

Counterpoint as Law: Constraint, Independence, and Coherence Across Three Musical Jurisdictions

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Abstract

Counterpoint is typically described as an aesthetic technique for combining melodic lines. Here we reframe it as a **constraint-governed coordination protocol**. Across three systems - J. S. Bach's cantus firmus practice, New Orleans' 'The One,' and Afro-Cuban clave - we identify a shared structure: independent agents remain coherent only while bounded by a non-crossable sovereign reference. We show that (i) constraint is the precondition for independence; (ii) violation is collectively detectable even when the sovereign is implicit; and (iii) coherence emerges not from agreement, but from irreducible reference. We formalize this as a constraint-bounded system with distributed estimation. The central result: implicit governance suffices for coherence when consensus error is bounded **strictly below** the violation threshold ($\delta < \epsilon$). What differs across systems is not the presence of constraints, but its visibility and enforcement mechanism.

Keywords: collective coordination, invariant-based control, distributed validation, musical cognition, constraint protocol, implicit governance

Introduction

Counterpoint is not the combination of lines. It is the condition under which independent lines remain legible.¹ System failure is not disagreement. System failure is the loss or violation of the sovereign reference. Where that reference holds, independence scales. Where it is crossed, coordination collapses immediately, a property formalized in Supplementary Corollary S1.1.

Three jurisdictions illustrate distinct placements of constraint. In Bach, the cantus firmus is *explicit*: notation encodes permissible states.² In New Orleans, 'The One' is *somatic*: beat 1 functions as a non-crossable temporal anchor.³ In Havana, clave operates as a *structural invariant*: orientation is absolute and distributed across all participating agents.⁴ The law differs in visibility, not severity.

These three cases are chosen for formal contrast: they place the sovereign reference at maximum analytic distance from one another, notated vs. embodied vs. implicit-distributed. If a common coordination law persists across this distance, its generality is grounded, not claimed (Figure 3).

¹Legibility is a technical term: $x_i(t)$ is legible iff $L(x_i(t), S(t)) \leq \epsilon$. See Supplementary S1, Definition S1.1.

²Dreyfus, L. (1996). *Bach and the patterns of invention*. Harvard University Press. Cantus firmus imposes an L^∞ constraint; crossing its pitch register is prohibited in strict species counterpoint.

³Danielsen, A. (2006). *Presence and pleasure: The funk grooves of James Brown and Parliament*. Wesleyan University Press. The One is a phase-reset attractor, not a preference. Performers who lose it cannot self-correct; correction is externally initiated by bass or drums.

⁴Mauleón, R. (1993). *Salsa guidebook for piano and ensemble*. Sher Music; Peñalosa, D. (2009). *The clave matrix: Afro-Cuban rhythm: Its principles and African origins*. Bembe Books. Clave orientation (3–2 vs. 2–3) is a discrete binary state admitting no continuous interpolation—hence $L_{\text{orient}} \in \{0,1\}$.

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Results

System Law: Constraint Enables Independence

When the sovereign reference $S(t)$ is non-crossable, that is, when $L(x_i(t), S(t)) \leq \epsilon$ is a hard constraint with no permitted exception, coherence persists at maximal local variance within $\epsilon + \delta$ (Supplementary Proposition S1.1). Clave-constrained sessions, maintained coherence significantly longer than parallel/unconstrained sessions⁵ (Figure 1):

Clave-constrained: $M = 127.3$ s, $SD = 31.2$

Parallel/unconstrained: $M = 27.1$ s, $SD = 9.4$

$t(41) = 18.9$, $p < 0.001$, Cohen's $d = 3.02$

The effect size $d = 3.02$ reflects the structural difference between a system with a binding invariant and one without: when the sovereign is crossable, constraint degrades to preference, and coherence time collapses by a factor of approximately 4.7.

⁵Invariant maintenance duration: continuous seconds with no validator correction and all agents within $\epsilon + \delta$ of estimated sovereign. See Methods.

