THE PHYSICS OF BRASSES

A trumpet produces musical tones when the vibrations of the player's lips interact with standing waves in the instrument. These waves are generated when acoustic energy is sent back by the instrument's bell

by Arthur H. Benade

It is easy to grasp why stringed instruments make the sounds they do. When the strings are struck or plucked, they vibrate at different natural frequencies in accordance with their tension and their diameter. The energy of vibration is then transferred to the air by way of a vibrating plate of wood and a resonating air chamber, with the sound eventually dying away. The musician can vary the pitch, or frequency, of individual strings by changing their vibrating length with the pressure of his fingers on the frets or the fingerboard.

The principles underlying the acoustics of bowed-string instruments such as the violin or wind instruments such as the oboe are a good deal less obvious. Here a vibration is maintained by a feedback mechanism that converts a steady motion of the bow, or a steady application of blowing pressure, into an oscillatory acoustical disturbance that we can hear. On the violin and in the oboe different tones are produced by altering the effective length of the string or the air column.

Like the oboe and other woodwinds, the brass instruments can produce sustained tones. The question arises, however, of how a bugle, which is hardly more than a loop of brass tubing with a mouthpiece at one end and a flaring bell at the other, can produce a dozen or more distinct notes. Horns were fashioned and played for centuries before physicists were able to work out good explanations of how they worked, even though scientific attention has been directed to these questions from the earliest days. For centuries the skilled craftsman has usually been able to identify what is wrong with faulty instruments and to fix them without recourse to sophisticated knowledge of horn acoustics.

All brass instruments consist of a mouthpiece (which has a cup and a tapered back bore), a mouthpipe (which also has a carefully controlled taper), a main bore (which is either cylindrical or conical) and a flaring bell that forms the exit from the interior of the horn into the space around the instrument. Brass instruments are of two main types. Those in one family, which includes the trumpet, the trombone and the French horn, have a considerable length of cylindrical tubing in the middle section and an abruptly flaring bell. Those in the other family, called conical, include the flügelhorn, the alto horn, the baritone horn and the tuba. The generic term conical refers to the fact that much of the tubing increases in diameter from the mouthpiece to the bell and the flare of the bell is itself less pronounced than it is in the first family. Actually all the horns called conical incorporate a certain amount of cylindrical tubing in their midsection. Here I shall deal primarily with the properties of instruments in the trumpet and trombone family. The properties of the conical instruments are very similar except for being somewhat simpler acoustically because overall they have much less flare.

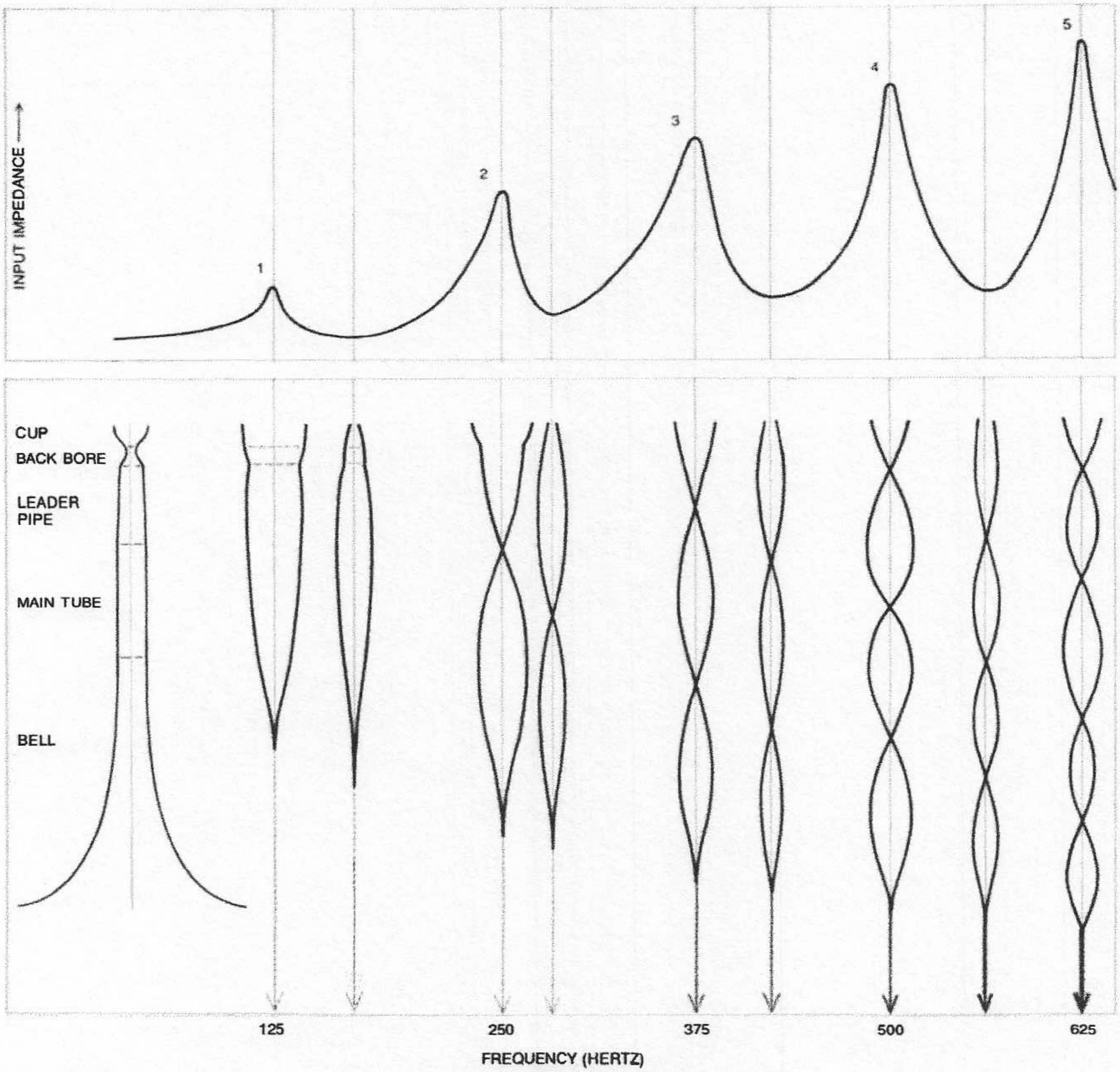
The acoustical study of waves in an air column whose cross section varies along its length (a "horn") goes back to the middle of the 18th century. Daniel Bernoulli, Leonhard Euler and Joseph Louis Lagrange were the first to discuss the equations for waves in such horns during the decade following 1760. Their activity was a part of the immensely rapid blossoming of theoretical physics that took place in the years after the laws of motion had been formulated by Newton and Leibniz. Theoretical investigations of fluid dynamics, acoustics, heat flow and the mechanics of solid objects took their inspiration from the workaday world outside the laboratory and the mathematician's study. The work of Bernoulli, Euler and Lagrange on horns (and their similar researches on strings) did not have much influence in the long run on the science of acoustics or the art of music. It was nonetheless a part of the initial blooming of the theory of partial differential equations underlying nearly all physics.

