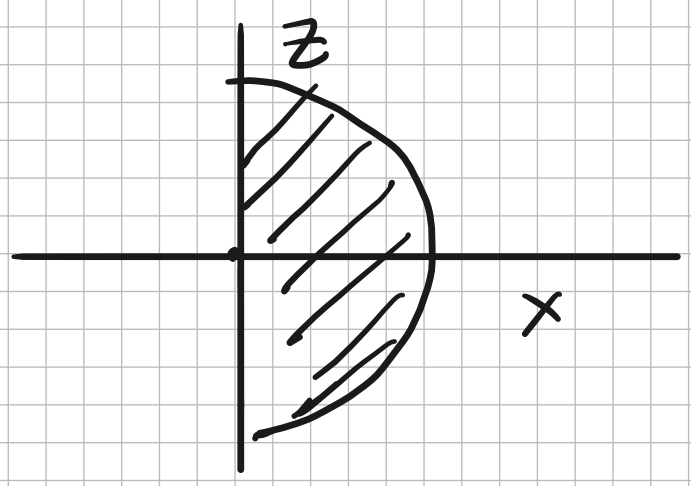
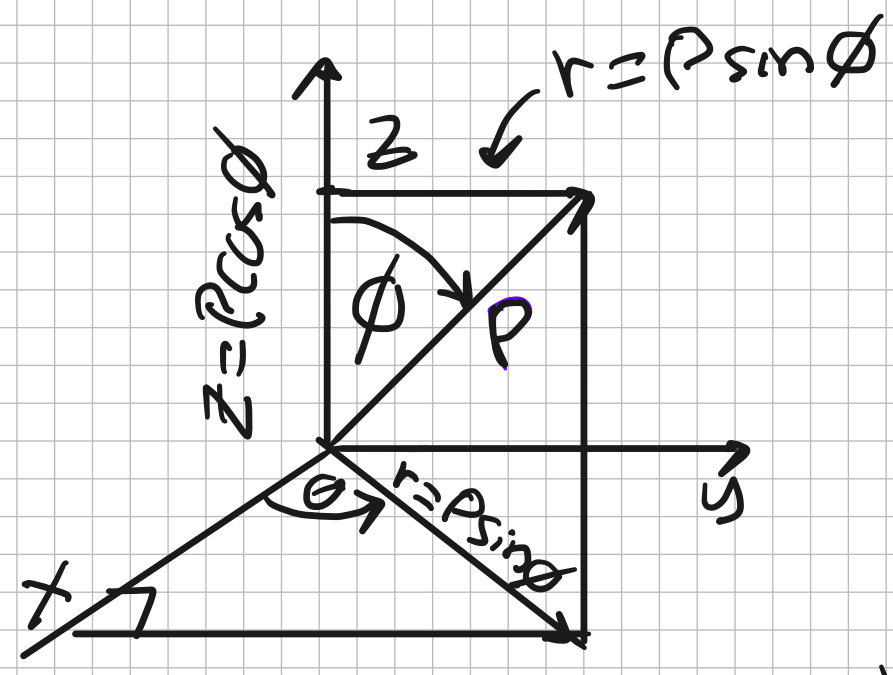
11

$$y = \sqrt{49 - x^2 - z^2}$$





$$\cos\theta = \frac{x}{\rho\sin\phi} \Rightarrow x = \overbrace{\rho\sin\phi}^{r}\cos\theta$$

$$y = \overbrace{\rho\sin\phi}^{r}\sin\theta$$

$$x^2 + y^2 + z^2 = r^2$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\pi} \int_{0}^{7} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\frac{\rho^3}{3}\Big|_0^7 \int_0^{\pi} \sin\phi \, d\phi \int_{-\frac{\pi}{2}}^{\pi/2} 1 \, d\theta$$

$$\frac{7^3}{3}\left[-\cos\phi\right]_0^{\pi}\cdot\theta\Big|_{-\frac{\pi}{2}}^{\pi/2}$$

$$-\frac{7^3}{3}\left[\cos\pi-\cos 0\right]\cdot\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$$

