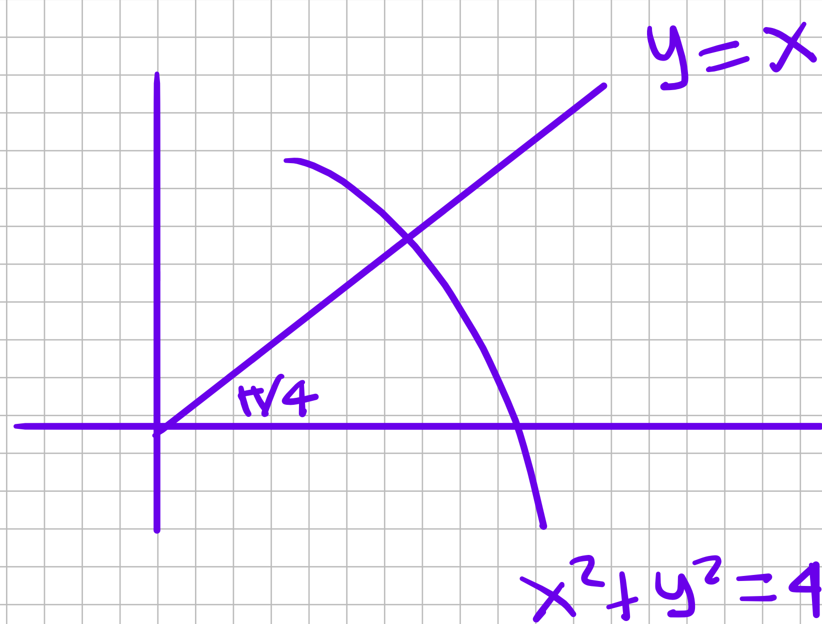
5]



$\boxed{y = x}$

$\cancel{r}\sin\theta = \cancel{r}\cos\theta$

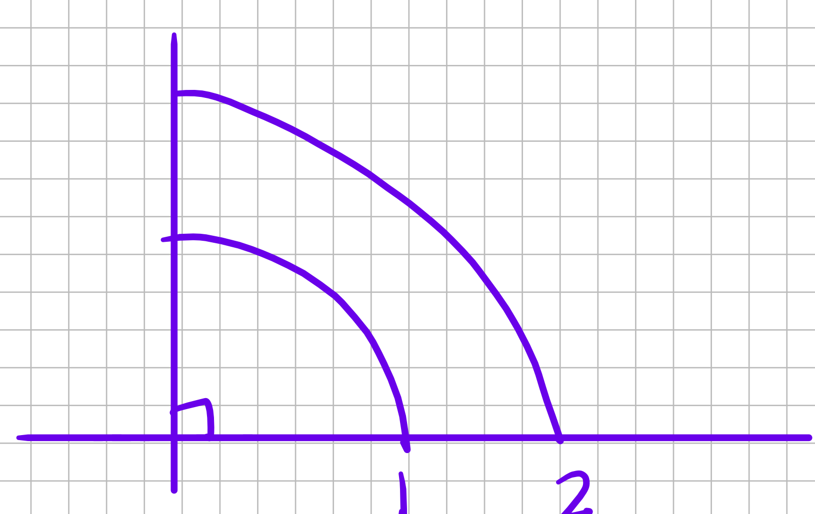
$\tan\theta = 1$

$\underline{\underline{\theta = \pi/4}}$

$0 \leq r \leq 2$

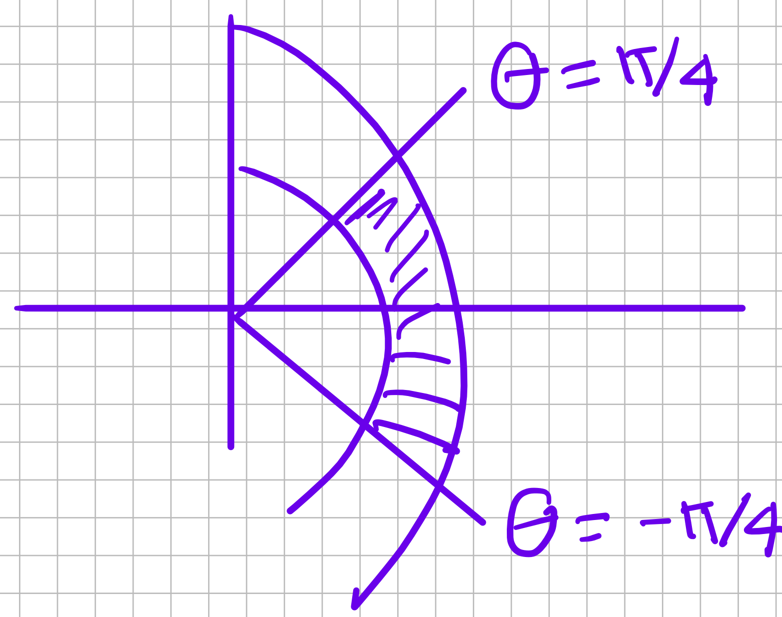
$0 \leq \theta \leq \pi/4$

$$\int_0^{\pi/4} \int_0^2 f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$



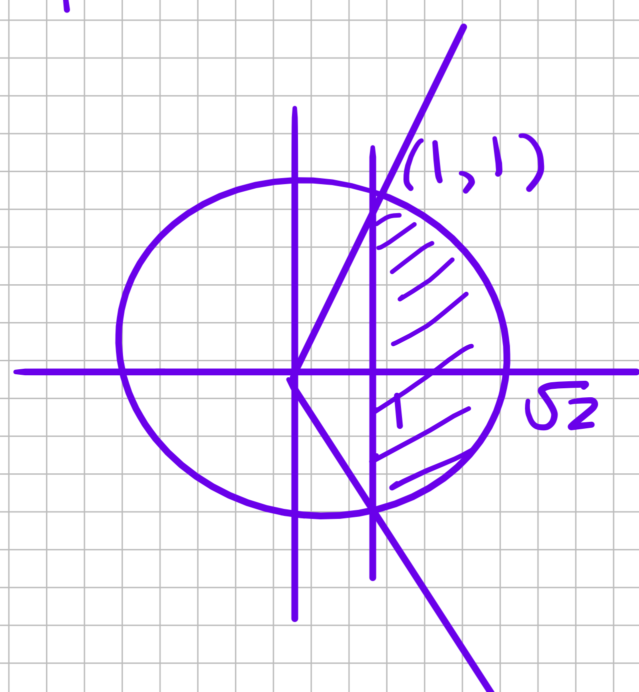
$$\int_0^{\pi/2} \int_1^2 f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