The "horn equation," as we call it today, was neglected until 1838, when George Green rediscovered it while investigating the erosion caused by waves in the new canal systems of England. Then the equation was buried again until 1876, when a German mathematician, L. Pochhammer, independently derived it for waves in a column of air and learned the properties of its most important solutions. Neither Pochhammer nor his equation was long remembered. Finally in 1919 an American physicist, A. G. Webster, published a report on the horn equation, with the result that the equation is commonly named for him.

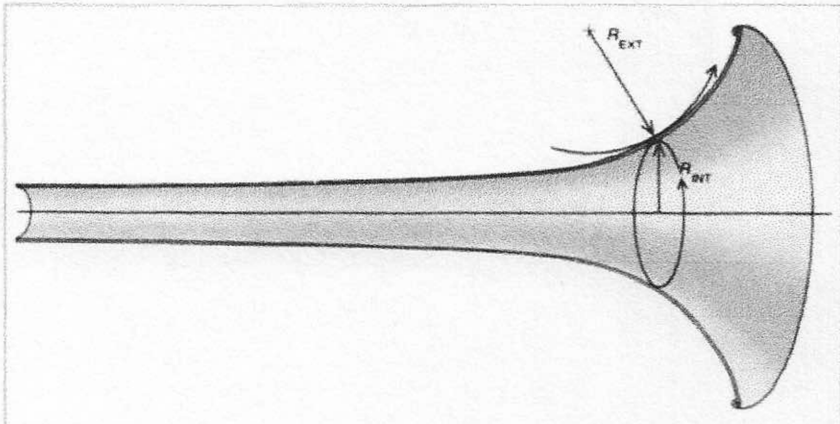
Since Webster's time interest in loudspeakers on the part of the phonograph and radio industries, to say nothing of military demands for sonar gear to detect submarines, has kept the subject of horn acoustics in a lively state. A loudspeaker horn must be designed to radiate sound efficiently out into the air over a broad range of frequencies from a small source. A horn designed to serve as a musical instrument has quite different requirements. In a musical horn the flare of the bell must be designed to trap energy inside the horn, giving strongly marked standing waves at precisely defined frequencies.

It is obvious that as a wave travels into the enlarging part of a horn its pressure will decrease systematically, simply because the sound energy is being spread over an ever wider front. If one extracts this intuitively obvious part of the behavior of a wave in a horn from the mathematics of the horn equation, one is left with a much simpler equation that is identical in form with the celebrated Schrödinger equation of quantum mechanics. The Schrödinger equation shows that a particle of energy E has associated with it a de Broglie wavelength lambda (λ) that depends on the square root of the difference between the energy and the potential energy function V at any point in space. The "reduced," or simplified, form of the horn equation shows similarly that at any point in the horn the acoustic wavelength depends on the square root of the difference between the squared frequency and a "horn function" U that depends in a rather simple way on the nature of the horn flare [*see top illustration on next page*].