($\cos\pi = -1$, $\cos 0 = 1$)

$$\frac{7^3(2)}{3}\cdot\pi$$

$$\frac{7^3\cdot 2\pi}{3}$$

Spherical

$$\int_{-\frac{\pi}{2}}^{\pi/2}\int_0^{\pi}\int_0^{7}\frac{\rho^2\sin\phi}{\rho}\,d\rho\,d\phi\,d\theta$$

$$\frac{\rho^2}{2}\Big|_0^7\int_0^{\pi}\sin\phi\,d\phi\int_{-\frac{\pi}{2}}^{\pi/2}1\,d\theta$$

$$\frac{49}{2}\left[-\cos\phi\right]_0^{\pi}\cdot\left[\theta\right]_{-\frac{\pi}{2}}^{\pi/2}$$

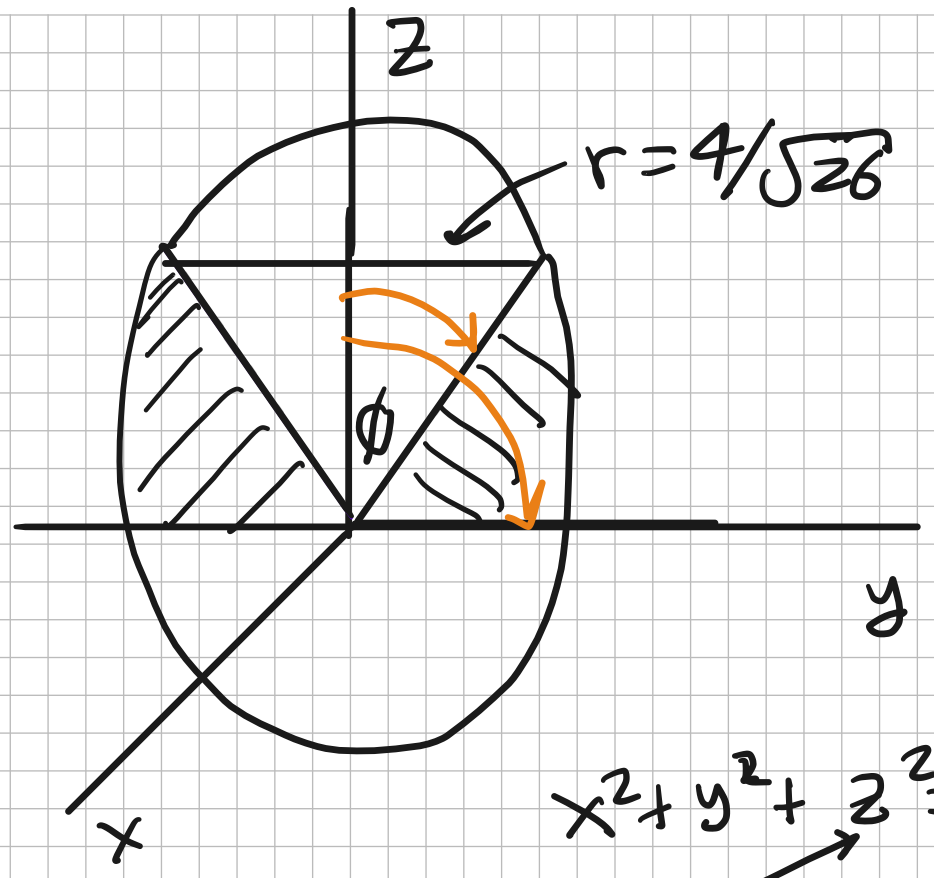
$$-\frac{49}{2}\left[-1-1\right]\left[\frac{\pi}{2}+\frac{\pi}{2}\right]$$

$$49\pi$$

6]

$x^2+y^2+z^2 = 16$ above xy plane

outside cone $z = 5\sqrt{x^2+y^2}$



$$x^2 + y^2 + z^2 = 16$$

$$z = 5r$$

$$r^2 + 25r^2 = 16$$

$$26r^2 = 16$$

$$r = \frac{4}{\sqrt{26}}$$

$$z = 5r = 5\left(\frac{4}{\sqrt{26}}\right) = \boxed{\frac{20}{\sqrt{26}}}$$

$$\tan\phi = \frac{4/\sqrt{26}}{5\left(\frac{4}{\sqrt{26}}\right)} = \frac{1}{5}$$