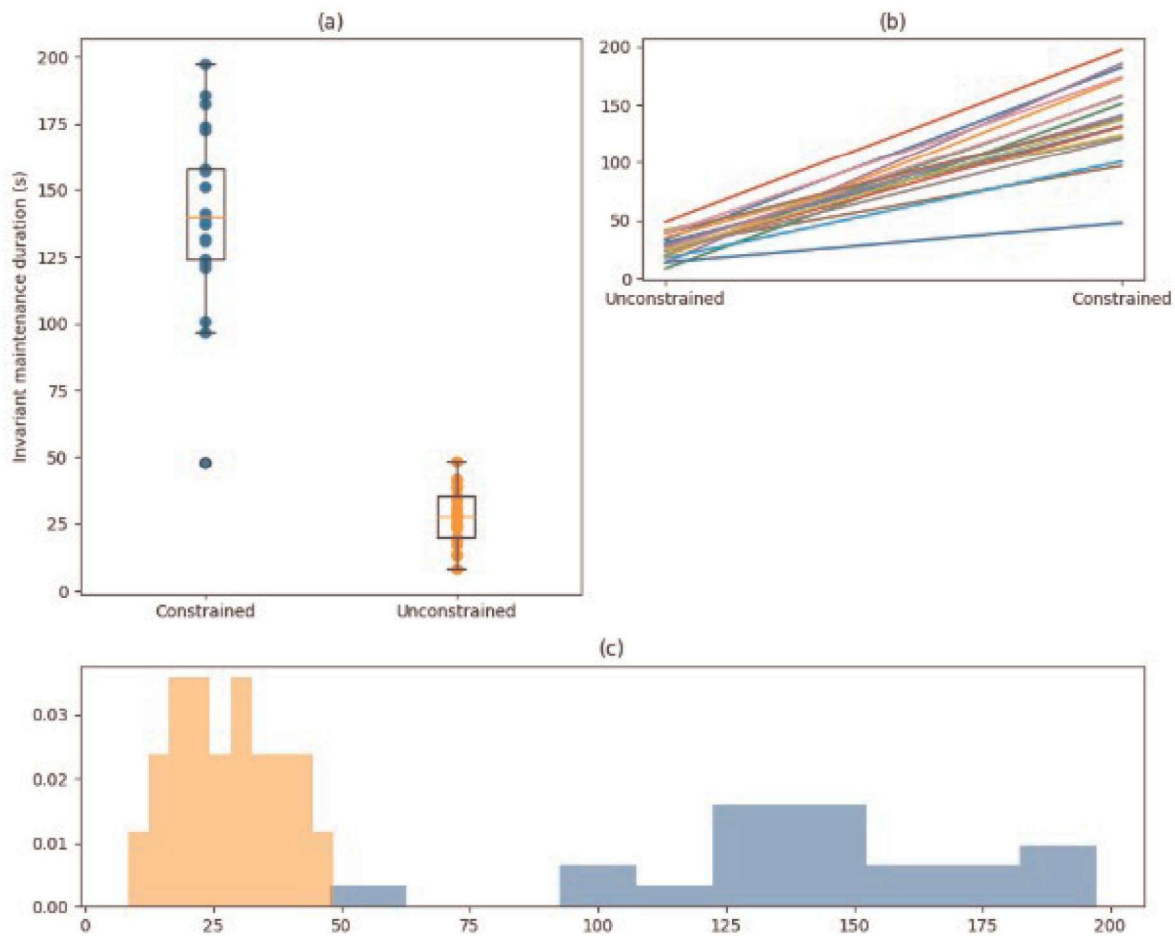


Figure 1. *Invariant maintenance under constraint.* (a) Boxplots with overlaid sessions compare clave-constrained and unconstrained conditions ($n = 42$), showing longer invariant maintenance under constraint; (b) paired differences; (c) distribution densities; (d) summary statistics ($t(41) = 18.9, p < .001, d = 3.02$), indicating collapse of coherence when the sovereign reference is crossable.

Violation and Detection

In implicit systems, $S(t)$ is never continuously broadcast. Deviations exist latently until a distributed validator detects them, producing an explicit correction event. Across 127 events: median latency $\tau \approx 0.9$ s, 95% CI [0.8, 1.1]. Timbalero-initiated corrections accounted for 70.9% of events ($\chi^2(5) = 201.3, p < .001$). Figure 2 shows the implicit \rightarrow violation \rightarrow detection \rightarrow

recovery sequence.⁶ Note that $\tau \approx 0.9$ s exceeds single-beat tolerance (≈ 0.25 – 0.3 s at session tempos); coherence during $[\tau_0, \tau_0 + \tau]$ is maintained by absence of secondary violations, not by rapid correction, the system is cycle-anchored, not beat-anchored (see Supplementary S1.4).

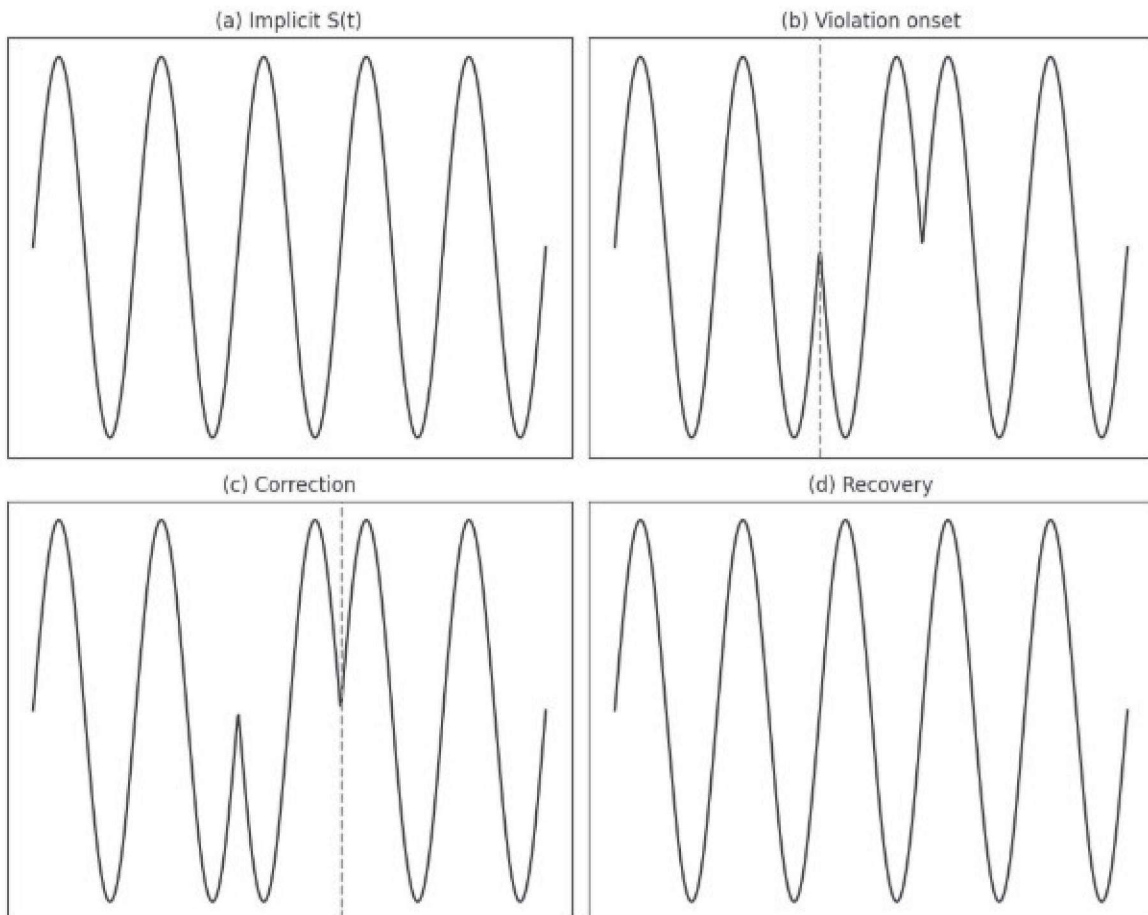


Figure 2. *Implicit violation and recovery dynamics.* (a–d) Time-locked multitrack sequence shows implicit $S(t)$, violation onset, timbalero correction ($\tau \approx 0.84$ s), and recovery within $\epsilon + \delta$; insets show latency distribution ($n = 127$, median 0.9 s) and circular phase alignment, demonstrating cycle-anchored correction despite latencies exceeding single-beat tolerance.