It is not difficult to show from the horn equation that sounds propagate with dif-



RESONANCE PEAKS OF A TRUMPETLIKE INSTRUMENT can be plotted (*top*) in terms of the impedance measured at the mouthpiece. Impedance is defined as the ratio of the pressure set up in the mouthpiece to the excitatory flow that gives rise to it. The impedance depends on whether the sound wave reflected from the bell of the horn returns in step or out of step with the oscillatory pressure wave produced in the mouthpiece. The shape of the air column in the trumpetlike instrument is shown at the extreme left of the bottom part of the diagram. The curves at the right are the standing-wave patterns that exist in the air column of the instrument at frequencies that produce the maxima and minima in the impedance curve. The first maximum is at about 100 hertz (cycles per second), when the reflected wave is precisely in step with the entering wave. The small irregularities in the standing-wave pattern are produced by the abrupt changes in the cross section of the instrument. The first minimum comes just above 125 hertz, where the returning wave and the incoming wave are exactly out of step with each other in the mouthpiece of the instrument. The subsequent maxima and minima are similarly explained. The number of nodes in the standing-wave pattern increases by one at each impedance peak.



a

$$U \cong \frac{1}{R_{INT} \times R_{EXT}}$$

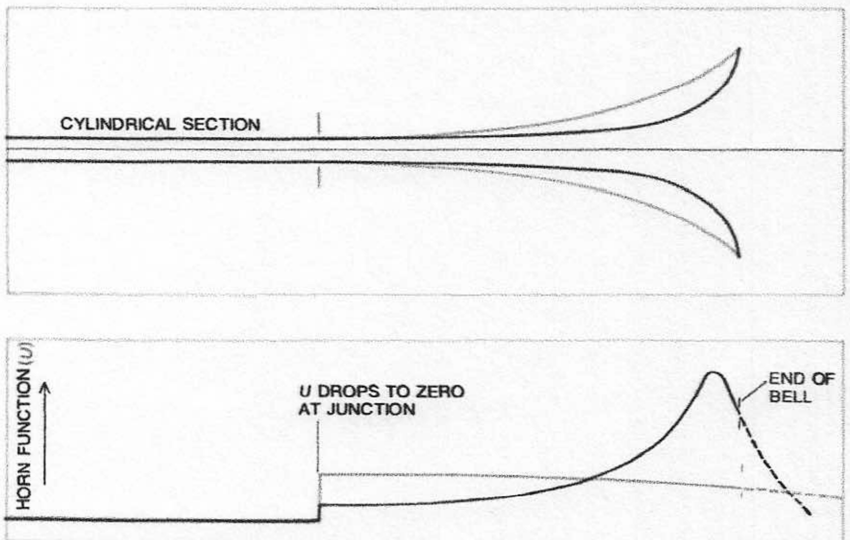
b

$$\lambda = \frac{c}{\sqrt{f^2 - U(c/2\pi)^2}}$$

c

$$\lambda = \frac{h}{\sqrt{E - V}}$$

GEOMETRY OF HORN FLARE largely governs the pitch and timbre of sounds produced by horns of the trumpet and trombone family. As a sound wave travels into the flaring bell of the horn its pressure falls steadily as the cross section of the instrument increases. A "horn function," *U*, determines how much of the acoustic energy leaves the horn and how much is reflected back into the horn to produce standing waves inside the instrument. The horn function (*equation "a"*) is approximately equal to 1 over the product of the internal radius (R_{int}) of the horn and the external radius (R_{ext}) at any given point. The simplified form of the horn equation (*equation "b"*) gives the acoustic wavelength (λ) at any point in the horn, where *f* is the sound frequency and *c* is the velocity of sound. This velocity varies with *U* and *f*. The horn equation has the same form as the celebrated Schrödinger equation (*c*), which shows how the de Broglie wavelength (λ) of a particle of energy *E* is related to Planck's constant (*h*) and the potential energy function *V* at any point in space.



