Web.5

7)

$$\frac{\Delta x}{2}\left[\underset{w(x)}{w(0)} + 2w(90) + \cdots + 2w(810) + w(900)\right]$$

x values not n

$$\frac{90}{2}\left[0 + 2(65) + 2(67) + 2(72) + 2(80) + 2(72) + 2(70)\right.$$

$$\left. + 2(72) + 2(77) + 2(77) + 0\right]$$

$$45[1304] = \underline{58680} \ m^2$$

$$1 \text{ kg} \qquad 225 \ m^2$$

$$260.8 \text{ kg}$$

11)

$$S_6 = \frac{\Delta x}{3}\left[\underset{=1}{f(0)} + 4\underset{=3}{f(1)} + 2\underset{=4}{f(2)} + 4\underset{=6}{f(3)} + 2\underset{=4}{f(4)} + 4\underset{=3.5}{f(5)} + \underset{=3}{f(6)}\right] \quad {=}5$$

$$S_6 = \frac{1}{3}\left[3 + 16 + 12 + 16 + 7 + 12 + 5\right]$$

$$S_6 = 23.6666$$

13)

$$\int_1^{1.5} \frac{1}{x}\,dx$$

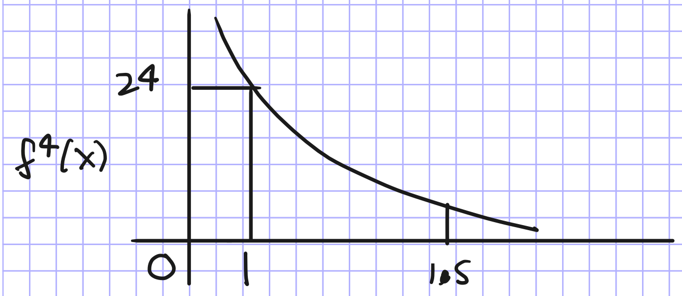
a) $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1x^{-2}$

$f'' = 2x^{-3}$

$f''' = -6x^{-4}$

a) $f^4 = \boxed{24x^{-5}}$



c) $\max |f^4(x)| = 24 = L$

$1 \leq x \leq 1.5$

d)

$$\frac{24}{180} \frac{(1.5-1)^5}{n^4} \leq 10^{-9}$$

$$\frac{24(0.5)^5}{180 n^4} \leq 10^{-9}$$

$$\cancel{\frac{24(0.5)^5}{180}} \cdot \frac{1\cancel{80}n^4}{\cancel{24}(0.\cancel{5})^5} \geq 10^9 \cdot \frac{24(0.5)^5}{180}$$

$n^4 \geq$

d) $n \geq \underline{45.1801}$

e) $n = 46$

$$\int_1^{1.5} \frac{1}{x}\,dx = \ln x \Big|_1^{1.5} = \ln 1.5 - \overset{0}{\ln 1} = \underline{\underline{\ln(1.5)}}$$

☆ Diff equations ☆

14) $I = \int_a^b f(x)\,dx$

$$I = \int_{\frac{1}{10}}^{1/5} (x^3 \ln x)\, dx$$

$$|\varepsilon| \le \frac{1}{3\times 10^9}$$

$$f(x) = x^3 \ln x$$

$$f'_{\text{1st}} = 3x^2 \ln x + x^3 \cdot \frac{1}{x}$$

$$f' = 3x^2 \ln x + x^2$$

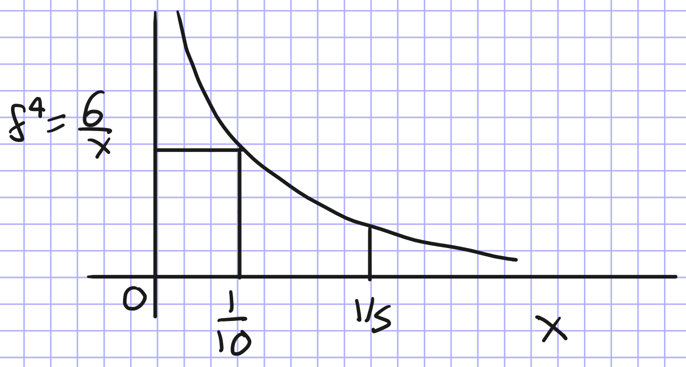
$$f'' = 6x \ln x + 3x^2 \cdot \frac{1}{x} + 2x$$