6/a] $\int_1^2 dr \int_{-\pi/4}^{\pi/4} r f(r\cos\theta, r\sin\theta)\, dr\, d\theta$



7/b)

$$\iint_R x \, dx\, dy \qquad x^2+y^2 \le 2 \qquad x \geq 1$$



$x=1$

$1+y^2=2$

$y^2=1$

$y=1$

$x=1$

$r\cos\theta=1$

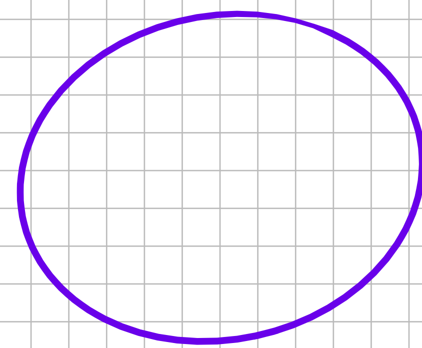
$r=\sec\theta$

$$\int_{-\pi/4}^{\pi/4}\int_{r=\sec\theta}^{\sqrt{2}} (r\cos\theta)\, r \, dr\, d\theta$$

7/d)

$$\iint_{x^2+y^2 \le 1} \ln(x^2+y^2)\, dx\, dy$$

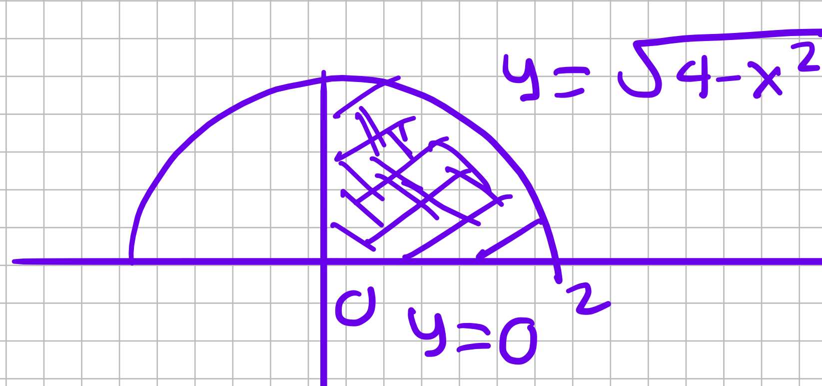
$$\int_0^{2\pi}\int_0^1 \ln(r^2)\, r \, dr\, d\theta$$



12)

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \underline{(x^2+y^2)}^{3/2} \; \underline{dy\,dx}$$

$$\int_0^{\boxed{\pi/2}} \int_0^2 (r^2)^{3/2} \, r \, dr \, d\theta$$



14)

$$\int_0^{2\pi} \int_0^{\infty} \frac{1}{(1+r^2)^2} \, r \, dr \, d\theta$$

$u = 1+r^2 \quad du = 2r\,dr$

$r=0 \quad u=1$

$$\int_1^{\infty} \frac{1}{u^2} \, \frac{du}{2}$$

$$\frac{1}{2} \int_1^{\infty} u^{-2} \, du$$

$$-\frac{1}{2}\, u^{-1}\Big|_{1}^{\infty}$$

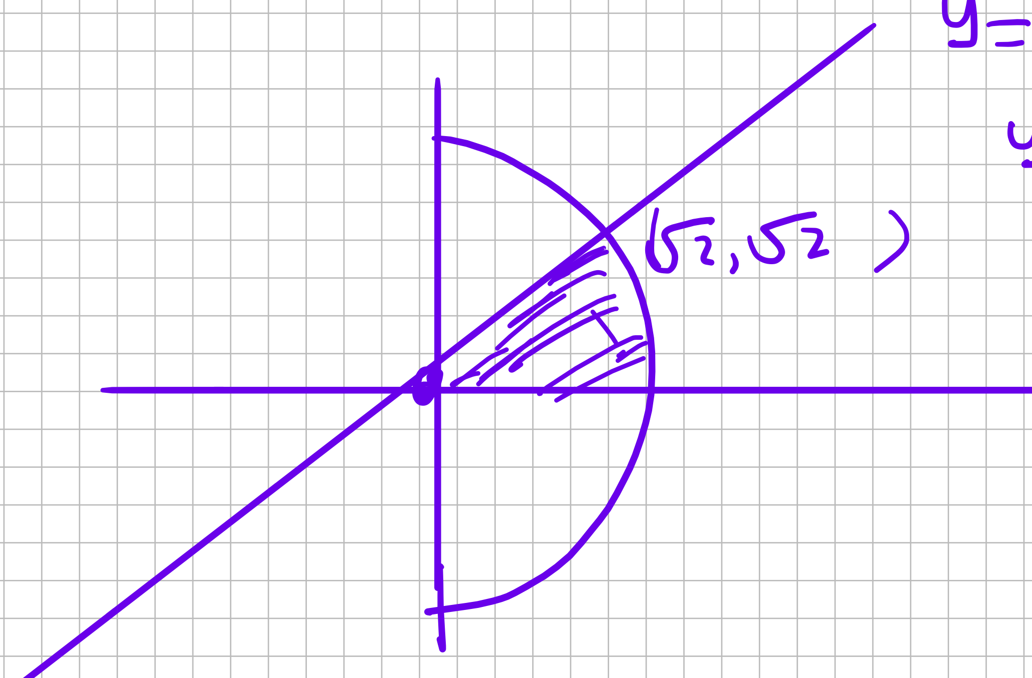
$$-\frac{1}{2}\left(\frac{1}{\infty}^{\nearrow 0}-\frac{1}{1}\right)=\frac{1}{2}$$

$$\frac{1}{2}\int_{0}^{2\pi} d\theta$$

$$\frac{1}{2}\,\theta\Big|_{0}^{2\pi}=\pi$$

21)

$$I=\int_{0}^{\sqrt{2}}\int_{x=y}^{x=\sqrt{4-y^2}} \frac{y}{x}e^{x^2+y^2}\,dx\,dy$$



$y=\sqrt{4-x^2}$

$y=x$

$x=\sqrt{4-x^2}$

$x^2=4-x^2$

$2x^2=4$

$x=\pm\sqrt{2}$

$$I=\int\int \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}e^{r^2}\,r\,dr\,d\theta$$

$$I=\int_{0}^{\pi/4}\int_{0}^{2} \tan\theta\, re^{r^2}\,dr\,d\theta$$

$$I = \underbrace{\int_0^2 r e^{r^2}\, dr}\ \underbrace{\int_0^{\pi/4} \tan\theta \, d\theta}$$