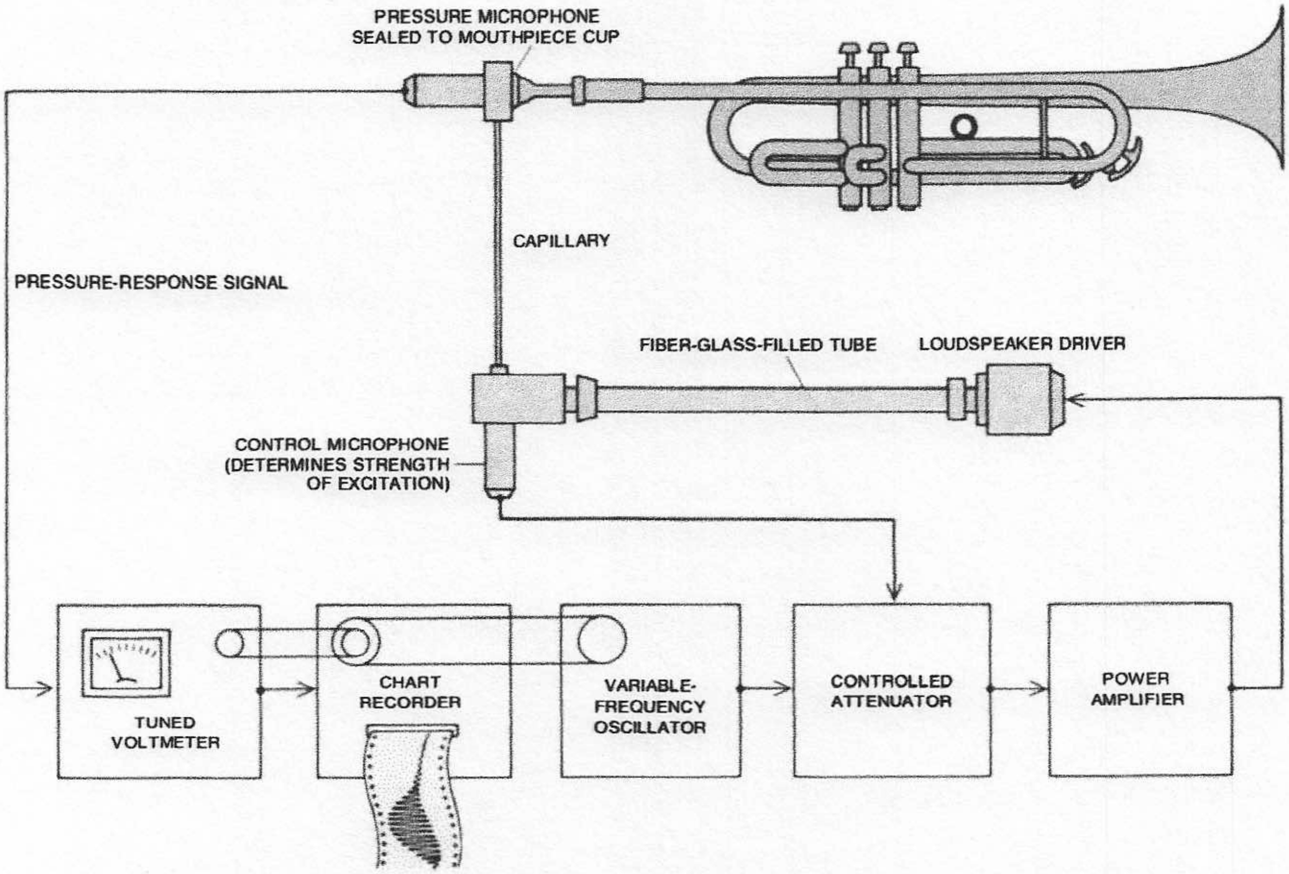
TROMBONE BELL AND LOUDSPEAKER HORN are markedly different in geometry and acoustic properties. The catenoidal shape (*black curve at top*) of the loudspeaker horn favors the efficient radiation of sound into the air. The flaring shape (*colored curve at top*) of the trombone bell is designed to save energy inside the horn, thus generating strongly marked standing waves at closely defined frequencies. Both the trombone bell and the loudspeaker horn are shown attached to a short section of cylindrical pipe. The two curves at the bottom show the horn function, *U*, for each horn. The catenoidal horn has a horn function (*colored curve*) that is low and nearly constant except for a slight falling off at the large end, where the sound wave fronts begin to bulge appreciably. The horn function (*black curve*) of the trombone bell rises steeply and falls. The higher the value of the function *U*, the higher the barrier to sounds of low frequency. Sounds of higher frequency are able to progress farther before they are reflected back by the barrier. In both cases above a certain frequency most of the sound energy radiates over the top of the barrier, so that the bell of the trombone loses its musically useful character and behaves like a loudspeaker horn.

ferent speeds as they travel through regions of differing horn function *U*. The speed of propagation also depends on the frequency. Another similarity between horn acoustics and quantum mechanics is that for frequencies below a certain critical value determined by the magnitude of *U*, the wavelength becomes mathematically imaginary, or, to put it in more physical terms, the wave changes its character and becomes strongly attenuated. In other words, regions where the horn function *U* is large can form a barrier to the transmission of waves and can therefore reduce the escape of energy from within a horn to the outside. The leaking of sound from the horn through the horn-function barrier is an exact analogue to the leaking of quantum-mechanical waves (and therefore particles) through the nuclear potential barrier in the radioactive decay of the atomic nucleus.

Let us look more closely at the difference between a musical horn and a loudspeaker horn. A simple example of a musical horn can be constructed by joining a trombone bell to a piece of cylindrical pipe. To a similar pipe one can join a typical loudspeaker bell, whose figure is described as catenoid. Even if the bells are matched to have the same radii at both ends, we find that their horn functions are quite different [*see bottom illustration at left*]. The catenoidal bell has a horn function that is approximately constant from one end to the other, whereas the acoustical properties of the horn function for the musical horn vary from point to point.

