8)

$$\frac{1}{x^2+1} y' + xy = 3 \qquad y(0) = 0$$

Paul Dawkins Notes

$$y' + x(x^2+1)\, y = 3(x^2+1)$$

ST1) $y' + P(x)\, y = Q(x)$

ST2) $I.F = e^{\int P(x)\,dx}$

ST3) $\frac{d}{dx}[y\ I.F] = Q(x)\ I.F$

ST4) $\int \frac{d}{dx} y\ I.F = \int Q(x)\ I.F\ dx$

$$y.(I.F) = \int Q(x)\ I.F\ dx$$

$$y(x) = \frac{\int Q(x)\ I.F\ dx}{I.F} + \frac{C}{I.F}$$

$$y' + x(x^2+1)y = 3(x^2+1)$$

$$y' + P(x)\, y = Q(x)$$

$$I.F = e^{\int x(x^2+1)\,dx}$$

$$= e^{\int (x^3+x)\,dx}$$

$$I.F = e^{x^4/4 + x^2/2}$$

$$\int \frac{d}{dx}\left[e^{x^4/4+x^2/2}\right].\, y = \int 3(x^2+1)\, e^{x^4/4+x^2/2}\, dx$$

$$y\, e^{x^4/4+x^2/2}\Big|_0^x = \int 3(x^2+1) e^{x^4/4+x^2/2}\, dx$$

$$y\, e^{x^4/4+x^2/2}\Big|_0^x = \int_0^x 3(s^2+1) e^{s^4/4+s^2/2}\, ds$$

$$y(x)\, e^{x^4/4+x^2/2} - \overset{0}{\overset{\shortparallel}{y(0)}} e^0 = 3\int_0^x (s^2+1) e^{s^4/4+s^2/2}\, ds$$

$$y(x)\, e^{x^4/4+x^2/2} = 3\int_0^x (s^2+1) e^{s^4/4+s^2/2}\, ds$$

$$y(x) = \frac{3\int_0^x (s^2+1) e^{s^4/4+s^2/2}\, ds}{e^{x^4/4+x^2/2}}$$

9) <u>1.4.102</u>

$$y' + 2\sin(2x)y = 2\sin(2x) \qquad y(\pi/2) = 3$$

$$y' + P(x)y = Q(x)$$

$$I.F = e^{\int 2\sin(2x)\,dx}$$

$$= e^{-\cancel{2}\frac{\cos(2x)}{\cancel{2}}} = e^{-\cos(2x)}$$

$$\int \frac{d}{dx}\left[y e^{-\cos(2x)}\right] = \int 2\sin(2x)\, e^{-\cos(2x)}\, dx$$

$$y e^{-\cos(2x)} = \int \overbrace{2\sin(2x)\, \underbrace{e^{-\cos(2x)}}_{e^u}\, dx}^{du}$$

$$u = -\cos(2x) \qquad du = 2\sin(2x)\,dx$$

$$y e^{-\cos(2x)} = \int e^u\, du \rightarrow e^u$$

$$\frac{y \cancel{e^{-\cos(2x)}}}{\cancel{e^{-\cos(2x)}}} = \frac{e^{-\cos(2x)} + C}{e^{-\cos(2x)}}$$

$$y = 1 + \frac{C}{e^{-\cos(2x)}}$$

$$y = 1 + Ce^{\cos(2x)}$$

$$\underline{y(\pi/2) = 3}$$

$$3 = 1 + Ce^{\cos(\cancel{2}\cdot\pi/\cancel{2})}$$

$$3 = 1 + Ce^{\boxed{\cos\pi}}$$

$$2 = Ce^{-1}$$

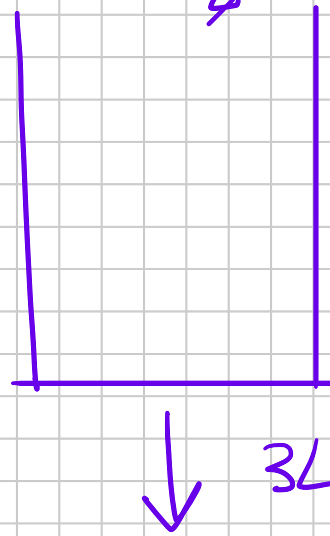
$$C = \frac{2}{e^{-1}} = 2e$$

$$\underline{\underline{y(x) = 1 + 2e\,e^{\cos(2x)}}}$$

10)

$\frac{3L}{min}$

$20t \frac{g}{L} \cdot \frac{2L}{min}$



10L

3L/min

$V(0) = 10$

$\frac{dV}{dt} = 2 - 3 = -1$

$V(t) = 10 - t$

$Q(t)$ = mass in grams of toxin

concentration of toxin

$$\frac{dQ}{dt} = 40t \frac{g}{min} - \boxed{\frac{Q(t)}{10-t}} \cdot \frac{3L}{min}$$

$$\frac{dV}{dt} = 2 - 3 = -1$$

$V(0) = 10L$

$V(t) = 10 - t$

$$\frac{dQ}{dt} = 40t - \frac{3}{10-t} Q$$

$Q(0) = 0$

$y' + P(x)y = Q(x)$

$$\frac{dQ}{dt} + \frac{3}{10-t}Q = 40t$$

$$I.F = e^{\int \frac{3}{10-t}dt} = e^{-3\ln|10-t|}$$

$$I.F = e^{\ln|10-t|^{-3}} \qquad 0 \leq t \leq 10$$

$$= e^{\ln(10-t)^{-3}}$$

$$\underline{I.F = (10-t)^{-3}}$$

$$\int \frac{d}{dt}\left[(10-t)^{-3} Q\right] = \int (10-t)^{-3} \cdot 40t$$

$$(10-t)^{-3} Q(t) = \int \frac{40t}{(10-t)^3} dt$$

$$u = 10-t \quad du = -dt$$

$$-\int \frac{40(10-u)}{u^3} du$$

$$-40\int(10u^{-3}-u^{-2})\,du$$

$$-40\left[\frac{10u^{-2}}{-2}-\frac{u^{-1}}{-1}\right]$$

$$-40\left[\frac{-5}{u^2}+\frac{1}{u}\right]$$

$$(10-t)^{-3}Q(t)=\frac{200}{(10-t)^2}-\frac{40}{10-t}+C$$

$$Q(t)=200(10-t)-40(10-t)^2+C(10-t)^3$$

$$Q(0)=0$$

$$0=200(10)-40(100)+C(10)^3$$

$$\frac{2000}{10^3}=C \qquad \underline{C=2}$$

$$\underline{Q(t)=200(10-t)-40(10-t)^2+2(10-t)^3}$$

When is tank half empty

$V(t) = 5\,L$

$$V(t) = V(0) + \int_0^t (2-3)\,ds$$

$$= 10 + \int_0^t -\,ds$$

$$= 10 - s\Big|_0^t = \underline{\underline{10-t}}$$

$10 - t = 5 \qquad \underline{t = 5}$

$$\underline{Q(t) = 200(10-t) - 40(10-t)^2 + 2(10-t)^3}$$

$$Q(5) = 200(5) - 40(5)^2 + 2(5)^3$$

$$= 1\cancel{0}\cancel{0}0 - 1\cancel{0}\cancel{0}0 + 250$$

$$\underline{\underline{Q(5) = 250}} \quad g$$