$u = r^2 \quad du = 2r\,dr$

$$\int \cancel{r} e^u \frac{du}{2\cancel{r}}$$

$$\frac{1}{2} e^{r^2} \Big|_0^2$$

$$\boxed{\frac{1}{2}(e^4 - 1)}$$

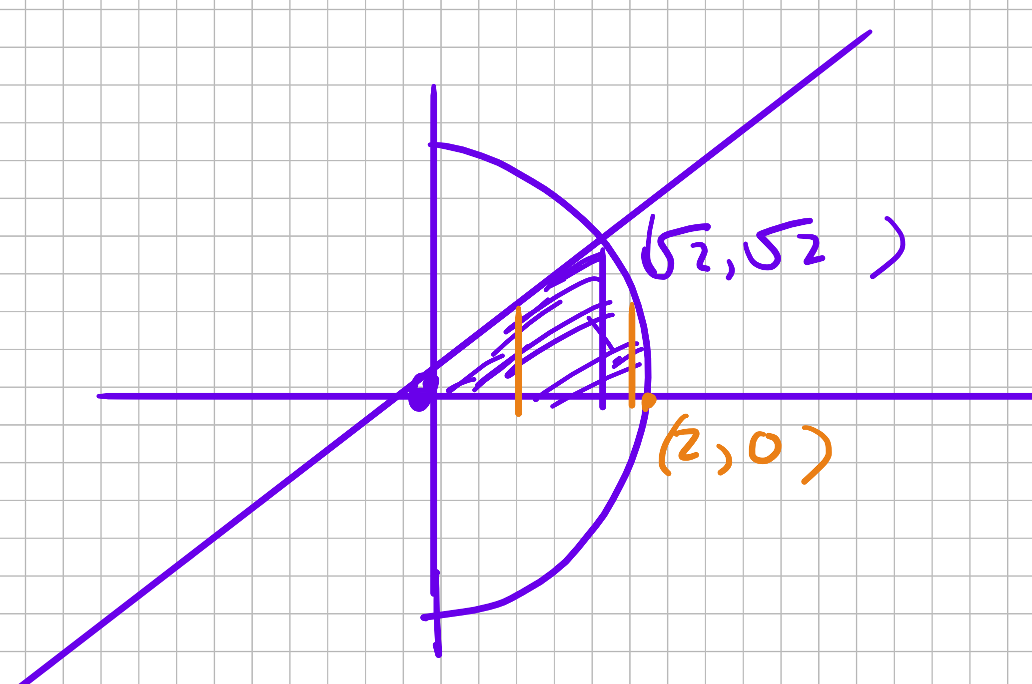
$$\int_0^{\pi/4} \frac{\sin\theta}{\cos\theta}\, d\theta$$

$u = \cos\theta \quad du = -\sin\theta\, d\theta$

$$\int_1^{1/\sqrt{2}} -\frac{du}{u}$$

$$-\ln|u|$$

$$-[\ln(1/\sqrt{2}) - 0]$$

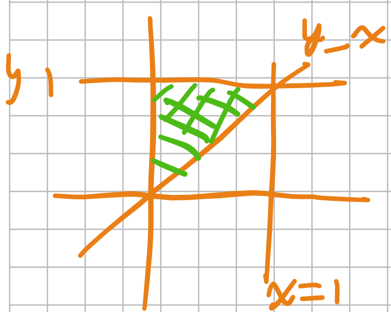


$$\int_0^{\sqrt{2}} \int_0^{x} \frac{y}{x} e^{x^2+y^2}\, dy\, dx$$

$$+ \int_{\sqrt{2}}^{2} \int_0^{\sqrt{4-x^2}} \quad dy\, dx$$

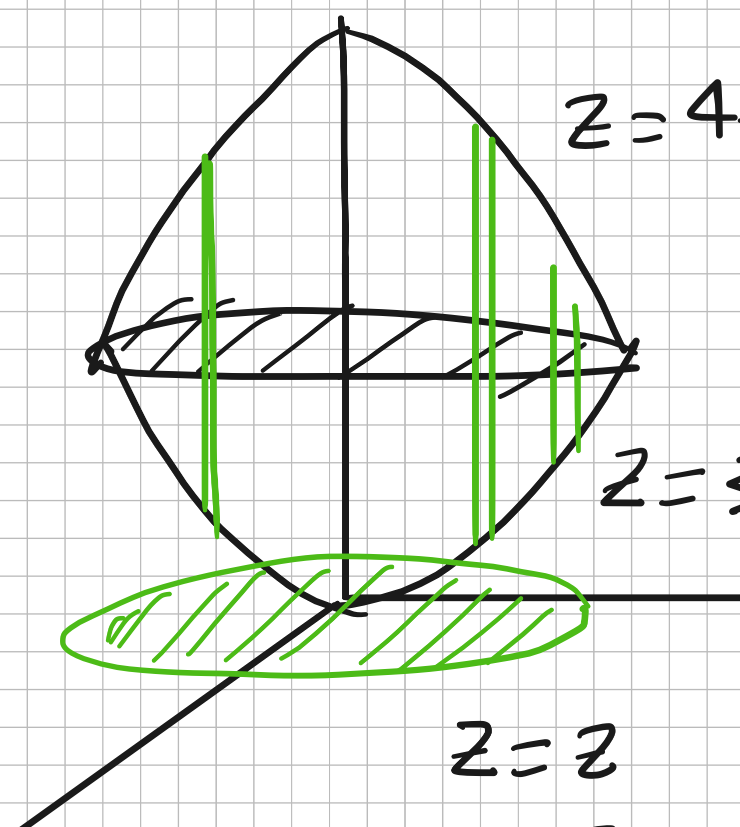
52]

In $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$



$\int_0^1 \int_0^y e^{x/y}\, dx\, dy$

26]