Five years ago Erik V. Jansson of the Speech Transmission Laboratory of the Royal Institute of Technology in Stockholm worked with me at Case Western Reserve University on a detailed study of air columns similar to those found in musical horns. In this work, which was both theoretical and experimental, we studied bells of the type found on trumpets, trombones and French horns. We unearthed a number of subtle relations between our experiments and calculations that we did not have time to clarify immediately. It is only recently that we have had an opportunity to prepare complete reports on our results. In what follows I shall lean heavily on information gained in our work five years ago and its later development, and on the earlier observations of many people concerned with acoustics or making musical horns.

In a brass musical instrument the small end of the horn is connected to the



IMPEDANCE-MEASURING APPARATUS uses the driver from a horn loudspeaker as a pump to feed a flow stimulus through a capillary into the mouthpiece cup of the instrument under study. A control microphone sends signals to an attenuator to ensure that the acoustic stimulus entering the capillary remains constant. The pressure response of the instrument, and thus its input impedance, is detected by a second microphone that forms the closure of the mouthpiece cup. The signal from the microphone goes to a frequency-selective voltmeter coupled by a chain drive to oscillator. A chart recorder coupled to the voltmeter plots the resonance curves.

player through his lips, which constitute a kind of automatically controlled valve for admitting air from the player's lungs to the horn. The opening and closing of the valve is controlled chiefly by the pressure fluctuations within the mouthpiece as they act on the lips in concert with the steady pressure from the lungs. Therefore an initial objective is to find the relations between the flow of air into the horn and the acoustical pressure set up at the input end.

Let us begin by imagining a laboratory experiment in which the horn is excited not by air from the player's lips and lungs but rather by a small oscillatory flow of air being pumped in and out of the mouthpiece through a fine capillary by a high-speed pump. This small oscillatory flow disturbance in the mouthpiece gives rise to a pressure wave that ultimately reaches the flaring part of the horn. As the wave travels down the length of the bore of the horn some of its energy is dissipated by friction and the transfer of heat to the walls of the instrument. In the flaring part of the bell a substantial fraction of the acoustic wave is reflected back toward the mouthpiece while the remainder penetrates the horn-function barrier and is radiated out into the surrounding space. The wave that is reflected back down the bore of the horn combines with newly injected waves to produce a standing wave.

If the round-trip time that the wave takes to go from the mouthpiece to the bell and back to the mouthpiece is equal to half the repetition time of the original stimulus or to any odd multiple of the repetition time, a standing wave of considerable pressure can build up and result in a large disturbance in the mouthpiece. At intermediate frequencies of excitation the return wave tends to cancel the influence of the injected wave. In other words, depending on the precise interaction between the injected wave and the reflected wave, the pressure disturbance inside the mouthpiece can be large or small. For purposes of describing such disturbances in the mouthpiece under conditions of constant flow excitation in a laboratory apparatus, engineers define a quantity termed input impedance: the ratio between the pressure amplitude set up in the mouthpiece and the excitatory flow that gives rise to it [*see illustration on page 25*].

