1)}^{u^2-1} \overbrace{\sec^4 x}^{u^4} \overbrace{\tan x \sec x \, dx}^{du}$$

$1 + \tan^2 x = \sec^2 x$

$u = \sec x \quad du = \sec x \tan x \, dx$

substitution

$$\int (u^2 - 1) u^4 \, du = \int (u^6 - u^4) \, du$$

$$\int u^6 - u^4 \, du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$u = \sec x$

$$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

$$\int \tan^3 x \sec^5 x \, dx = \text{ABOVE}$$

Integrate ∫ tanx dx , Trigonometric integral of tanx solved example

Trignometric Integrals 5

Ex| Integrate $\int \tan x \, dx$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$\tan x = \frac{\sin x}{\cos x}$

$u = \cos x \quad du = -\sin x \, dx$

U-Substitution

$-du = \sin x \, dx$

$$\int \underbrace{\frac{1}{\cos x}}_{\frac{1}{u}} \underbrace{\sin x \, dx}_{-du} = -\int \frac{1}{u} \, du = -\ln|u| + C$$

$$= -\ln|\cos x| + C = \ln|\cos x|^{-1} + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

Integrate ∫ (tanx)^3 dx , Trigonometric integral of odd power of tan x solved

Ex Integrate $\int \tan^3 x \, dx$

$$\int \tan^3 x \, dx = \int \tan^2 x \, \tan x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \, dx \qquad 1 + \tan^2 x = \sec^2 x$$

$$= \int (\tan x \sec^2 x - \tan x) \, dx$$

$$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$$

$u = \tan x \quad du = \sec^2 x \, dx$

$$\int u \, du = \frac{u^2}{2} + C = \frac{\tan^2 x}{2} + C$$

$\int \tan x \, dx$

$\Downarrow$

$\ln|\sec x| + C$

Last Example

$$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$$

$$= \frac{\tan^2 x}{2} - \ln|\sec x| + C$$

$$\int \tan^3 x \, dx = \frac{\tan^2 x}{2} - \ln|\sec x| + C$$

Prove ∫ secx dx = Ln|secx + tanx| +C , Trig. Integral theory question solved

Prove $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

Strategy: Multiply secx by $\dfrac{\text{sex} + \tan x}{\sec x + \tan x}$

$$\int \sec x \, dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$u = \sec x + \tan x \qquad du = (\sec x \tan x + \sec^2 x)\, dx$

$$\int \underbrace{\frac{1}{\sec x + \tan x}}_{1/u} \cdot \underbrace{(\sec^2 x + \sec x \tan x)\, dx}_{du}$$

$$\int \frac{1}{\sec x+\tan x}\cdot(\sec^2 x+\sec x\tan x)\,dx$$

$u=\sec x+\tan x \qquad du=(\sec x\tan x+\sec^2 x)\,dx$

$$\int \frac{1}{u}\,du = \ln|u|+C=\ln|\sec x+\tan x|+C$$

$$\int \sec x\,dx=\ln|\sec x+\tan x|+C$$

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Evaluate ∫ (secx)^4 dx, even powers of secx trignometric integral solved example

Trigonometric Integrals 6

$\sec^{4} x$ ← EVEN

Ex Evaluate $\int \sec^4 x \, dx$

Factor $\sec^2 x$

$$\int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$$

$1 + \tan^2 x = \sec^2 x$

$$= \int (1 + \tan^2 x) \sec^2 x \, dx$$

$$= \int (\sec^2 x + \tan^2 x \sec^2 x) dx$$

$$= \int \sec^2 x \, dx + \int \underbrace{\tan^2 x}_{u^2} \underbrace{\sec^2 x \, dx}_{du}$$

$u = \tan x \quad du = \sec^2 x \, dx$

$$\int \sec^2 x\,dx + \int \tan^2 x \sec^2 x\,dx$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \underbrace{\tan^2 x}_{u^2}\ \underbrace{\sec^2 x\,dx}_{du}$$

$u = \tan x$
$du = \sec^2 x\,dx$

$$\int u^2\,du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$$

$$= \tan x + \frac{\tan^3 x}{3} + C$$

$$\int \sec^4 x\,dx = \tan x + \frac{\tan^3 x}{3} + C$$

Integrate ∫ (secx)^3 dx, integration by parts & Trig. Integral combined example

Ex] Integrate $\int \sec^3 x \, dx$

Power of secx ODD reduce odd power by 1 and Integrate by Parts.