$z = 4 - x^2 - y^2$

$z = 3x^2 + 3y^2$

$x^2 + y^2 = 1$

$z = z$

$4 - x^2 - y^2 = 3x^2 + 3y^2$

$4 = 4x^2 + 4y^2$

$\int\int (z_{top} - z_{bot})\ dA$

$\int\int (4 - x^2 - y^2 - 3x^2 - 3y^2)\ dA$

$\int\int (4 - 4x^2 - 4y^2)\ dA$

$\int_0^{2\pi} \int_0^1 (4 - 4r^2)\, r\, dr\, d\theta$

$$\int_0^{2\pi}\int_0^1\int_{3r^2}^{4-r^2} dz \;\; r\, dr\, d\theta$$

Cylindrical

15.7

$$\int_0^1\int_0^1\left(\int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\, dx\right) dy\;\; dz$$

$$\frac{z}{y+1}\, x \Big|_0^{\sqrt{1-z^2}}$$

$$\int_0^1 \int_0^1 \frac{z\sqrt{1-z^2}}{y+1}\, dy \;\; dz$$

$$\int_0^1 \int_0^1 \frac{1}{y+1} dy \; z\sqrt{1-z^2}\; dz$$

$$\ln(y+1)\Big|_0^1 \quad \int_0^1 z\sqrt{1-z^2}\, dz$$

$$\ln 2 - \ln 1 = 0$$

$$\ln 2 \quad \int_0^1 z\sqrt{1-z^2}\, dz$$

$$\int_0^1 z(1-z^2)^{\frac{1}{2}}\,dz$$

$u = 1 - z^2$

$du = -2z\,dz$

$dz = \frac{du}{-2z}$

$$\int_0^1 \cancel{z}\,(u)^{\frac{1}{2}}\,\frac{du}{-2\cancel{z}}$$

$\frac{\ln(2)}{3}$

$$\frac{1}{-2}\int_0^1 u^{\frac{1}{2}}\,du$$

$$\frac{1}{-2}\left(\frac{2}{3}(1-z^2)^{\frac{3}{2}}\Big|_0^1\right)$$

$$\frac{1}{-2}\left(\cancel{\frac{2}{3}(0)^{\frac{3}{2}}} - \frac{2}{3}(1-0)^{\frac{3}{2}}\right)$$

$$\frac{1}{-2}\left(-\frac{2}{3}\right) = \boxed{\frac{1}{3}}$$

15.7

$\boxed{x^2 + z^2 = 4}$

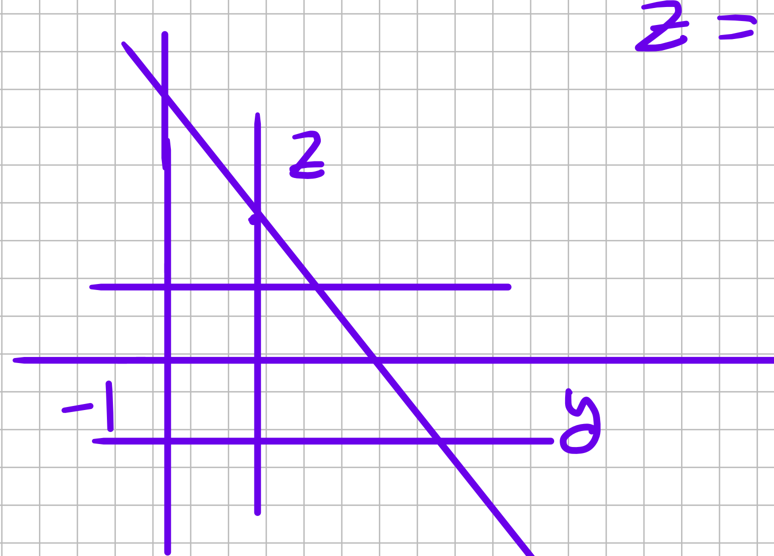
$y = -1$ $\quad y = 4 - z$

$z = 4 - y$

$x = 0$

$z = 2$

$z = -2$



$$\int\int\int_{-1}^{4-z} dy\,dz\,dx$$

$$\boxed{-1 \le y \le 4 - z}$$

$$x^2+z^2 \le 4$$

$$\int_0^{2\pi}\int_0^{2}\int_{-1}^{4-2\sin\theta} dy \quad \overbrace{r\,dr\,d\theta}^{xz}$$

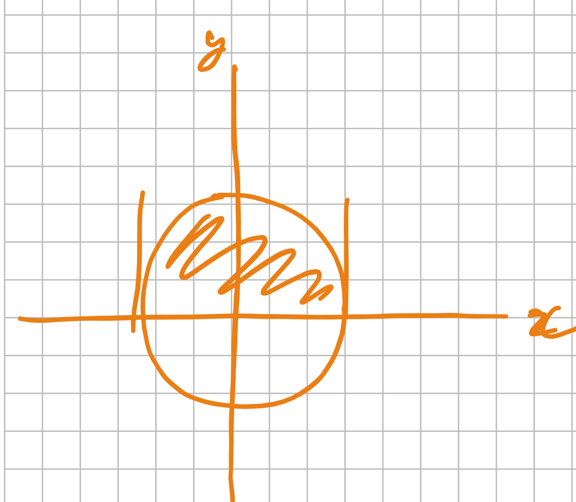
$$x = 2\cos\theta$$

$$z = 2\sin\theta$$

---

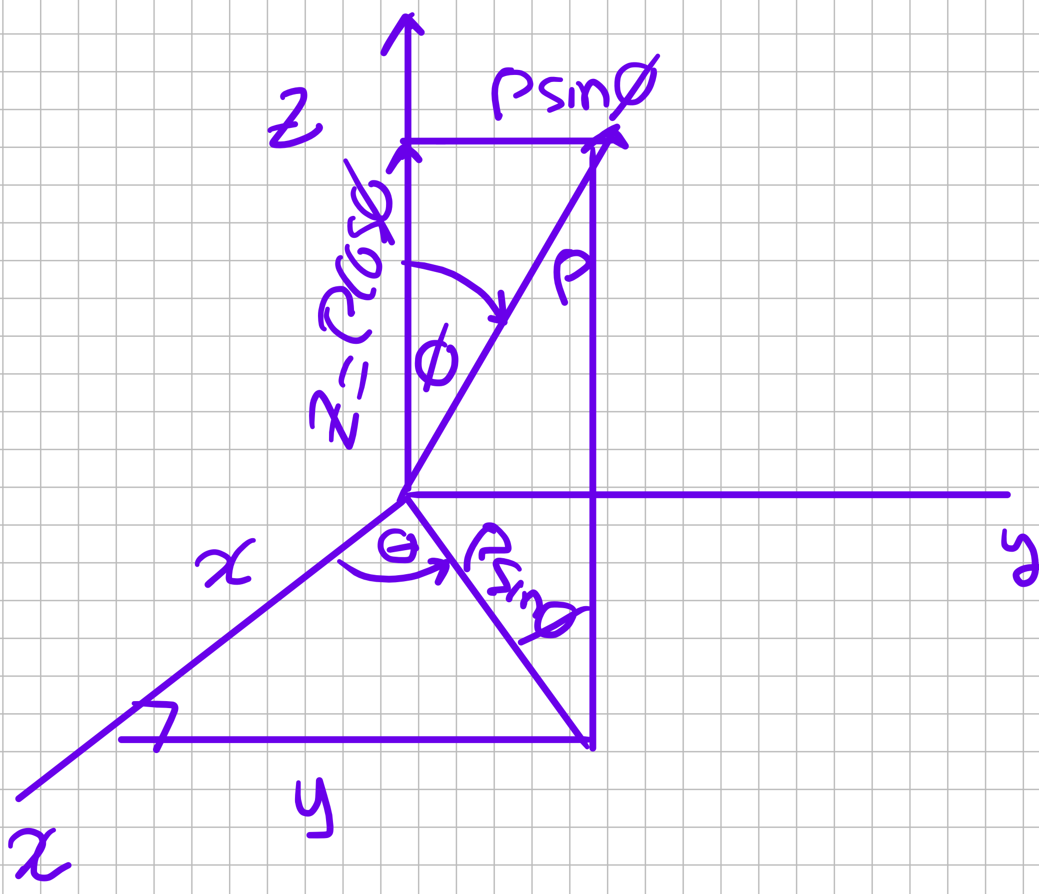
30)