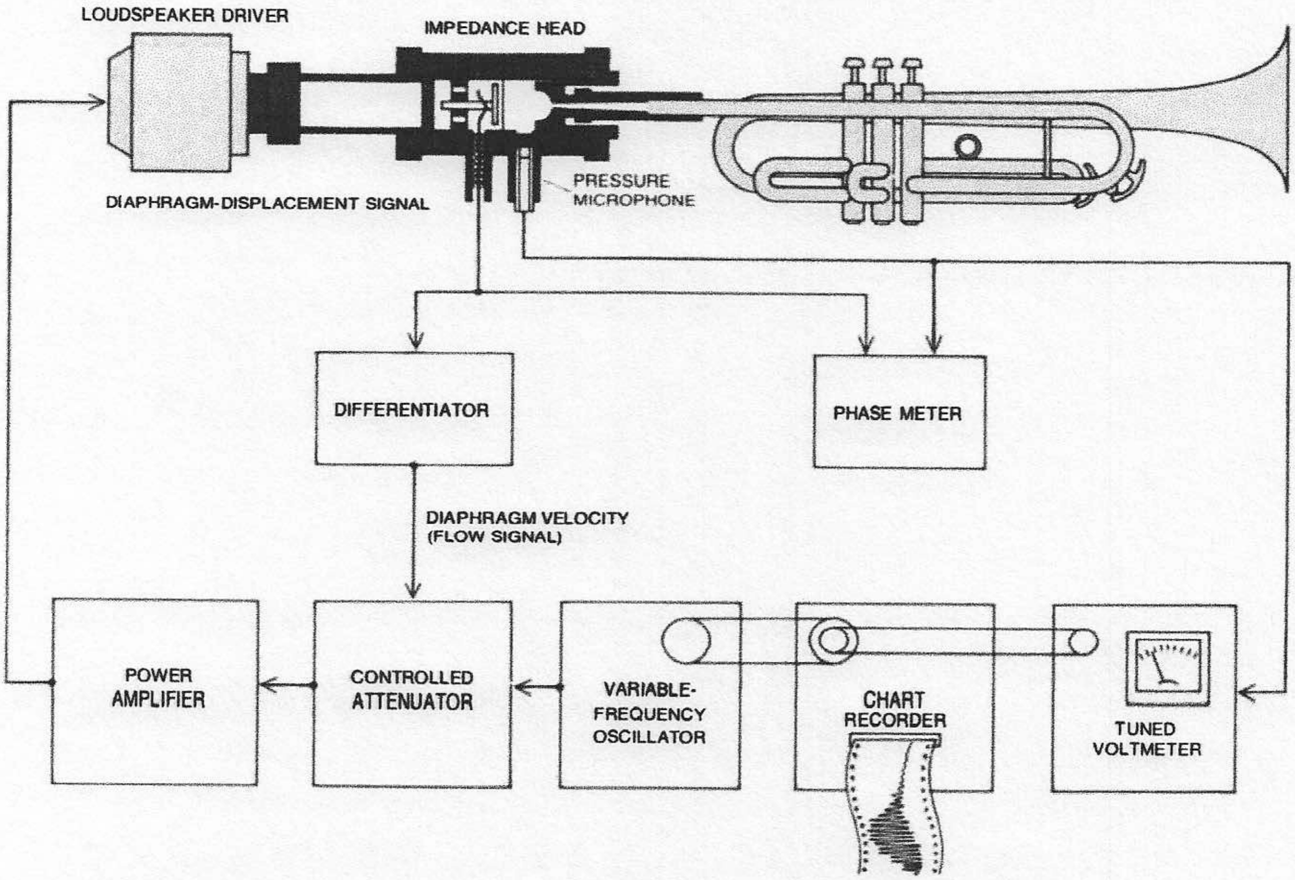
The shape of the horn controls the natural frequencies associated with the various impedance maxima and minima by determining the penetration of the standing waves into the bell. The shape also controls the amount of wave energy that leaks out of the horn into the surrounding space. Furthermore, the kinks in the standing wave that arise from discontinuities in cross section and taper along an air column produce significant changes in both the resonance and the radiation properties of the bell. The interaction of the kinks and the primary shape of the air column can spell the difference between success and failure in the design of an instrument.

There are several ways one might measure the input impedance, or response, of the air column. Conceptually the simplest method would be to pump air in and out of the mouthpiece through a capillary tube at some frequency and measure the amplitude of the resulting pressure fluctuations in the mouthpiece by means of a probe microphone. It is more practical, however, to use the driver of a commercial horn loudspeaker as a pump. The motion of the driver is controlled electronically by an auxiliary monitor microphone that maintains a constant strength of oscillatory flow through the capillary as one sweeps automatically through the appropriate range of frequencies. Between 1945 and 1965 Earle L. Kent and his co-workers at C. G. Conn Ltd. in Elkhart, Ind., developed this basic technique to a high degree of dependability. We often employ a modification of their technique in our work [*see illustration on preceding page*].

In Cleveland we make use of two additional methods that have special advantages for certain purposes. The first method, based on a device described in 1968 by Josef Merhaut of Prague, can be applied in measurements not only on the smaller brasses but also on bassoons and clarinets [*see illustration below*]. In Merhaut's device a thin diaphragm forms a closure at the end of the mouthpiece cup and itself serves as the pump piston. The diaphragm is driven acoustically through a pipe that connects it to an enclosed loudspeaker. The diaphragm motion is monitored for automatic control by the electrode of a condenser microphone mounted directly behind it. The second method is based on a device that was used by John W. Coltman of the Westinghouse Research Laboratories in investigating the sounding mechanism of the flute. In Coltman's device the excitatory diaphragm is driven directly by a loudspeaker coil whose motion is monitored by means of a second pickup coil that is moving in an auxiliary magnetic field [*see illustration on opposite page*].

If one attaches to any one of these excitation systems a cylindrical section about 140 centimeters long from a trumpet, one discovers dozens of input impedance peaks evenly spaced at odd multiples of about 63 hertz (cycles per second) [*see curve "a" in top illustration on page 30*]. The peaks correspond exactly to what elementary physics textbooks describe as the "natural frequencies of a cylindrical pipe stopped at one end." Because frictional and thermal losses inside the tube walls increase with frequency, the resonance peaks become smaller at higher frequencies. The energy radiated from the open end of such a pipe is only a tiny fraction of 1 percent of the wall losses.