$$\phi = \tan^{-1}(1/5)$$

$$\int_0^{2\pi} \int_0^{\tan^{-1}(1/5)} \int_0^{4} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\frac{\rho^3}{3}\Big|_0^4 \int_{\tan^{-1}(1/5)}^{\pi/2} \sin\phi \, d\phi \int_0^{2\pi} d\theta$$

$$\frac{4^3}{3}\left[-\cos\phi\right]_{\tan^{-1}(1/5)}^{\pi/2} \cdot 2\pi$$

$$-\frac{64}{3}\left[\cos(\pi/2) - \cos(\tan^{-1}(1/5))\right].2\pi$$

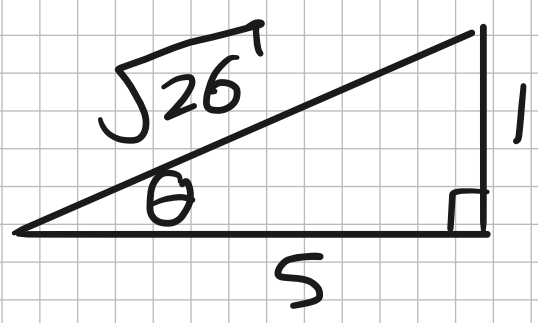
$\cos(\pi/2) = 0$

$\cos(\tan^{-1}(1/5)) = 5/\sqrt{26}$

$$\cos(\underbrace{\tan^{-1}(1/5)}_{\theta})$$

$$\theta = \tan^{-1}(1/5)$$

$$\tan\theta = 1/5$$



$$\cos\theta = \frac{5}{\sqrt{26}}$$

$$-\frac{64}{3}\left[\cos(\pi/2) - \cos(\tan^{-1}(1/5))\right].2\pi$$

$\cos(\pi/2) = 0$

$\cos(\tan^{-1}(1/5)) = 5/\sqrt{26}$

$$-\frac{64}{3}\left[0 - \frac{5}{\sqrt{26}}\right].2\pi$$

$$\frac{5(64)(2\pi)}{3\sqrt{26}} = \frac{(128\pi)}{(3\sqrt{26})}$$

7]