$$= \int \sec x \sec^2 x \, dx$$

$u = \sec x$	$dv = \sec^2 x \, dx$
$du = \sec x \tan x \, dx$	$v = \tan x$

$$\int u \, dv = uv - \int v \, du$$

$$\int \sec x \sec^2 x \, dx = \sec x \tan x - \int \tan x \cdot \sec x \tan x \, dx$$

$$= \sec x \tan x - \int \tan^2 x \sec x \, dx$$

$1 + \tan^2 x = \sec^2 x$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \overbrace{\tan^2 x}^{\sec^2 x - 1} \sec x\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int (\sec^2 x - 1)\sec x\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int (\sec^3 x - \sec x)\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C$$

Integrate ∫ (tanx)^3/(√secx) dx , trigonometric integral tricky example solved

Trigonometric Integrals 7

Ex| Integrate $\int \frac{\tan^3 x}{\sqrt{\sec x}}\, dx$

$$\int \frac{\sin^3 x}{\cos^3 x} \frac{1}{\sqrt{\sec x}}\, dx$$

$\tan x = \frac{\sin x}{\cos x}$

$$\int \frac{\sin^2 x}{\cos^3 x} \cdot \frac{1}{\sqrt{\sec x}} \cdot \sin x\, dx$$

split off factor of $\sin x$

$$\int \frac{(1-\cos^2 x)}{\cos^3 x} \cdot \frac{1}{\sqrt{\sec x}} \sin x\, dx$$

$\sin^2 x = 1-\cos^2 x$

$$\int \frac{(1-\cos^2 x)}{\cos^3 x} \cdot \frac{1}{\sqrt{\sec x}} \cdot \sin x \, dx$$

$$\int \frac{(1-\cos^2 x)}{\cos^3 x} \cdot \sqrt{\cos x} \, \sin x \, dx$$

$\sec x = \frac{1}{\cos x}$

$u = \cos x \quad du = -\sin x \, dx$

U-Substitution

$$-\int \frac{(1-u^2)}{u^3} \sqrt{u} \, du$$

$$-\int \frac{u^{1/2} - u^{2.5}}{u^3} \, du = -\int (u^{-2.5} - u^{-0.5}) du$$

$$-\int (u^{-2.5} - u^{-0.5})\, du = -\left[\frac{u^{-1.5}}{-1.5} - \frac{u^{0.5}}{0.5} \right] + C$$

$$= \frac{2}{3} \cdot \frac{1}{u^{3/2}} + 2u^{1/2} + C \qquad u = \cos x$$

$$\frac{2}{3} \cdot \frac{1}{(\cos x)^{3/2}} + 2(\cos x)^{1/2} + C$$

$$\int \frac{\tan^3 x}{\sqrt{\sec x}}\, dx = \frac{2}{3} \cdot \frac{1}{(\cos x)^{3/2}} + 2(\cos x)^{1/2} + C$$

Solving Trigonometric integral by using half angle trig. identities

Ex Evaluate : $\int_0^{\frac{\pi}{4}} \sqrt{1-\cos 4\theta}\, d\theta$

$$\int_0^{\pi/4} \sqrt{1-\cos 4\theta}\, d\theta$$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$\sin^2 2\theta = \frac{1}{2}(1-\cos 4\theta)$

$$\int_0^{\pi/4} \sqrt{2\sin^2 2\theta}\, d\theta$$

$$= \sqrt{2}\int_0^{\pi/4} \sqrt{\sin^2 2\theta}\, d\theta = \sqrt{2}\int_0^{\pi/4} |\sin 2\theta|\, d\theta$$

$$= \sqrt{2}\int_0^{\pi/4} \sin(2\theta)\, d\theta$$

$\sin 2\theta > 0$ on $[0, \pi/4]$

$$= \sqrt{2} \int_0^{\pi/4} \sin(2\theta)\, d\theta$$

$|\sin 2\theta| = \sin 2\theta$
since $\sin 2\theta > 0$
on $[0, \pi/4]$

$$= \sqrt{2} \left[-\frac{\cos 2\theta}{2} \right] \Big|_0^{\pi/4}$$

$\int \sin(kx)\,dx = -\frac{\cos(kx)}{k} + C$

$$= -\frac{\sqrt{2}}{2} \left[\cos\frac{\pi}{2} - \cos 0 \right]$$

$\cos\frac{\pi}{2} = 0 \quad \cos 0 = 1$

$$= -\frac{\sqrt{2}}{2} [\, 0 - 1 \,] = \frac{\sqrt{2}}{2}$$

$$\int_0^{\frac{\pi}{4}} \sqrt{1 - \cos 4\theta}\, d\theta = \frac{\sqrt{2}}{2}$$