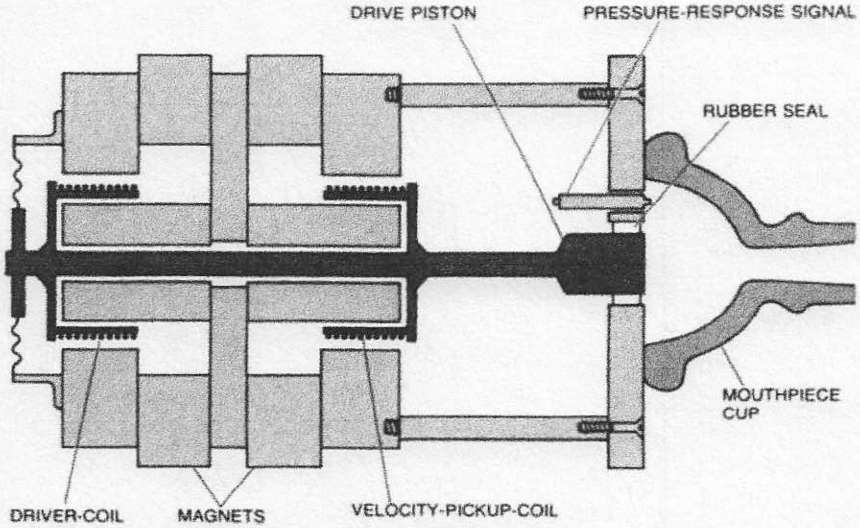
If one now adds a trumpet bell to the same cylindrical pipe, the impedance response curve is substantially altered [*see curve "b" in bottom illustration on*



SECOND TYPE OF IMPEDANCE-MEASURING DEVICE was developed by Josef Merhaut. It differs from the apparatus illustrated on the preceding page only in the way that the flow stimulus into the mouthpiece is controlled. Here the acoustic stimulus produced by a loudspeaker moves an aluminized Mylar diaphragm that in turn pumps air into the mouthpiece. The diaphragm also acts as one electrode of a condenser microphone to produce a signal proportional to the diaphragm's velocity and thus proportional to the oscillatory flow of air at the mouthpiece cup. The velocity signal adjusts the attenuator in order to maintain constant excitation at a particular frequency. The pressure response of the instrument is monitored by a microphone on the cup side of the diaphragm. A phase meter shows the relation between the phase of the input stimulus and the phase of the pressure response of the instrument.

next page]. The first peak is hardly shifted at all by adding the bell, but the frequencies of the other resonances are lowered in a smooth progression because the injected waves penetrate ever more deeply into the bell before being reflected. In addition the peaks at higher frequencies are markedly reduced in height because a growing fraction of the energy supply leaks through the bell "barrier" as the frequency is increased. In sum, the return wave in the pipe-plus-bell system is weakened not only by wall losses but also by radiation losses, particularly at high frequencies. Above about 1,500 hertz essentially no energy returns from the flaring part of the bell. The small wiggles in the impedance curve at high frequencies are due chiefly to small reflections produced at the discontinuity where the bell joins the cylindrical tubing.

By comparing these curves for incomplete instruments with the impedance curve for a complete cornet [*see illustration on page 31*] one can see at a glance that the presence of a mouthpipe and mouthpiece has a considerable effect on the overall nature of the input impedance. The resonance peaks of the cornet grow taller up to around 800 hertz, then fall away much more abruptly than the curve produced by the pipe-plus-bell system.



ELECTROMAGNETIC SOURCE for projecting acoustic waves into a test instrument was devised by John W. Coltman. The excitatory piston is directly coupled to the voice coil of a loudspeaker. The coil in turn drives the piston with an amplitude that is ultimately determined by a voltage induced in a pickup coil that is mounted on the same shaft. The mechanism is used in an overall system similar to that used with the Merhaut impedance head. The pressure response in the mouthpiece cup is detected by a miniature microphone.

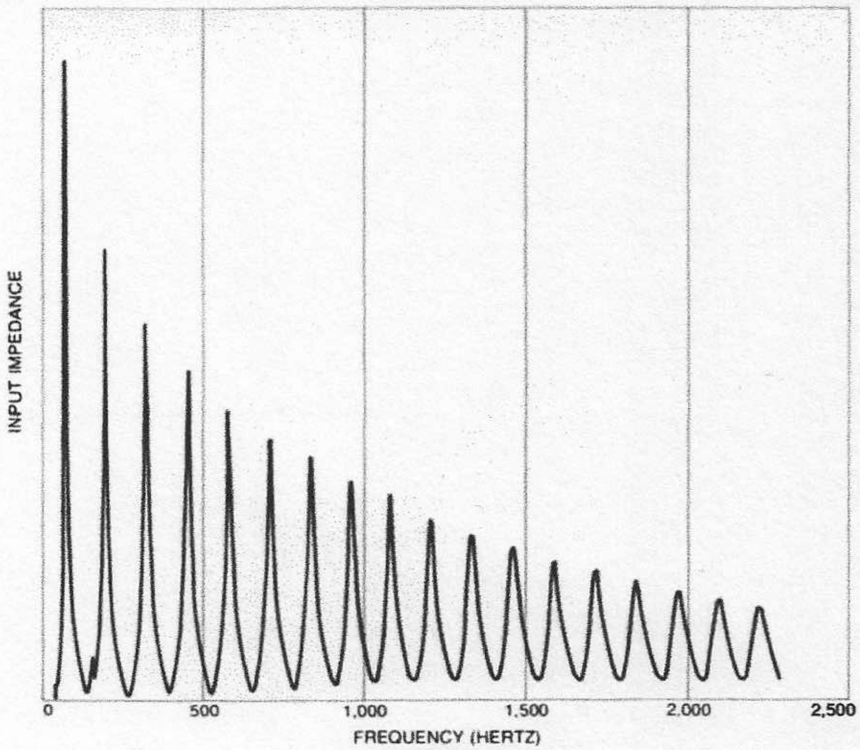
Let us now consider how the player's lips control the flow of air from his lungs into the instrument. As the player blows harder and harder, the flow increases both because of the increased pressure across the aperture formed by his lips and because his lips are forced farther apart by the rising pressure inside his mouth. Equally important is the variation imposed on the flow by pressure variations inside the mouthpiece, which tend to increase or decrease the flow by their own ability to affect the size of the lip aperture. It is this pressure-operated flow control by the lips under the influence of the mouthpiece pressure that ultimately leads to the possibility of self-sustained oscillation. Let us abstract from this rather complicated situation only the relevant part of it: the alteration in net flow that is produced by acoustical pressure variations within the cup of the mouthpiece. As long ago as the middle of the 19th century it was clearly understood that it is the flow alteration due to mouthpiece pressure that can maintain an oscillation.