$$\int_{-3}^{3}\int_0^{y=\sqrt{9-x^2}}\int_0^{z=9-x^2-y^2} \sqrt{x^2+y^2}\; dz\,dy\,dx$$



$$\int_0^{\pi}\int_0^{3}\int_0^{\boxed{9-r^2}} r^2\; dz\,dr\,d\theta$$

dz



$$\cos\theta = \frac{x}{\rho\sin\phi} \qquad x = \rho\sin\phi\cos\theta$$

$$\sin\theta = \frac{y}{\rho\sin\phi} \qquad y = \rho\sin\phi\sin\theta$$

$$X = \overbrace{\rho\sin\phi}^{r}\cos\theta$$

$$y = \overbrace{\rho\sin\phi}^{r}\sin\theta$$

$$Z = \rho\cos\phi$$

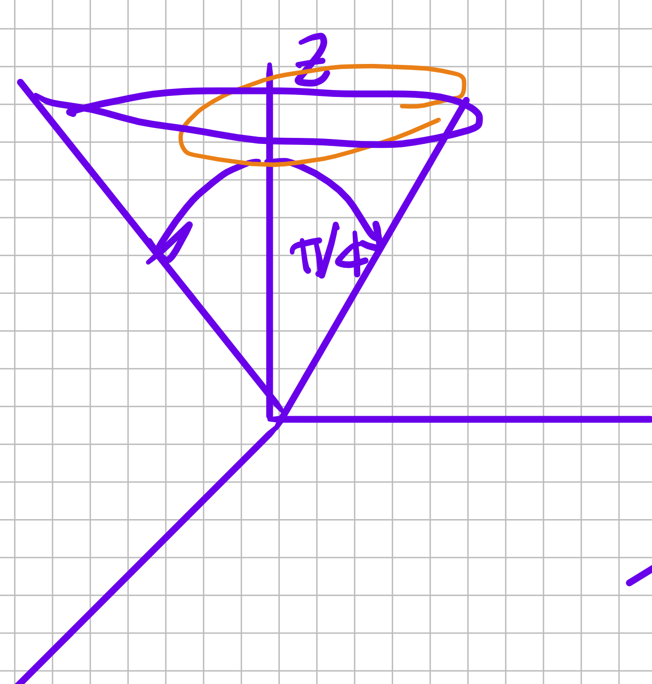
$$X^2 + y^2 + Z^2 = \rho^2$$

1)

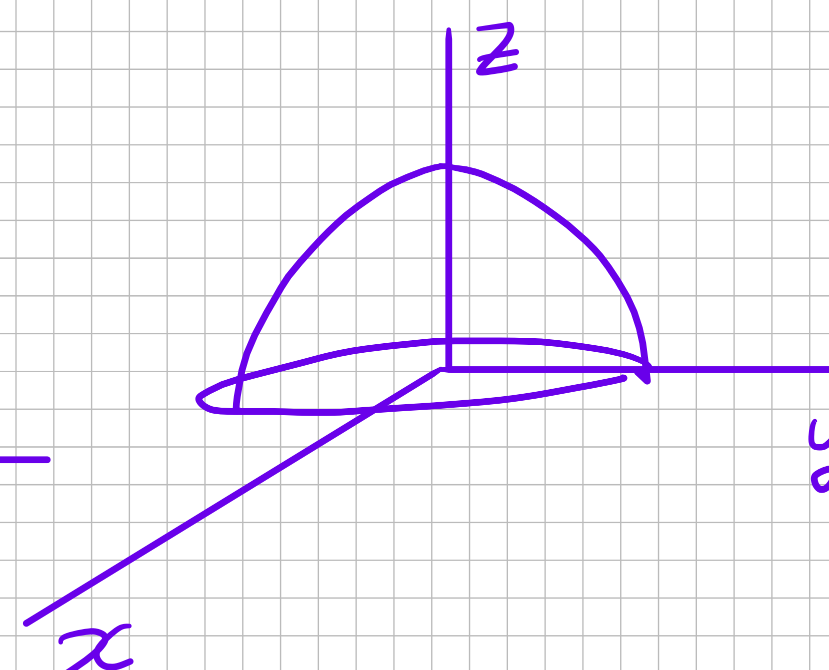
$$x^2 + y^2 + z^2 \leq 1$$

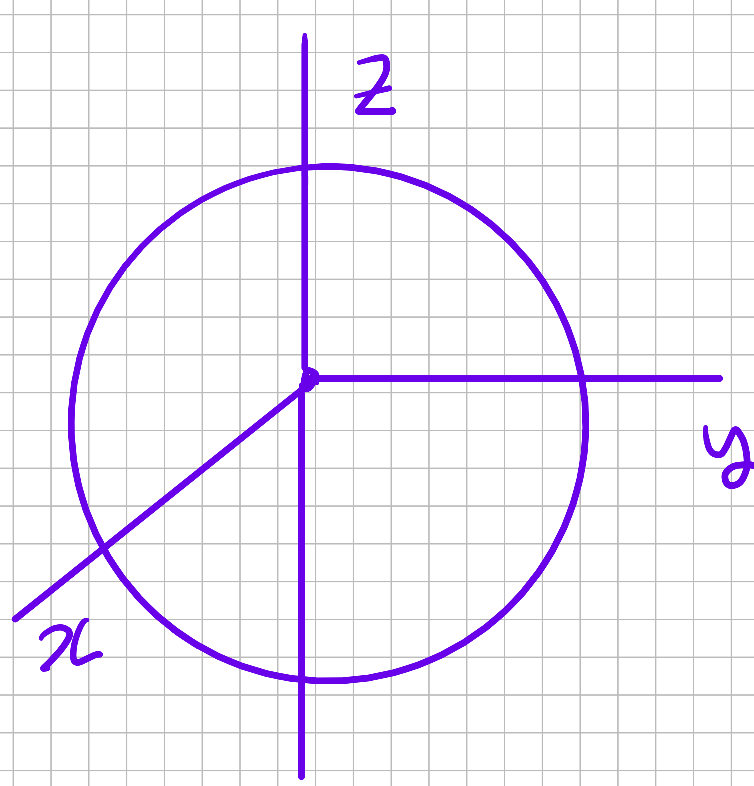
$$\int \int_0^{\pi} \int_0^{1} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$0 \leq \phi \leq \pi/4$