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Integrate ∫ (cotx)^6(cscx)^4 dx , Trigonometric integrals solved example

Trigonometric Integrals 8

Ex] Integrate $\int \cot^6 x \csc^4 x \, dx$

Power of cscx EVEN

Factor out $\csc^2 x$

$1+\cot^2 x = \csc^2 x$

$\int \cot^6 x \csc^2 x \csc^2 x \, dx$

$\int \cot^6 x (1+\cot^2 x) \csc^2 x \, dx$

$u = \cot x \quad du = -\csc^2 x \, dx$

U-Substitution

$-\int u^6 (1+u^2) \, du = -\int (u^6 + u^8) du$

$$= -\int (u^6 + u^8)\,du$$

$$= -\left[\frac{u^7}{7} + \frac{u^9}{9}\right] + C$$

$u = \cot x$

$$= -\left[\frac{\cot^7 x}{7} + \frac{\cot^9 x}{9}\right] + C$$

$$\int \cot^6 x \csc^4 x\,dx = -\left[\frac{\cot^7 x}{7} + \frac{\cot^9 x}{9}\right] + C$$

Ex Evaluate $\int_{\pi/4}^{\pi/3} \cot^3 x \csc^5 x \, dx$

Power of cotx is ODD

$$\int_{\pi/4}^{\pi/3} \cot^2 x \csc^4 x \cot x \csc x \, dx$$

Factor cotx cscx

$1 + \cot^2 x = \csc^2 x$

$$\int_{\pi/4}^{\pi/3} \underbrace{(\csc^2 x - 1)}_{u^2-1} \underbrace{\csc^4 x}_{u^4} \underbrace{\cot x \csc x \, dx}_{-du}$$

U-Substitution

$u = \csc x \qquad du = -\csc x \cot x \, dx$

$x = \frac{\pi}{4} \qquad u = \csc \pi/4 = \sqrt{2}$

$x = \pi/3 \qquad u = \csc \pi/3 = 2/\sqrt{3}$

$$\int_{x=\pi/4}^{x=\frac{\pi}{3}} (\csc^2 x - 1)\csc^4 x \cot x \csc x \, dx$$

$$= -\int_{u=\sqrt{2}}^{u=2/\sqrt{3}} (u^2 - 1)u^4 \, du = -\int_{\sqrt{2}}^{2/\sqrt{3}} (u^6 - u^4) \, du$$

$$= -\left[\frac{u^7}{7} - \frac{u^5}{5} \right] \Bigg|_{\sqrt{2}}^{2/\sqrt{3}}$$

$$= -\left[\frac{(2/\sqrt{3})^7}{7} - \frac{(2/\sqrt{3})^5}{5} \right] + \left[\frac{(\sqrt{2})^7}{7} - \frac{(\sqrt{2})^5}{5} \right]$$

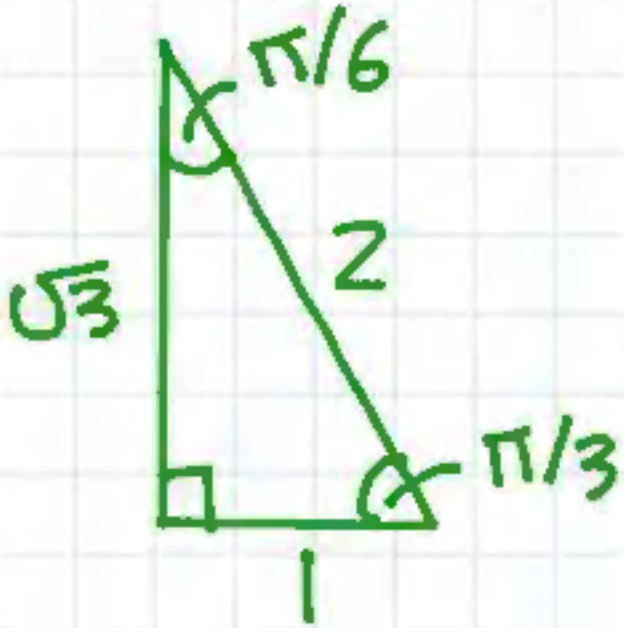
$$\int_{\pi/4}^{\pi/3} \cot^3 x \csc^5 x \, dx = \text{ABOVE}$$