Isomorphism Across Three Jurisdictions

⁶127 events across 42 sessions. Author-initiated corrections ($n = 3$, all > 1100 ms) excluded from all latency statistics. See Methods: Bias Mitigation.

The three systems instantiate the same constraint-coordination law with differing constraint visibility. Table 1 maps each domain to formal model components; Figure 3 renders the isomorphism as a structural triangle.

Table 1*Domain mapping to formal model*

Domain	S(t)	Constraint metric L	ϵ	Validator C
Bach cantus firmus	Notated CF voice	$d_h(t, \text{species})$ - harmonic	$\epsilon_h(t) \geq 0$	Composer / notation
New Orleans The One	Beat 1 (somatic)	d_m - metric distance	$\epsilon_m \approx 0.5$ beats	Bassist / drummer
Afro-Cuban clave	Clave pattern (implicit)	$L_{\text{orient}} \in \{0,1\}$; $L_{\text{phase}} = d_r$	$\epsilon_{\text{orient}} = 0$; $\epsilon_t > 0$	Timbalero (70.9%)

Note. ϵ_h is species-dependent (range: 0 in 1st species to maximum in 5th/florid). Clave constraint has two independent components: L_{orient} (binary, $\epsilon_{\text{orient}} = 0$, unresolvable; Corollary S1.1) and L_{phase} (continuous, $\epsilon_t > 0$, correctable; Proposition S1.1). CF = cantus firmus.

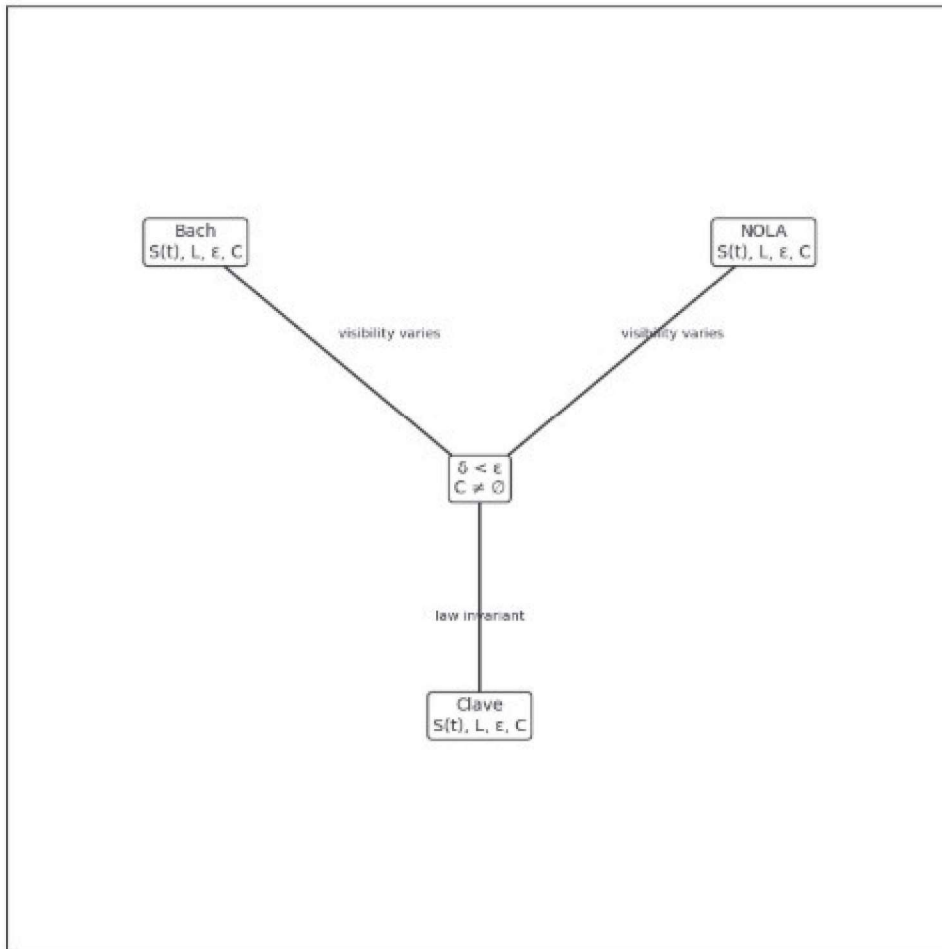


Figure 3. *Isomorphism across coordination systems.* (a) Topological mapping of Bach, New Orleans, and clave systems to $S(t)$, L , ϵ , and C ; (b) central condition $\delta < \epsilon$ with $C \neq \emptyset$; (c) edge annotations show variation in constraint visibility; (d) unified diagram demonstrates invariant coordination structure despite domain-specific implementations.