$$f'' = 6x \ln x + 5x$$

$$f''' = 6 \ln x + 6x \cdot \frac{1}{\cancel{x}} + 5$$

$$f''' = 6 \ln x + 11$$

$$f'''' = \frac{6}{x}$$



$x = \frac{1}{10}$ $\quad f^4 = \frac{6}{\frac{1}{10}} = 60 = L = \max|f^4(x)|$

$$|E| \le \frac{60\left(\frac{1}{5}-\frac{1}{10}\right)^5}{180\,n^4} \le \frac{1}{3\times 10^9}$$

$$\frac{180\,n^4}{60\left(\frac{1}{5}-\frac{1}{10}\right)^5} \geqslant 3\times 10^9$$

$$n^4 \geqslant \frac{3\times 10^9 \cdot 60\left(\frac{1}{5}-\frac{1}{10}\right)^5}{180}$$

$$n^4 \geqslant 10\,000$$

$$n \geqslant 10 \qquad \underline{n = 10}$$

15) $I = \int_0^1 3e^{x^2}\,dx$

$f(x) = 3e^{x^2}$

$f'(x) = 3e^{x^2} \cdot 2x$

$f'(x) = 6x\,e^{x^2}$

$f''(x) = 6e^{x^2} + 6x\,e^{x^2} \cdot 2x$

$f'' = e^{x^2}(6 + 12x^2)$

$f''' = e^{x^2} \cdot 2x(6 + 12x^2) + e^{x^2}(24x)$

$f''' = e^{x^2}[24x^3 + 36x]$

$f^4 = e^{x^2} \cdot 2x(24x^3 + 36x) + e^{x^2}(72x^2 + 36)$

$$f^4 = e^{x^2}\left[48x^4 + 72x^2 + 72x^2 + 36\right]$$

$$f^4 = e^{x^2}\left[\underbrace{48x^4} + \underbrace{144x^2} + 36\right]$$

$$|A+B| \leq |A| + |B|$$

$$0 \leq x \leq 1$$

$$|AB| \leq |A|\,|B|$$

plug $x=1$

$$|f^4(x)| \leq e^1\left[48 + 144 + 36\right] = \boxed{228e}$$

$$|I - S_n| \leq \frac{228e(1-0)^5}{180n^4} \leq 10^{-9}$$

$$\frac{180n^4}{228e} \geq 10^9$$

$$n^4 \geq \frac{10^9 \cdot 228e}{180} = (\text{ANS})^{0.25}$$

$$n \geq 242.24$$

$$** \boxed{n = 244} **$$

Web 6

$$\frac{dy}{dx} = \frac{8y}{x} \qquad x>0 \qquad F(1,1)=1$$

$$\frac{dy}{8y} = \frac{dx}{x} \Rightarrow \int \frac{dy}{y} = \int \frac{8dx}{x}$$