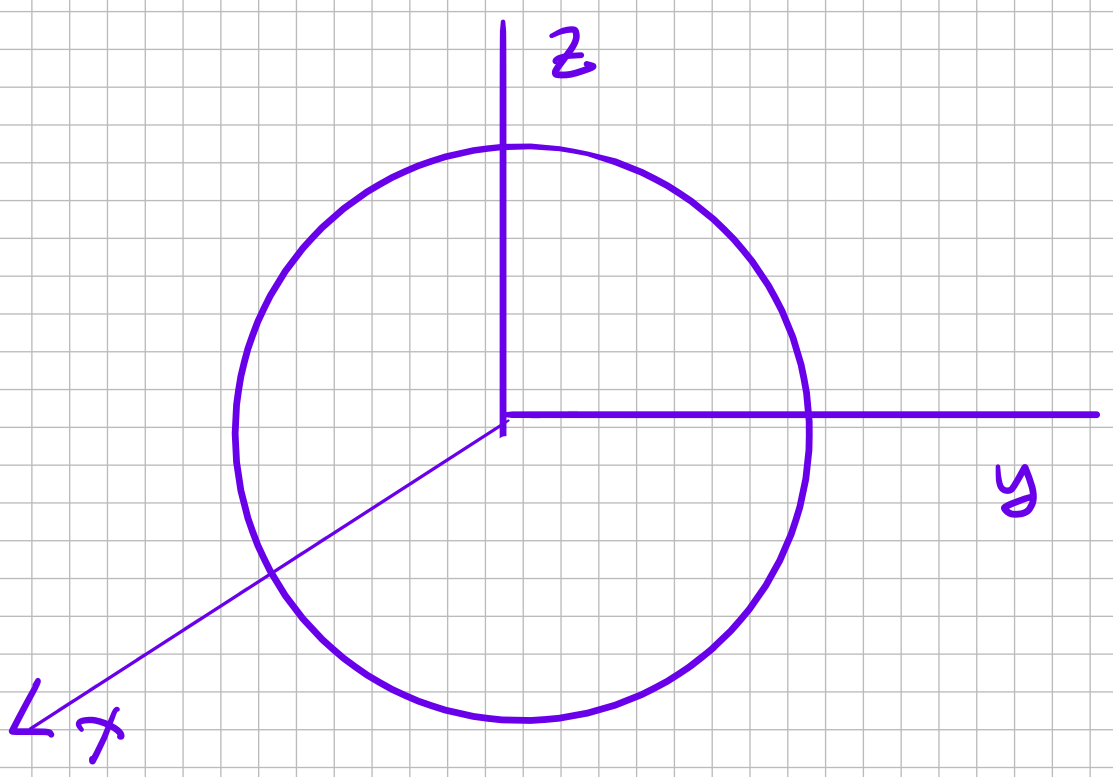
$x^2+y^2+z^2=4$

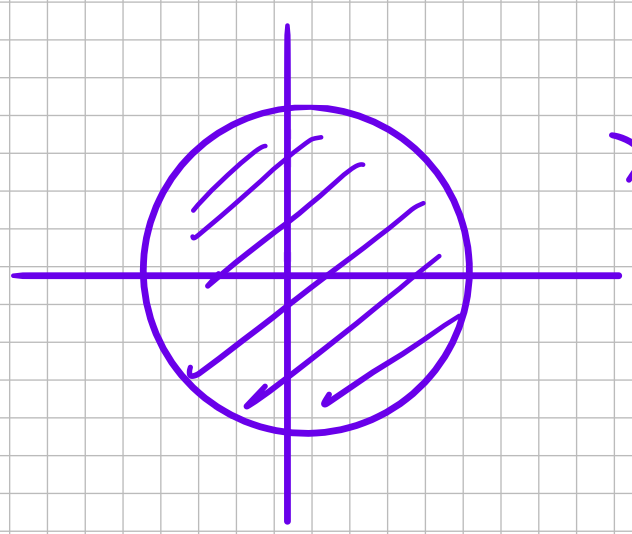
$x^2+y^2=4$

$$\sqrt{4-x^2-y^2} \le z \le \sqrt{4-x^2-y^2}$$

$$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$$

$$-2 \le x \le 2$$





$x^2+y^2=4$

r^2

$r^2+z^2=4$

$z=\sqrt{4-r^2}$

$0 < \theta < 2\pi$

$0 < r < 2$

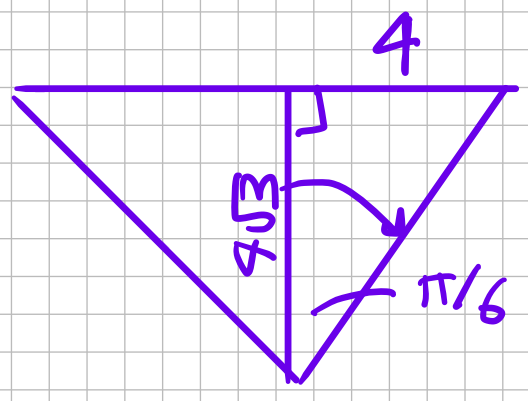
$-\sqrt{4-r^2} < z < \sqrt{4-r^2}$

$0 < \phi < \pi$

$0 < \rho < 2$

$0 < \theta < 2\pi$

14)