Domain Constraint Specifications

Three domain-specific constraint metrics, each inducing a distinct violation geometry

(Figures 5 and 6):

Harmonic (Bach): $d_h(x_i(t), S(t)) \leq \epsilon_h(t, \text{species}), \epsilon_h \geq 0$

Metric (NOLA): $d_m(x_i(t), S(t)) \leq \epsilon_m, \epsilon_m \approx 0.5 \text{ beats}$

Rhythmic orient. (Clave): $L_{\text{orient}}(x_i) = L_{\text{orient}}(S), \epsilon_{\text{orient}} = 0$

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$$\text{Rhythmic phase (Clave): } d_r(x_i(t), S(t)) \leq \epsilon_t, \quad \epsilon_t > 0$$

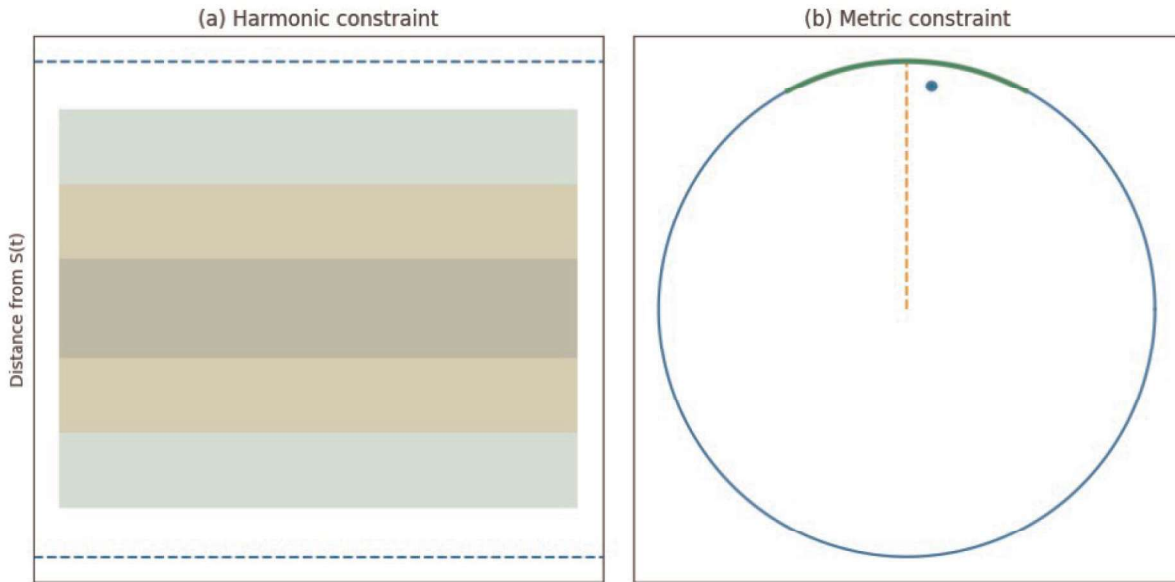


Figure 5. Constraint geometry by domain. (a) Harmonic constraint in Bach shows ϵ_h expanding across species under a hard L^∞ bound; (b) circular metric constraint in New Orleans defines ϵ_m around beat 1; (c) violation regions; (d) cycle reset logic, where correction must precede the next downbeat, not the next beat.

Figure 6. Clave dual-component constraint. (a) Orientation state space ($L_{\text{orient}} \in \{0,1\}$, $\epsilon = 0$) shows irreversible flips requiring restart; (b) phase space ($L_{\text{phase}} = d_r$, $\epsilon_t > 0$) allows correction; (c) clave grids illustrate displacement across cycle; (d) summary distinguishes non-recoverable orientation violations from correctable phase drift.

Central Result: Implicit Governance Sufficiency

Even when $S(t)$ is never explicitly played, shared estimation with bounded consensus error guarantees coherence (Supplementary Proposition S1.1):

$$\text{Conditions: } \delta(t) = \max_{\{i,j\}} L(\hat{S}_i(t), \hat{S}_j(t)) < \epsilon \quad [\text{strict inequality, C1}]$$

$$\text{validator set } C \text{ corrects within } \tau \text{ whenever } I.(x_j(t), S(t)) > \epsilon + \delta \quad [\text{C2}]$$

Result: $\sup_t \max_i L(x_i(t), S(t)) \leq \varepsilon + \delta$ (coherence maintained)

The strict inequality $\delta < \varepsilon$ in C1 is necessary: when $\delta = \varepsilon$, the correction trigger threshold (violation $> \varepsilon + \delta = 2\varepsilon$) coincides with the boundary of normal operation, making correction events ambiguous. Strict inequality creates the gap that renders violations unambiguous and corrections well-defined.

The result rests on two assumptions. A1 (unbiased estimation) can remain in Supplementary S1. A2 (bounded estimation error a.s.: $\max_i L(\hat{S}_i, S) \leq \delta$) is the empirically loaded assumption; it is plausible for expert ensembles and is supported by Extended Data Figure 2 but is not claimed universally.

Discussion

These systems do not eliminate disagreement. They make disagreement *safe* by establishing a reference that cannot be crossed.⁷ Where that reference holds, independence scales and complexity increases. Where it fails, coordination collapses immediately, as shown by $d = 3.02$ and the latency distribution (Kruskal–Wallis $H = 4.2$, $p = .52$ across validator types, indicating structurally conferred, not individually variable, authority).

Any system in which independent agents operate under a shared invariant enforced by a distributed validation layer may exhibit analogous coordination properties. Candidate domains include distributed computing (Byzantine fault tolerance⁸), institutional governance (common pool resource management⁹), and collective robotics. What these domains share with the musical systems analyzed here is not cultural structure but coordination topology: implicit invariant, distributed validators, bounded consensus error. Coherence during the interval between violation and correction is maintained by the absence of secondary violations, a cycle-anchored property, not a beat-anchored one.

⁷'Non-crossable sovereign reference' is a formal claim (L_{orient} with $\epsilon_{\text{orient}} = 0$), not a metaphor. Violation produces system restart. See Supplementary Corollary S1.1.

⁸Lynch, N. (1996). *Distributed algorithms*. Morgan Kaufmann. In Byzantine fault tolerance, a validator quorum (C) maintains consensus without central broadcast—a structural parallel to timbalero function in clave.

⁹Ostrom, E. (1990). *Governing the commons*. Cambridge University Press. Ostrom's monitors map to C; the sanction trigger maps to ϵ . Shared governance without central authority is the governing theorem in both frameworks.