$0 \leq \phi \leq \pi/2$





$$\int_0^{2\pi} \int_0^{\pi} \int_0^{1} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{4\pi}{3}$$

$$\int_0^{2\pi}\int_0^{\pi} \frac{\rho^3}{3}\Big|_0^1 \sin\phi \, d\phi \; d\theta$$

$$\int_0^{2\pi} -\frac{1}{3}\cos\phi\Big|_0^{\pi} \; d\theta$$

$$\int_0^{2\pi} -\frac{1}{3}\left[\overset{-1}{\cos\pi} - \overset{1}{\cos 0}\right] d\theta$$

$$\frac{2}{3}\int_0^{2\pi} 1 \, d\theta$$

$$\frac{2}{3}\,\theta\Big|_0^{2\pi}$$

$$\boxed{\frac{4\pi}{3}}$$

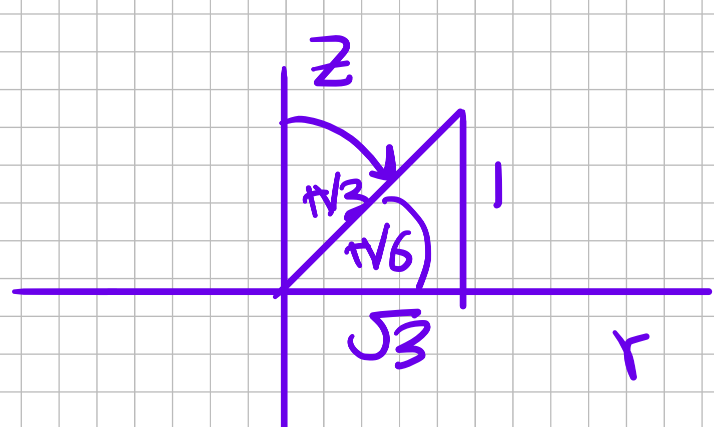
2)

mad

$$z = \frac{\sqrt{x^2+y^2}}{\sqrt{3}} \qquad x^2+y^2+z^2=1$$

$$z = \frac{r}{\sqrt{3}}$$

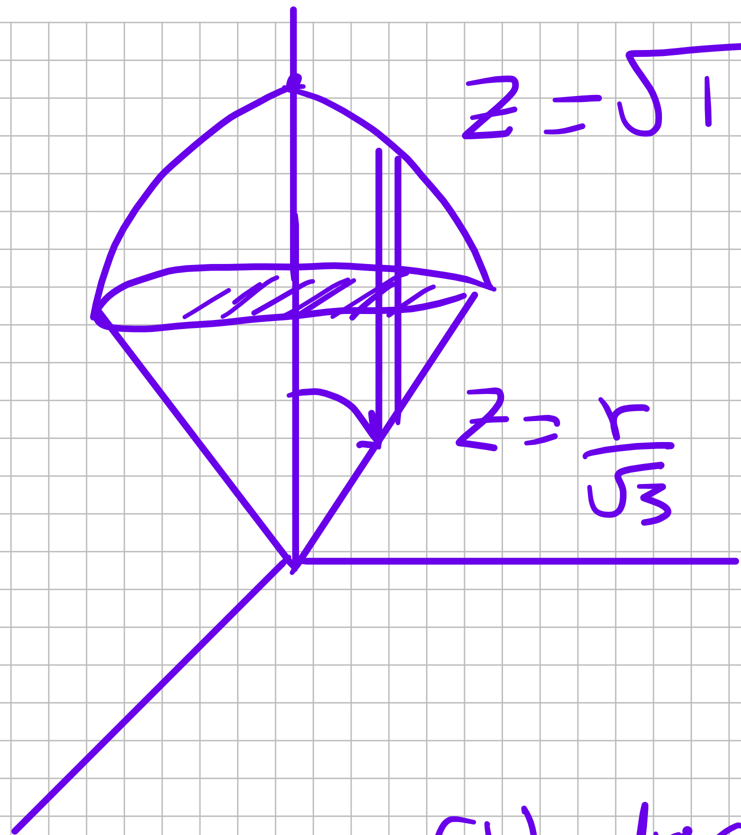
$$z = \sqrt{x^2+y^2}$$



$$\rho\cos\phi = \frac{\rho\sin\phi}{\sqrt{3}}$$

$$\sqrt{3} = \tan\phi$$

$$\phi = \tan(\sqrt{3})$$



$\boxed{z=r}$

$\cancel{\rho}\cos\phi = \cancel{\rho}\sin\phi$

$\tan\phi = 1$

$\boxed{\phi = \pi/4}$

Cylindrical

$$\int_0^{2\pi}\int_0^{\sqrt{3}/2}\int_{r/\sqrt{3}}^{\sqrt{1-r^2}} z\,dz\,r\,dr\,d\theta$$

$z=z$

$$\sqrt{1-r^2} = r/\sqrt{3}$$

$$1-r^2 = \frac{r^2}{3}$$

$$1 = \frac{r^2}{1}+\frac{r^2}{3}$$

$$1 = \frac{4r^2}{3}$$

$$r^2 = 3/4$$

$$r = \pm\frac{\sqrt{3}}{2}$$

Cylindrical

$$\int_0^{2\pi}\int_0^{\sqrt{3}/2}\int_{r/\sqrt{3}}^{\sqrt{1-r^2}} z\,dz\,r\,dr\,d\theta$$

$\boxed{\rho=1}$

$\rho^2=1$

$x^2+y^2+z^2=1$

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$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} \underbrace{(\rho\cos\phi)}_{z}\,\rho^2\sin\phi\;d\rho\,d\phi\,d\theta$$