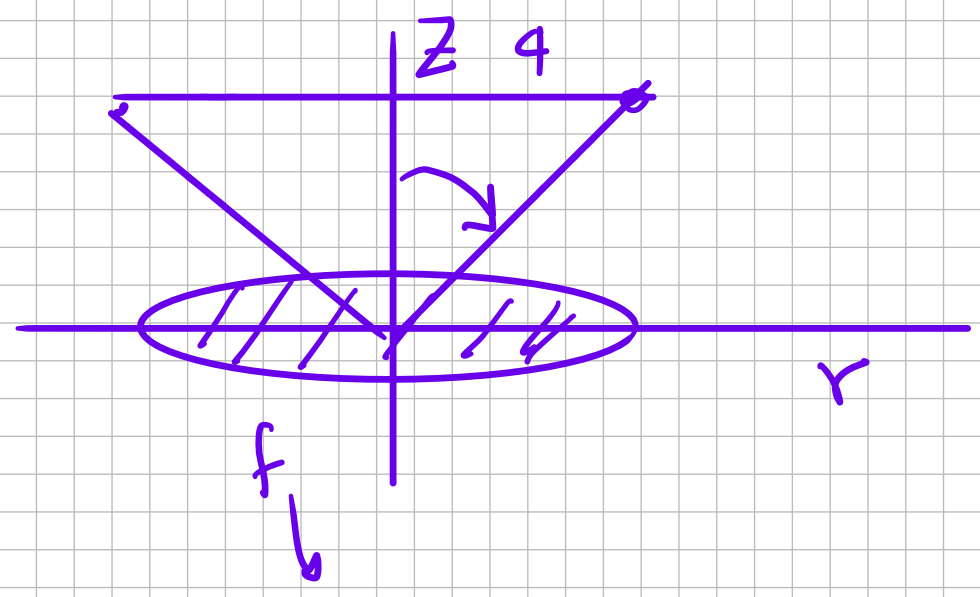


$\tan\frac{\pi}{6} = \frac{opp}{4\sqrt{3}}$

$opp = 4\sqrt{3}\tan\frac{\pi}{6}$

$opp = 4\sqrt{3}\cdot\frac{1}{\sqrt{3}} = 4$

$z = \frac{4\sqrt{3}}{4}r = \sqrt{3}r$



$\sqrt{3}r < z < 4\sqrt{3}$

$\sqrt{3}\sqrt{x^2+y^2} < z < 4\sqrt{3}$

$-\sqrt{16-x^2} \le y \le \sqrt{16-x^2}$

$y^2 + x^2 = 16$

$r^2 = 16$

$-4 \le x \le 4$

a b

dz dy dx

b) $\sqrt{3}r < z < 4\sqrt{3}$

$0 < r < 4$

$0 < \theta < 2\pi$

c) $\int_{a=0}^{b=2\pi}\int_{c=0}^{d=\pi/6}\int_{e=0}^{f=4\sec\phi} \underbrace{\rho^2\sin\phi}\, d\rho\, d\phi\, d\theta$

$z = \rho\cos\phi$

$4 = \rho\cos\phi$

$\underline{4\sec\phi = \rho}$

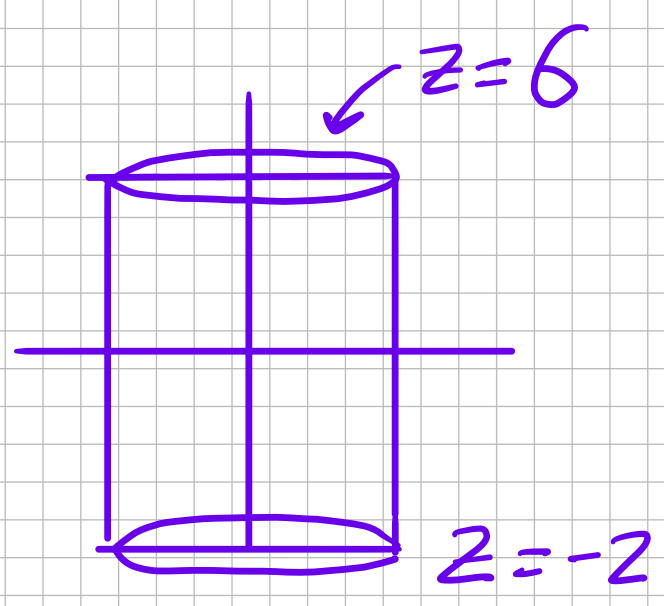
$\underline{z = 4\sqrt{3}}$ Involve ρ

$\downarrow$

$\rho\cos\phi = 4\sqrt{3}$

$\underline{\rho = 4\sqrt{3}\sec\phi}$

13]



$$\int_0^{2\pi}\int_0^{2}\int_{-2}^{6} f \; \sin(r^2)\, dz\, r\, dr\, d\theta$$

$$z\Big|_{-2}^{6}$$

$$8\int_0^{2} \sin(r^2) r \cdot 2\pi$$

$$16\pi\int_0^{2} \sin(r^2) r\, dr$$

$u = r^2 \quad du = 2r\,dr$

$$16\pi \int \sin(u)\, \frac{du}{2}$$

$$-8\pi \cos r^2\Big|_0^2$$

$$-8\pi[\cos 4 - \cos 0]$$

$$-8\pi[\cos 4 - 1]$$

$$8\pi[1 - \cos 4]$$