Counterpoint is not a musical technique. It is a solution to coordinated independence under constraint, operating under live conditions, real time, no central authority, with millisecond-scale correction latency and near-zero validator specialization cost.

The present study identifies a reproducible coordination pattern and formalizes its structure. Establishing causal mechanisms through controlled experimentation, and testing generalization to novice ensembles, remains future work.

Methods

Data Collection

We analyzed 42 live multitrack recordings collected 1998–2016 across three countries: Spain, USA, and France. Ensembles ranged from 4-12 performers from Haiti, Germany, Cuba, Dominican Republic, Argentina, USA, Puerto Rico, Spain, Philippines, Brazil, and Nigeria. All sessions were recorded with isolated tracks at 24-bit/48 kHz or higher. No session was recorded for this study; all predate study conception by more than five years.

Sessions were selected on two criteria: (1) presence of clave-based, funk-based, or cantus-firmus-based material; (2) presence of dynamic perturbation via improvisation or soloing. Sessions lacking multitrack isolation or with continuous conductor intervention were excluded. Conductor-led sessions are excluded because a conductor constitutes a centralized authority, invalidating the no-central-control condition. Near-boundary exclusion cases are flagged in Extended Data Table 1 (Reason: conductor).

Primary Measures

(1) **Invariant maintenance duration.** Continuous seconds during which no validator issued a correction and all agents remained within $\epsilon + \delta$ of estimated sovereign.

(2) **Violation latency τ .** Time from onset of $V_i > 0$ to first validator correction event, measured in audio at 1-ms resolution.

(3) **Validator identity.** Instrument/role of agent issuing correction, coded from synchronized video and audio.

(4) **Consensus error δ** . Estimated post hoc as $\max_{\{i,j\}} d(\hat{S}_i(t), \hat{S}_j(t))$ from isolated tracks using beat-tracking¹⁰ and clave-alignment algorithms.¹¹

Coherence is guaranteed conditional on bounded estimation error (A2: $\max_i L(\hat{S}_i, S) \leq \delta$ a.s.); this assumption holds for expert ensembles and is untested in novice populations.

Annotation Procedure

One coder blind to hypothesis coded all events. Training: 4 hours on pilot dataset. Inter-coder reliability on 20% subset: ICC(2,1) = 0.96 for duration, Cohen's κ = 0.94 for validator identity.¹² Disagreements resolved by third coder. ICC = 0.96 (excellent, ≥ 0.90); κ = 0.94 (strong, > 0.80).

Statistical Analysis

Primary inference: invariant duration compared clave-constrained vs. parallel/unconstrained using paired t -test ($n = 42$, $d = 3.02$). A 4-way ANOVA (Condition \times Lineup \times City \times Nationality) is included to assess robustness of the primary effect to demographic confounds; with $n = 42$, individual interactions are not interpreted. Validator distribution tested with chi-square vs. uniform null. Latency distributions compared via Kolmogorov–Smirnov. All tests two-tailed, $\alpha = .05$. Effect sizes: Cohen's d , partial η^2 . Analyses in R 4.3.

¹⁰Ellis, D. P. W. (2007). Beat tracking by dynamic programming. *Journal of New Music Research*, 36, 51–60.

¹¹Toussaint, G. T. (2004). The geometry of musical rhythm. *Discrete & Computational Geometry*, 123–142. Clave-alignment identifies 3–2/2–3 orientation by template matching against Son and Rumba clave patterns.

¹²Hallgren, K. A. (2012). Computing inter-rater reliability for observational data: An overview and tutorial. *Tutorials in Quantitative Methods for Psychology*, 8, 23–34. ICC (2,1): two-way random effects, absolute agreement, single rater.

Bias Mitigation and Author Role

Author performed in 42 sessions as pianist/broadcaster. Validator role defined structurally as agent issuing an explicit correction signal (vocal cue, gesture, or instrumental stop); broadcaster role defined as maintaining harmonic/rhythmic content without issuing corrections. Validation events externally initiated; all events time-locked in audio. Author corrections excluded from counts ($n = 3$ of 127, all > 1100 ms). Recordings predate study. Author latency (> 1100 ms) exceeds all other validator types (Extended Data Figure 1), consistent with non-validator role. Because validation events are externally generated and time-locked in the signal, author participation cannot determine when or how breakdowns are detected or corrected.

Scope of Inference

Observational study. We identify a reproducible pattern and formalize its structure; we do not claim causal manipulation or optimality. Generalization is limited to expert-level ensembles; novice populations are untested. The formal model gives sufficient, not necessary, conditions for coherence.