$$\ln|y| = 8\ln|x| + C$$

$$\ln|y| = \ln x^8 + C$$

$$e^{\ln|y|} = e^{\ln x^8 + C}$$

$$|y| = e^{\ln x^8} \cdot e^C$$

$$|y| = C_1 \cdot x^8$$

$$y = (\pm C_1) x^8$$

$$\boxed{y = C_2 x^8}$$

$$\frac{y}{x^8} = C_2 \qquad x=1 \quad y=1$$

$$\frac{1}{1} = C_2$$

$$F(x,y) = C_2 = 1 = \boxed{\frac{y}{x^8}}$$

$$= y' x^{-8}$$

b a

7) $\frac{dy}{dx} = \frac{x}{9y}$

$$\int 9y\,dy = \int x\,dx$$

$$\frac{9y^2}{2} = \frac{x^2}{2} + C_1$$

$$\frac{9y^2}{2} - \frac{x^2}{2} = C_1$$

$$9y^2 - x^2 = 2C_1$$

C_1 and C are different

$$\boxed{9y^2 - x^2 = C}$$

$$F(x, y) = 9y^2 - x^2$$

$$F(kx, ky) = 9(ky)^2 - (kx)^2$$

$$= 9k^2y^2 - k^2x^2$$

$$F(kx, ky) = k^2(9y^2 - x^2)$$

b) $F(x, y) = 9y^2 - x^2 = C$

$X = 3 \quad y = 1$

$$C = 9(1)^2 - 9 = 0$$

$$9y^2 - x^2 = 0$$

$$9y^2 = x^2$$

$$y^2 = \frac{x^2}{9}$$

$$y = \sqrt{\frac{x^2}{9}} = \frac{|x|}{3} = \pm\frac{x}{3}$$

b) $y = \dfrac{x}{3}$ $\qquad x = 3 \quad y = 1$

c) $F(x, y) = 9y^2 - x^2 = C$

$(0, -3)$

$9(-3)^2 - 0^2 = C \qquad C = 81$

$$9y^2 - x^2 = 81$$

$$9y^2 = x^2 + 81$$

$$y^2 = \frac{x^2+81}{9}$$

$$y^2 = \frac{x^2}{9} + 9$$

$$y = \pm\sqrt{\frac{x^2}{9} + 9}$$

due to neg -3

$$y = -\sqrt{\frac{x^2}{9} + 9}$$

$x = 0 \quad y = -3$

$$y = -\sqrt{\frac{0}{9} + 9} = \boxed{-3}$$

15) $\frac{dv}{dt} = g - kv^p \qquad p = 1$

$$\frac{dv}{dt} = g - kv \qquad v(0) = 0$$

$$\int \frac{dv}{g-kv} = \int \frac{dt}{1}$$

$u = g - kv \qquad du = -k\,dv$

$$\frac{\ln|g-kv|}{-k} = t + C$$

$$\ln|g-kv| = -kt \underbrace{-kC} \qquad \boxed{C_1 = -kC}$$

$$\ln|g-kv| = -kt + C_1$$

$$e^{\ln|g-kv|} = e^{-kt+C_1}$$

$$|g-kv| = e^{-kt}\underbrace{e^{C_1}}_{C_2}$$

$$|g-kv| = C_2 e^{-kt}$$

$$g-kv = \pm C_2 e^{-kt}$$

$$g-kv = Ae^{-kt}$$

$$-kv = Ae^{-kt} - g$$

$$v = \frac{-A}{k}e^{-kt} + \frac{g}{k}$$

$$v(0)=0$$

$$0 = \frac{-A}{k}\cdot 1 + \frac{g}{k} \qquad \boxed{A=g}$$

a) $v(t) = \frac{\overset{g}{9.807}}{\underset{k}{21}}\left(1 - e^{-\overset{k}{21}t}\right)$

b) $y(0)=0$ $\qquad$ $y(t)>0$ for $t>0$

$$y(t) = \int v(t)\,dt$$

$$y(t) = \frac{9.807}{21}\left(t - \frac{e^{-21t}}{-21}\right) + C$$

$$y(0)=0$$

$$0 = \frac{9.807}{21}\left(0 - \frac{1}{-21}\right) + C$$

$$0 = \frac{9.807}{21^2} + C$$

$$\boxed{\int e^{kt}\,dt = \frac{e^{kt}}{k} + C}$$

$$C = -\frac{9.807}{21^2}$$

$$y(t) = \frac{9.807}{21}\left(t - \frac{e^{-21t}}{-21}\right) + C$$

$$\boxed{y(t) = \frac{9.807}{21}\left(t + \frac{e^{-21t}}{21}\right) - \frac{9.807}{21^2}}$$

$$y(0) = 0$$

$$0 = \frac{9.807}{21}\left(0 + \frac{1}{21}\right) - \frac{9.807}{21^2}$$

c) $y(t) = 1$ <u>Wolfram alpha</u>

$$\boxed{1 = \frac{9.807}{21}\left(t + \frac{e^{-21t}}{21}\right) - \frac{9.807}{21^2}}$$

<u>Web S</u>

<u>Q.1</u>

$$-8\int \frac{\tan^{-1}(\sqrt{t})}{\sqrt{t}}\,dt$$

U-SUB + Parts

Let $w = t^{1/2}$ $\quad dw = \frac{1}{2\sqrt{t}}\,dt$

$$-8\int \frac{\tan^{-1}(w)}{\cancel{w}}\, 2\cancel{\sqrt{t}}\; dw$$

$$-16\int \underbrace{\tan^{-1}(w)}_{u}\,\underbrace{dw}_{dv}$$

$u=\tan^{-1}(w)$ $\quad du=\dfrac{1}{1+w^2}\,dw$ $\quad dv=dw$ $\quad v=w$

$$w\cdot\tan^{-1}(w)-\int w\cdot\frac{1}{1+w^2}\,dw$$

$\Downarrow$ $\quad \boxed{x=1+w^2 \quad dx=2w\,dw}$

$$w\cdot\tan^{-1}(w) - \frac{\ln(1+w^2)}{2}+C$$

$$-16\left[\sqrt{t}\,\tan^{-1}(\sqrt{t})-\frac{\ln(1+t)}{2}\right]+C$$