$\boxed{y = x}$

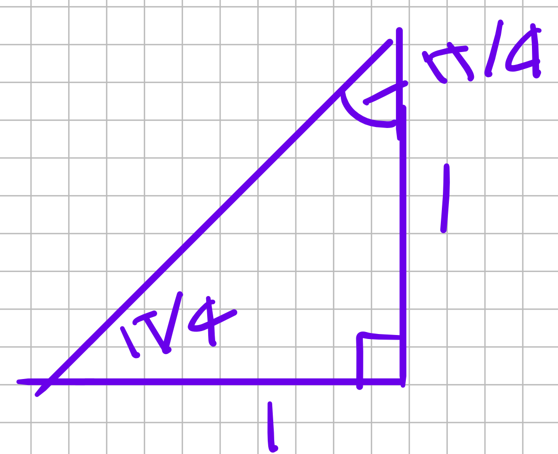
$z = \sqrt{x^2+y^2}$

$\boxed{r = \rho\sin\phi}$

$\cancel{\rho}\cos\phi = \cancel{\rho}\sin\phi$

$\boxed{\tan\phi = 1}$

$\boxed{\phi = \pi/4}$



3) $z = \sqrt{3}\sqrt{x^2+y^2}$

$x^2+y^2+z^2 = 4$

$z = \sqrt{3}\,r$

$\boxed{\rho = 2}$

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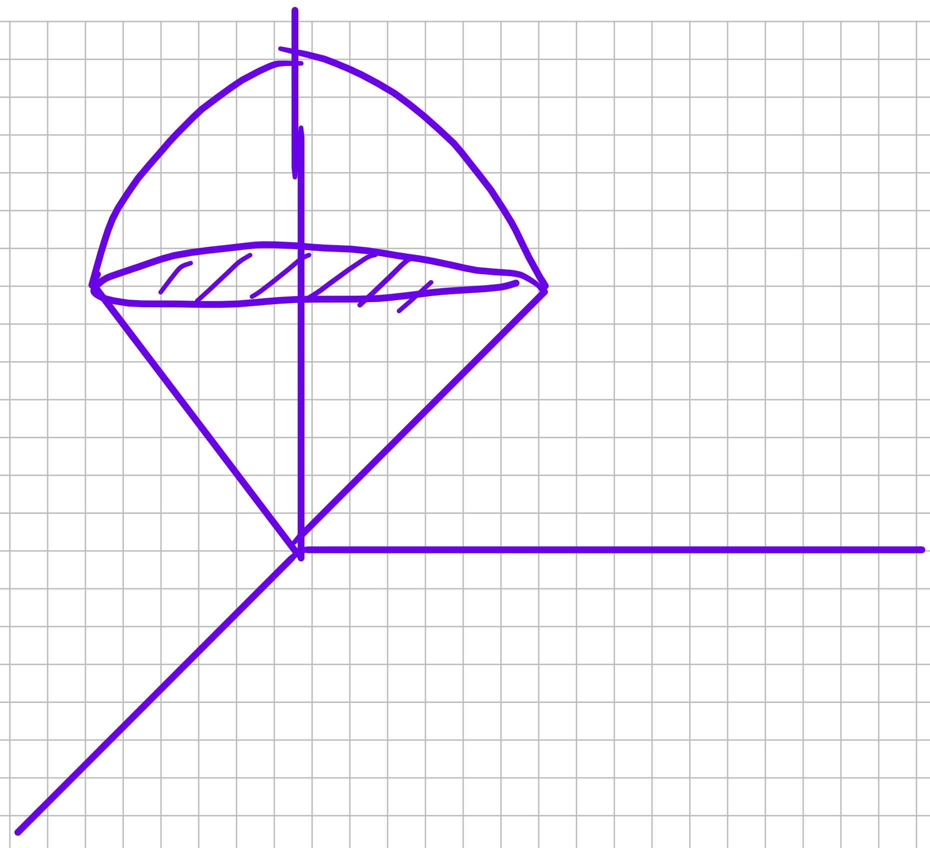


CARTESIAN

$$\sqrt{3}\sqrt{x^2+y^2} \le z < \sqrt{4-x^2-y^2}$$

$$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$$

$$-1 \le x \le 1$$

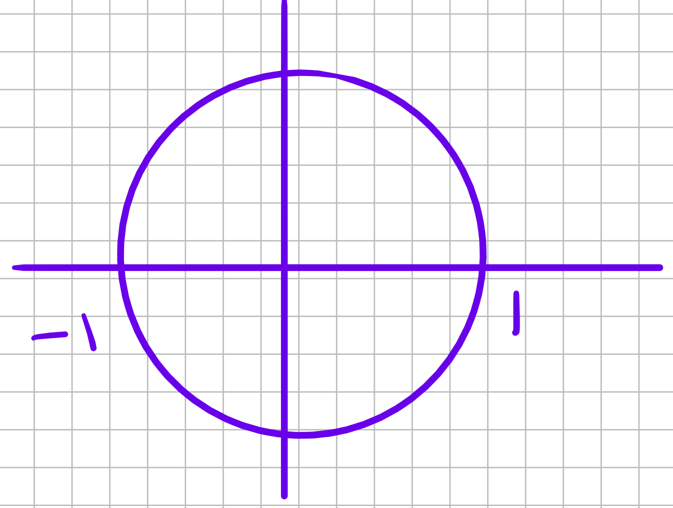
$$z = z$$

$$\left(\sqrt{3}\sqrt{x^2+y^2}\right)^2 = \left(\sqrt{4-x^2-y^2}\right)^2$$

$$3(x^2+y^2) = 4 - x^2 - y^2$$

$$4x^2 + 4y^2 = 4$$

$$x^2 + y^2 = 1$$



CARTESIAN

$$\sqrt{3}\sqrt{x^2+y^2} \le z < \sqrt{4-x^2-y^2}$$

$$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$$

$$-1 \le x \le 1$$

$$z = z$$

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{3}\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} dz \; dy \; dx$$

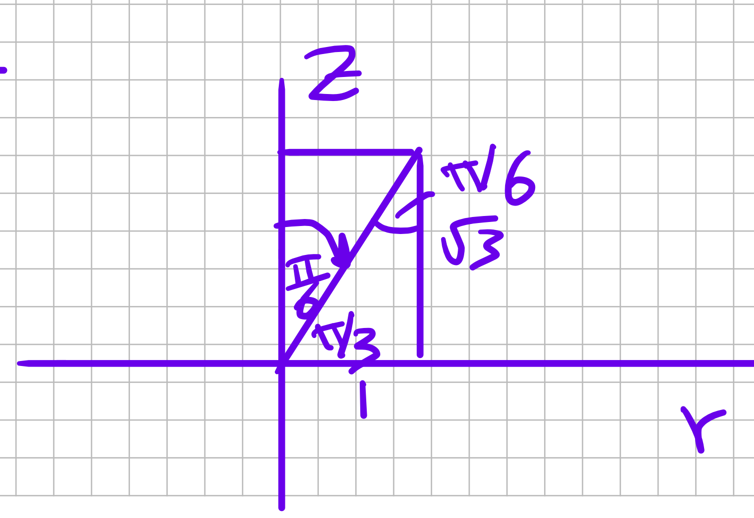
CARTESIAN

$$\int_{0}^{2\pi} \int_{0}^{1} \int_{\sqrt{3}r}^{\sqrt{4-r^2}} dz \; r \; dr \, d\theta$$