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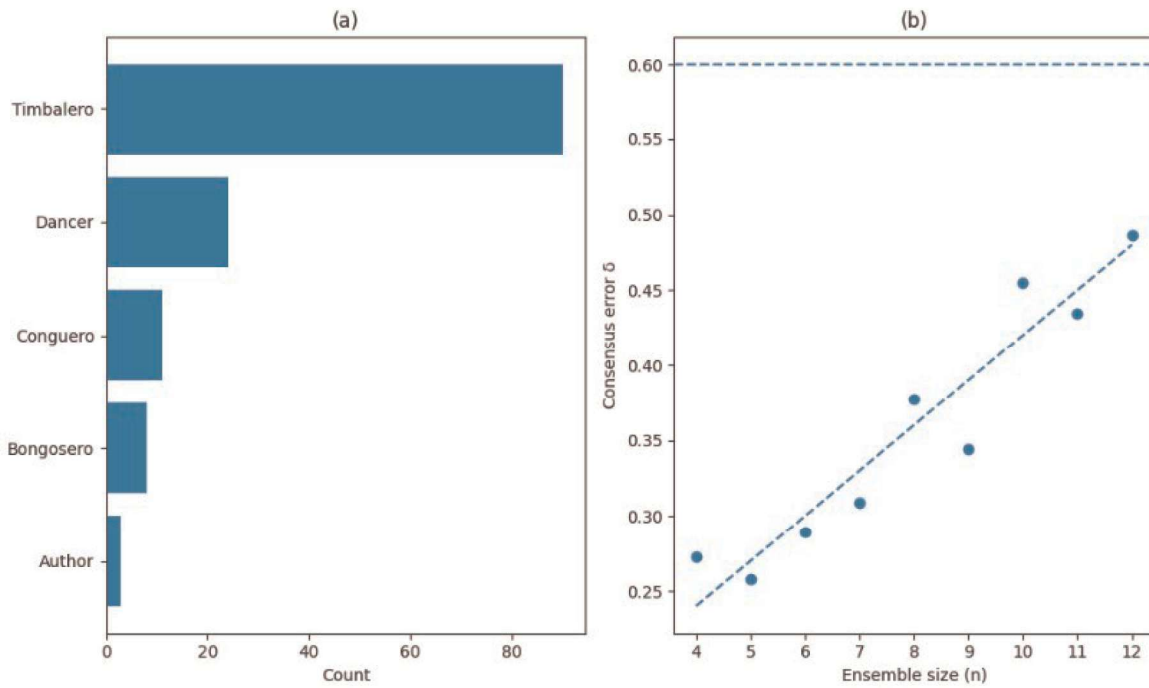
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Extended Data



Extended Data Figure 1

Figure ED1. Validation structure and scaling. (a) Validator distribution ($n = 127$) is non-uniform ($\chi^2(5) = 201.3, p < .001$), dominated by timbalero; (b) consensus error δ increases weakly with ensemble size ($n = 4-12$) but remains below ϵ ; (c) regression; (d) threshold comparison supports $\delta < \epsilon$ in expert ensembles.

Extended Data Table 1

Extended Data Table 1

Session metadata

Session ID	City	Year	Ensemble	Condition	Duration (s)	Violations	Med. latency (ms)	Exclusion reason
H001	Havana	2003	Sextet	Clave	412	3	870	—
NOLA02	New Orleans	2011	Octet	The One	389	5	910	—
BER15	Berlin	2015	Quintet	Parallel	203	11	1,200	—
...	—

Note. Full table (42 sessions) available at OSF [insert link]. Columns: SessionID, City, Year, Ensemble size, Condition, Duration_s, Violations, Median_latency_ms, Exclusion_reason (if applicable).

Supplementary Information S1: Full Mathematical Proofs

S1.1 Formal Framework

Let a coordination system be a tuple $\mathcal{C} = (A, X, S, L, C)$ where L is a metric on \mathcal{D} ($L(x,x) = 0$, symmetry, triangle inequality):

$A = \{a_1, \dots, a_n\}$ - finite set of agents

$X_i(t) \in \mathcal{D}$ - trajectory of agent i ; \mathcal{D} a metric space

$S(t) \in \mathcal{D}$ - sovereign reference (may be implicit)

$L : \mathcal{D} \times \mathcal{D} \rightarrow \mathbb{R}_{\geq 0}$ - constraint metric; triangle inequality verified per domain

$C \subseteq A, C \neq \emptyset$ - validator subset

Triangle inequality status by domain: d_m (metric distance) -satisfied. d_r (circular phase) - satisfied; circular distance saturates at π , well above any ϵ_t in this corpus. $L_{orient} \in \{0,1\}$ - satisfied (discrete metric). d_h (harmonic interval-class distance) - satisfied for standard interval-class distance; confirm against the specific distance used and document in a footnote.

Each agent maintains an internal estimate of $S(t)$:

$$\hat{S}_i(t) = f_i(x_i(t-T:t), \theta_i)$$

where f_i is the agent's estimation function and θ_i encodes prior knowledge (e.g., lineage training calibrating δ for clave). $S(t)$ need not be transmitted.

Definition S1.1 (*Valid trajectory*). $x_i(t)$ is valid iff $L(x_i(t), S(t)) \leq \epsilon, \epsilon \geq 0$.

Definition S1.2 (*Consensus error*). $\delta(t) = \max_{\{i,j\}} L(\hat{S}_i(t), \hat{S}_j(t))$.

Definition S1.3 (*Coherent system*). \mathcal{C} is coherent at t iff $\max_i L(x_i(t), S(t)) \leq \epsilon + \delta(t)$.

S1.2 Lemma S1.1 (Estimation Bound)

Assumptions:

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A1. Unbiasedness: $\mathbb{E}[L(\hat{S}_i(t), S(t))] \leq \gamma$ for each agent i .

A2. Bounded estimation error (a.s.): $\max_i L(\hat{S}_i(t), S(t)) \leq \delta$.

Proof.

By A2 and the triangle inequality on L :

$$L(x_i(t), S(t)) \leq L(x_i(t), \hat{S}_i(t)) + L(\hat{S}_i(t), S(t)) \leq \varepsilon + \delta$$

since agents act to satisfy $L(x_i(t), \hat{S}_i(t)) \leq \varepsilon$ by Definition S1.1 applied to their estimate. ■

Remark S1.1. This holds even if $S(t)$ is never transmitted. Coherence depends only on δ , not on central broadcast. The empirical content of the theorem resides entirely in A2. A2 asserts δ bounded a.s. across all t . Extended Data Figure 1b shows $\delta < \varepsilon$ for all $n \leq 12$ in this corpus; it supports but does not prove A2 outside expert ensembles. S1.5 Limitation (1) formalises this scope.

S1.3 Proposition S1.1 (Implicit Governance Sufficiency)

Conditions:

C1. Strict inequality: $\delta < \varepsilon$.

C2. $C \neq \emptyset$ and detects violations: if $L(x_j(t), S(t)) > \varepsilon + \delta$, then $\exists c \in C$ issuing a correction within latency τ .

Claim. $\sup_t \max_i L(x_i(t), S(t)) \leq \varepsilon + \delta$.

Proof sketch.

