5]

$$m_1 x_1'' + c x_1' = -k_1 x_1 + k_2 (x_2 - x_1)$$

$$m_2 x_2'' + c x_2' = -k_2 (x_2 - x_1) - k_3 x_2$$

c_i, m_i, k_i are positive

$$y_1 = x_1 \quad y_2 = x_2 \quad y_3 = x_1' \quad y_4 = x_2'$$

$$m_1 y_3' + c y_3 = -k_1 y_1 + k_2 (y_2 - y_1)$$

$$m_2 y_4' + c y_4 = -k_2 (y_2 - y_1) - k_3 y_2$$

$$m_1 y_3' = -k_1 y_1 - k_2 y_1 + k_2 y_2$$

$$m_2 y_4' = k_2 y_1 - k_2 y_2 - k_3 y_2$$

$$y_1' = y_3$$

$$y_2' = y_4$$

$$y_3' = \frac{y_1(-k_1 - k_2) + k_2 y_2}{m_1}$$

$$y_4' = \frac{k_2 y_1 + y_2(-k_2 - k_3)}{m_2}$$

$$y_1' = 0 y_1 + 0 y_2 + 1 y_3 + 0 y_4$$

$$y_2' = 0 y_1 + 0 y_2 + 0 y_3 + 1 y_4$$

$$y_3' = -\frac{(k_1 - k_2)}{m_1} y_1 + \frac{k_2}{m_1} y_2 + 0 y_3 + 0 y_4$$

$$y_4' = \frac{k_2}{m_2} y_1 + \left(\frac{-k_2 - k_3}{m_2}\right) y_2 + 0 y_3 + 0 y_4$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1-k_2)}{m_1} & \frac{k_2}{m_1} & 0 & 0 \\ \frac{k_2}{m_2} & \frac{-k_2-k_3}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

2)

$$x'' + 4x' + 5x = \delta(t-\pi)$$

$$x(0) = 0 \qquad x'(0) = 2$$

$$s^2X - s(0) - 2 + 4sX - 0 + 5X = e^{-\pi s}$$

$$X[s^2 + 4s + 5] = e^{-\pi s} + 2$$

$$X(s) = \frac{e^{-\pi s} + 2}{s^2 + 4s + 5}$$

$$X(s) = \frac{e^{-\pi s}}{s^2+4s+5} + \frac{2}{s^2+4s+5}$$

$$X(s) = e^{-\pi s} \cdot \frac{1}{s^2+4s+5} + 2 \cdot \frac{1}{s^2+4s+5}$$

$$X(s) = e^{-\pi s} \cdot \frac{1}{(s+2)^2+1} + 2\, \frac{1}{(s+2)^2+1}$$

$$x(t) = U(t-\pi)\, f(t-\pi) + 2\, f(t)$$

$$F(s) = \frac{1}{(s+2)^2+1}$$

$$f(t) = e^{-2t}\sin t$$

$$x(t) = u(t-\pi)\, e^{-2(t-\pi)} \sin(t-\pi) + 2\, e^{-2t}\sin t$$

3]

$$x'' + 4x' + 3x = 2\,\delta(t-\pi)$$

$$x(0) = 2 \qquad x'(0) = 0$$

$$s^2 X - \underline{2s} + 4sX - \underline{2} = 2e^{-\pi s}$$

$$X(s^2+4s) = 2e^{-\pi s} + 2s + 2$$

$$X(s) = \frac{2e^{-\pi s} + 2s + 2}{s^2+4s}$$

$$X(s) = \frac{2}{s^2+4s}\, e^{-\pi s} + \frac{2s+2}{s^2+4s}$$

$$X(s) = F(s)\, e^{-\pi s} + G(s)$$

$$x(t) = u(t-\pi)\, f(t-\pi) + g(t)$$

$$F(s) = \frac{2}{s^2+4s} = \frac{2}{s(s+4)}$$

$$\frac{A}{s} + \frac{B}{s+4} = \frac{A(s+4)+Bs}{s(s+4)} = \frac{2}{s(s+4)}$$

$$A(s+4) + Bs = 2$$

$s=0$ $\quad 4A = 2 \quad A = 1/2$

$s=-4$ $\quad B(-4) = 2 \quad B = -1/2$

$$F(s) = \frac{1/2}{s} + \frac{-1/2}{s+4}$$

$$\underline{f(t) = 1/2 - \frac{1}{2}e^{-4t}}$$

$$X(s) = \frac{2}{s^2+4s}e^{-\pi s} + \frac{2s+2}{s^2+4s}$$

$$X(s) = F(s)e^{-\pi s} + G(s)$$

$$x(t) = u(t-\pi)f(t-\pi) + g(t)$$

$$\frac{2s+2}{s^2+4s}$$

$$G(s) = \frac{2s+2}{s(s+4)}$$

$$\frac{A}{s} + \frac{B}{s+4} = \frac{A(s+4)+Bs}{s(s+4)} = \frac{2s+2}{s(s+4)}$$

$$A(s+4) + Bs = 2s+2$$

$s=0$ $\quad 4A = 2 \quad A = 1/2$

$s=-4$ $\quad B(-4) = -8+2 \quad -4B = -6$

$$\boxed{B = 3/2}$$

$$G(s) = \frac{1/2}{s} + \frac{3/2}{s+4}$$

$$g(t) = 1/2 + \frac{3}{2} e^{-4t}$$

$$f(t) = \frac{1}{2} - \frac{1}{2} e^{-4t}$$

$$X(s) = F(s) e^{-\pi s} + G(s)$$

$$x(t) = u(t-\pi) f(t-\pi) + g(t)$$

$$x(t) = u(t-\pi)\left[\frac{1}{2} - \frac{1}{2} e^{-4(t-\pi)}\right] + \frac{1}{2} + \frac{3}{2} e^{-4t}$$