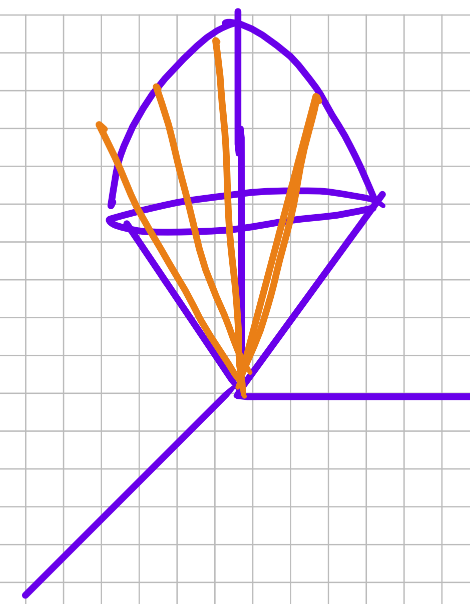
CYLINDRICAL

SPHERICAL

$$z = \frac{\sqrt{3}}{1} r$$



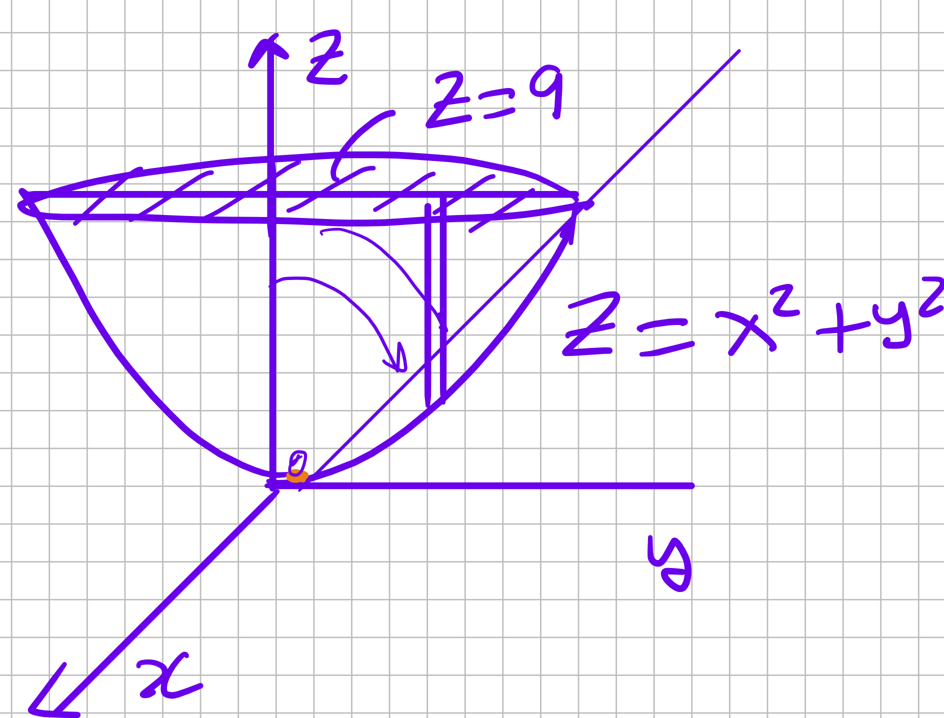
$$0 \le \rho \le 2$$
$$0 \le \phi \le \pi/6$$
$$0 \le \theta \le 2\pi$$



$$\int_0^{2\pi} \int_0^{\pi/6} \int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

5)

$z = x^2 + y^2$ $\quad z = 9$



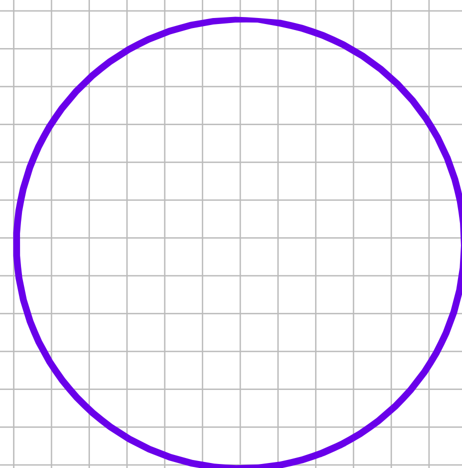
$z = z$

$x^2+y^2=9$

$$\iiint x \, dV$$

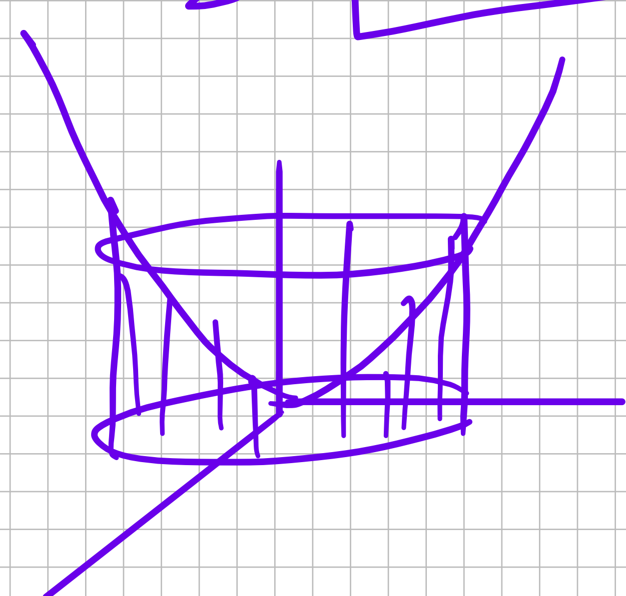
$$r^2 < z < 9$$
$$0 < r < 3$$
$$0 < \theta < 2\pi$$

$$\int_0^{2\pi}\int_0^{3}\int_{r^2}^{9} r\cos\theta \, dz \, r\,dr\,d\theta$$



$x^2+y^2=1$

$z=\boxed{x^2+y^2}$



$z = f(x,y)$ objective

$g(x,y)=0$ constraint

$$\boxed{z_x = \lambda g_x} \qquad \boxed{z_y = \lambda g_y}$$