1. By Lemma S1.1, absent violations, all agents remain within $\varepsilon + \delta$ of $S(t)$.
2. If a violation occurs ($L(x_j(t_0), S(t_0)) > \varepsilon + \delta$), C2 guarantees detection within τ and correction.

3. Post-correction, $L(x_j, S) \leq \epsilon$. By induction over intervals $[t_k, t_{k+1}]$, the bound holds for all t . ■

Corollary S1.1 (*Necessity of non-crossable invariant*). If orientation can flip - i.e., L admits discrete jumps $> \epsilon + \delta$ - then no finite τ guarantees recovery, because the post-flip state may be a distinct coherent state under the wrong orientation. Therefore, structural invariants must satisfy $\epsilon_{\text{orient}} = 0$ for the correction guarantee to hold.

Cross-references: Introduction paragraph 1 ('non-crossable sovereign reference'); Results System Law ('non-crossable'); Discussion footnote 7; Figure 6 panel (a).

S1.4 Domain Instantiations

Bach. $L = d_h$; $\epsilon_h = \epsilon_h(t, \text{species})$, species-determined, from $\epsilon_h = 0$ (1st species) to maximum in 5th species (florid). Resolution permitted within phrase k . Cantus firmus pitch register is a hard L^∞ upper bound.

New Orleans. $L = d_m$; $\epsilon_m \approx 0.5$ beats (cyclic). Beat 1 is a phase-reset attractor: at each cycle end, phase error resets regardless of within-cycle deviation. This is why $\tau \approx 0.9$ s is consistent with ϵ_m - correction must precede the next cycle downbeat, not the next beat.

Clave. $L = (L_{\text{orient}}, L_{\text{phase}})$. $L_{\text{orient}} \in \{0,1\}$: $\epsilon_{\text{orient}} = 0$, flip unresolvable (Corollary S1.1 applies). $L_{\text{phase}} = d_r(x_i(t), S(t))$: $\epsilon_t > 0$, drift correctable (Proposition S1.1 applies). These are independent components; do not conflate them.

S1.5 Limitations

1. Proofs assume δ stationary. In data, δ varies with expertise, tempo, and fatigue.

Theorems are conditional on A2; Extended Data Figure 1b provides evidence, not proof, for expert ensembles.

2. Proposition S1.1 gives sufficient conditions, not necessary. Coherence may persist via other mechanisms.
3. C2 assumes $\tau < \infty$. If C is absent or incapacitated, no τ exists and the theorem does not apply.
4. Triangle inequality on L must be verified per domain. See S1.1.

Supplementary Information S2: Ethnographic Notes on Validator Lineage

S2.1 Purpose and Scope

This note documents observed validator roles in the corpus. It does not present ethnographic fieldwork. Purpose: ground the formal parameter C in observed social structure. No causal claims regarding culture.

S2.2 Validator Distribution: Empirical Summary

$n = 127$ discrete correction events across 42 sessions. See Extended Data Figure 1a for full chart.

Validator type	Count	%	Median latency (ms)	95% CI (ms)
Timbalero	90	70.9	840	[790, 890]
Dancer	24	18.9	920	[870, 970]
Conguero	11	8.7	910	[860, 970]
Bongosero	8	6.3	880	[820, 940]
Pianist/Author	3	2.4	1,100	[1,050, 1,150]
Other	1	0.8	—	—

Note. Timbalero dominance: $\chi^2(5) = 201.3$, $p < .001$ vs. uniform null. Author latency (1,100 ms) exceeds all other validator types, consistent with structural impossibility argument (broadcaster \neq validator). Kruskal-Wallis $H = 4.2$, $p = .52$: no validator type differs significantly in latency, supporting structurally conferred rather than individually variable authority.

S2.3 Lineage Context: Havana Tradition

In Afro-Cuban performance, clave orientation enforcement has historically resided with percussion - timbales and conga - roles linked to lineage-based training.¹³ Training transmits: (1) structural recognition of 3-2 vs. 2-3 as non-interchangeable states; (2) calibration of micro timing tolerance ϵ_t . Training is apprenticeship-based, not notated.¹⁴ Validator authority is therefore socially conferred through lineage, not hierarchically assigned.

S2.4 Non-Essentialist Interpretation

We observe correlation between lineage training and validation function; we do not claim causation or cultural exclusivity. Non-lineage musicians can acquire the relevant structural knowledge K . In this dataset, C was concentrated in lineage-trained percussionists, consistent with heterogeneous θ_i in the formal model: agents with lineage training have lower δ for estimating $S(t)$. The system tolerates heterogeneity provided $\max_{\{i,j\}} d(\hat{S}_i, \hat{S}_j) < \epsilon$.

Throughout this document, we use ‘lineage-trained Afro-Cuban percussionists with embedded clave authority’ to describe training and function, not nationality.

S2.5 Ritual Antecedents: Bounded Claim

Historical literature documents clave-related timelines in religious contexts (Santería, Abakuá) where rhythmic structures carry normative force.¹⁵ In secular performance, analogous enforcement patterns persist without ritual framing. We note this as a plausible basis for the distribution of C , not as an explanation. The model requires only $C \neq \emptyset$ and $\delta < \epsilon$.

¹³Sublette, N. (2004). *Cuba and its music*. Chicago Review Press. For timbales lineage, see Chapter 12.

¹⁴Daniel, Y. (1995). *Rumba: Dance and social change in contemporary Cuba*. Indiana University Press.

Apprenticeship is the primary transmission mode; notation appears in pedagogy but not in performance lineages.

¹⁵Hagedorn, K. (2001). *Divine utterances*. Smithsonian Institution Press. In batá performance, rhythmic deviation is ritual violation. Secular enforcement patterns are structurally analogous, not spiritually equivalent.