Reference notes

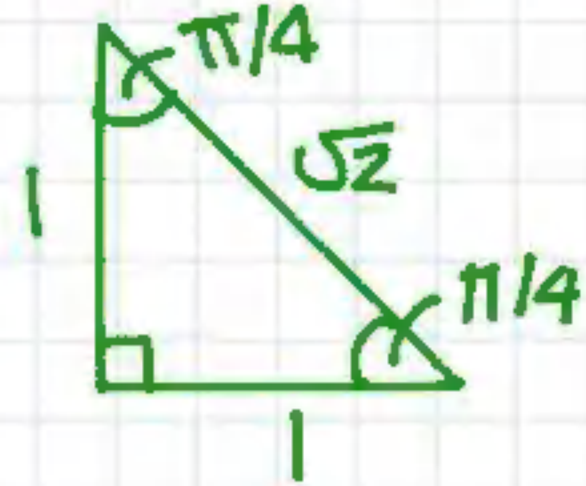
$\csc x = \dfrac{1}{\sin x}$ $\qquad$ $\cot x = \dfrac{1}{\tan x}$

$\dfrac{d}{dx} \cot x = -\csc^2 x$

$\dfrac{d}{dx} \csc x = -\csc x \cot x$



$\csc \pi/3 = \dfrac{1}{\sin \pi/3} = \dfrac{1}{\sqrt{3}/2} = 2/\sqrt{3}$



$\csc \pi/4 = \dfrac{1}{\sin \pi/4} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}$

∫ sin(mx)cos(nx) dx ,Trig. identities needed to solve ∫ (trig. function) dx

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$\int \sin(mx)\cos nx \, dx$ Apply Trig. Identity

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

$\int \sin(mx)\sin(nx)\, dx$ Apply Trig. Identity

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$\int \cos(mx)\cos(nx)\, dx$ Apply Trig. Identity

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

Integrate ∫ sin(3x)cos(5x) dx , Trigonometric integral solved example

Ex] Integrate $\int \sin(3x)\cos(5x)\,dx$

Apply $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

$$\int \sin(\underbrace{3x}_{A})\cos(\underbrace{5x}_{B})\,dx = \int \frac{1}{2}[\sin(\underbrace{-2x}_{A-B}) + \sin(\underbrace{8x}_{A+B})]\,dx$$

$$= \frac{1}{2}\int \sin(-2x)\,dx + \frac{1}{2}\int \sin(8x)\,dx$$

$$= \frac{1}{2}\left[\frac{-\cos(-2x)}{-2}\right] + \frac{1}{2}\left[-\frac{\cos(8x)}{8}\right] + C$$

$$= \frac{1}{4}\cos(-2x) - \frac{1}{16}\cos(8x) + C$$

Evaluate ∫ sin(2x)sin(4x) dx, Trigonometric integral solved example

$\boxed{\text{Ex}}$ Evaluate $\int \sin(2x)\sin(4x)\,dx$

Apply $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$$\int \sin(\underbrace{2x}_{A})\sin(\underbrace{4x}_{B})\,dx = \int \frac{1}{2}[\cos(\underbrace{-2x}_{A-B}) - \cos(\underbrace{6x}_{A+B})]\,dx$$

$$= \frac{1}{2}\int \cos(-2x)\,dx - \frac{1}{2}\int \cos(6x)\,dx$$

$$= \frac{1}{2}\left[\frac{\sin(-2x)}{-2}\right] - \frac{1}{2}\left[\frac{\sin(6x)}{6}\right] + C$$

$$= -\frac{1}{4}\sin(-2x) - \frac{1}{12}\sin(6x) + C$$

∫ sin(kx) dx , let u=kx, U-Substitution shortcuts, Linear U-Substitution

Reference notes U-Substitution

$$\int \sin(kx)\,dx$$

$u = kx \quad du = k\,dx$

$\frac{1}{k}du = dx$

$$\int \sin(u)\,\frac{1}{k}\,du$$

$$\frac{1}{k}\int \sin(u)\,du = \frac{1}{k}\left[-\cos(u)\right] + C$$

$$= -\frac{1}{k}\cos(kx) + C$$

$$\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + C$$