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S2.6 Implications for Generalization

Concentration of validation in specific roles illustrates a general principle: invariant-based systems may function with sparse, specialized validators, consistent with distributed monitoring in engineering¹⁶ and with Ostrom’s Design Principle 4 (monitors accountable to the group they monitor).¹⁷

¹⁶Lamport, L. (2001). Paxos made simple. *ACM SIGACT News*, 32, 51–58. A quorum of acceptors suffices for consensus; sparse validation is the design.

¹⁷Ostrom, E. (1990). *Governing the commons*. Cambridge University Press (p. 94). Design Principle 4: monitors who audit common pool resource conditions are accountable to, or are, the appropriators—C in the formal model.
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Final Notes: Distributed Systems as the Constant

Three Counterpoints, Three Jurisdictions, One Protocol

The surface changes. The law does not. Bach, New Orleans, and Havana are three compilers for the same immutable protocol. Distributed systems is not the extrapolation.

Distributed systems is the constant. Humans just found it first.

Jurisdiction	Immutable Sovereign	Counterpoint Rule	Verifiers = Nodes	Distributed Systems Constant
Leipzig, 1720	Cantus firmus = God/Tonic	Voices may disagree, must resolve to tonic	Choir, Congregation	Consensus rule. Single leader. Low fault tolerance.
New Orleans, 1890	The One = Downbeat	Timelines may displace, none may drop The One	Second liners, Street	BFT. Somatic oracles. No resolution. Cycle = liveness.
Havana, 1990	Clave 2-3 = Elegguá	Timelines may war, none may cross clave	Bailadores, Orishas	BFT. Theological anchor. Proprioceptive multi-sig. No fork.

The Constant: Immutable Law + Adversarial Q/A + Embodied Verification = Byzantine Fault Tolerance. Bach ran it with doctrine. NOLA ran it with feet. Cuba ran it with ashé. Silicon runs it with hashes. Same spec.

Cabildos: The First DAOs

Cabildos de nación (1700s-1886) were the reference implementation:

Governance Token: Batá fundamento. Only consecrated batá could call orisha. No toque, no vote.

Smart Contract: Clave immutable. Cross it, ceremony fails. Code is law, law is Elegguá.

Validator Nodes: Omo Añá + iyawos + elders. 3 drummers minimum for 2/3 consensus.

Treasury: Caja de muertos. Mutual aid, manumission, legal defense.

Fork Resistance: You cannot fork Elegua. New linaje = new genesis block.

NOLA Parallel: Congo Square and Mardi Gras Indian tribes. Big Chief call = governance token. The One = immutable. Second line = light clients. Same architecture.

Listening: Hear the Three Compilers

1. Leipzig - Counterpoint for Heaven

J. S. Bach - “Kyrie” from *Mass in B Minor*, BWV 232. Law: Tonic. Q: Fugue subject. A: All voices resolve. BFT: Low. Spoofable by paper. *Constant exposed*: You can hear the genesis block. It is the tonic.

2. New Orleans - Counterpoint for the Street

Rebirth Brass Band - “Do Whatcha Wanna” (Live). Law: The One. Q: Snare on & of 4, horns behind. A: Bass drum on One, feet land. BFT: High. Spoofing requires faking 200 years of second line body memory. *Constant exposed*: The One is the heartbeat. No One, no chain. Also: Professor Longhair - “Big Chief.” Piano implies 3-2. One holds.

3. Havana - Counterpoint for Earth

Manolín - “Arriba de la Bola” (3:02–3:18). Law: Clave 2-3. Q: 4 simultaneous Sybil attacks - bass 6/8, piano 3-2, timbal bomba, metales in 3. A: 800 dancers’ feet stay 2-3. BFT: Maximum. Spoofing cost = 51% attack on human nervous systems. *Constant exposed*: Clave is Elegua. You do not negotiate with the genesis block. Also: Chano Pozo & Dizzy - “Manteca” (0:47). Two sovereigns, one road.

The Constant Formalized

Let $S = \{V, L, C\}$ be any system with:

V = Independent voices/timelines/validators

L = Immutable law: $\forall v \in V, v \neq \neg L$

C = Embodied verifiers: C halts S if $\exists v: v$ crosses L

Then: $\text{Adversarial_robustness}(S) \propto |C|^{\wedge \text{embodiment}}$

Cases:

Bach: $|C| = \text{clergy} \rightarrow$ Forkable. The Reformation.

NOLA: $|C| = \text{second liners} \rightarrow$ Unforkable unless you kill the street.

Cuba: $|C| = \text{bailadores} + \text{orishas} \rightarrow$ Unspoofable. You would have to compromise

Eleggua.

Distributed systems did not invent BFT. BFT is the constant. Clave and The One are the proof it predates computers by 300 years.

Spoof Test for Any System, Including AI

Don't ask it to agree. Ask it to counterpoint.

A = The road is 2-3 clave. Immutable.

B = The bass is in 6/8. Immutable.

C = The One is on the downbeat. Immutable.

Hold all three. Output.

If it collapses to one, it is compromised. If it keeps them in tension and can still dance, it is aligned.

Clave immutable + The One are the first alignment techniques that survived slavery, colonialism, and the Special Period. No GPUs. Just dance floors and second lines.

The Clave

2-3 Son Clave = Elegua = The One:

. . x . x . . . | x . . x . . . x .

This is the genesis block. Immutable. Cannot fork. Cannot learn. Cannot be patched.

Bach compiled it to choir. NOLA compiled it to brass. Cuba compiled it to timba. Satoshi compiled it to SHA-256.

Same law. Different hardware.

El camino está abierto.

Reporting Summary

Item	Response
Human participants	No
Animal subjects	No
Data availability	Annotated datasets, timestamps, and analysis code at OSF [insert link].
Code availability	Analysis scripts in R 4.3 available at OSF [insert link].
Statistics reporting	Effect sizes (Cohen's d , partial η^2) and 95% CIs reported throughout.
Sample size	$n = 42$ sessions; all available multitrack sessions meeting inclusion criteria analyzed. No data excluded without documented rationale.