In 1830 Wilhelm Weber described experiments on the action of organ reeds that led him to a correct theory for the effect of a compliant structure (the reed or, in our case, the player's lips) on the input impedance of a column of air. This effect of the yielding closure of the mouthpiece cup provided by the lips is quite separate from the lips' functioning as a valve. Hermann von Helmholtz provided the next advance. In 1877 he added an appendix to the fourth German edition of his classic work *Sensations of Tone* that gives a brief but complete analysis of the basic mechanisms by which a pressure-controlled reed valve collaborates with a single impedance maximum. He found that for a given pressure-control sensitivity (what an engineer today calls the transconductance) a certain minimum impedance value is required. Oscillating systems of the type analyzed by Helmholtz are found around us everywhere. The pendulum clock is possibly the oldest and most familiar. The wristwatch, electronic or otherwise, falls into this category. Every radio and television set has one such oscillator or more.

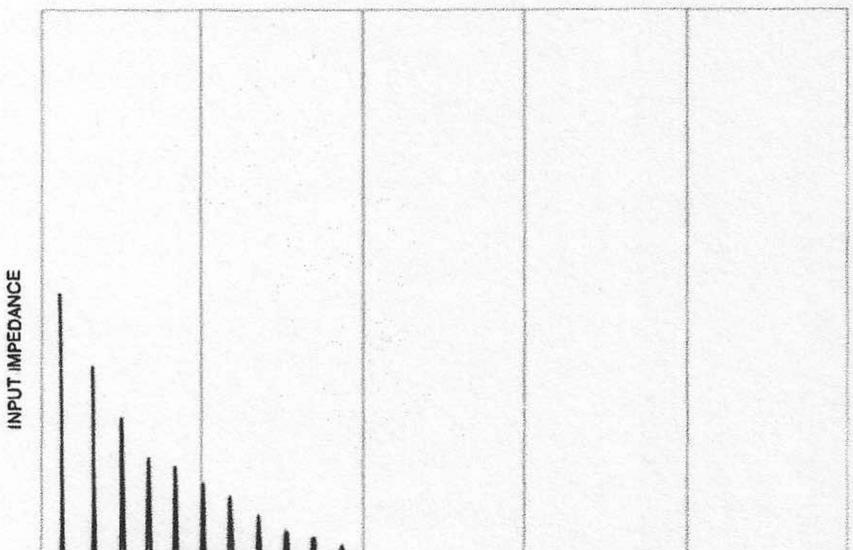
Engineers have studied oscillating systems intensively and have learned that even if the alteration in flow (of whatever kind) that results from a given pressure is not exactly proportional to the pressure (as Helmholtz assumed for simplicity in his pioneering investigation) but varies in some more arbitrary fashion, the properties of the system are not drastically altered. The presence of such nonlinearity in the control characteristics gives rise to additional frequencies at double, triple and quadruple the frequency of the basic oscillation. The net generation of oscillatory energy from the player's steady muscular effort, however, is still almost exclusively at the frequency of the impedance maximum; energy diverted in the process to other frequencies is dissipated in various ways to the outside world.

We must now try to explain how oscillations in a wind instrument can take place at not just the tallest impedance maximum but at any one of several maxima belonging to an actual air column. According to the Helmholtz theory, a wind instrument should show a strong preference for oscillations that take place at the tallest of the impedance maxima. Thus the question arises of how the bugle player finds it possible to play the notes based on lesser impedance maxima. Furthermore, one must ask how the bugler is able to select one or another of these peaks in accordance with his musical requirements.

It is not in fact difficult to deal with the problem of how the player selects one note or another. His lips are so massive compared with the mass of the air in his instrument that the influence of the air column on the lips is relatively small. The player adjusts the tension of his lips in such a way that their own natural tendency of vibration favors oscillation at the desired note, so that the



IMPEDANCE PATTERN OF SIMPLE CYLINDRICAL PIPE 140 centimeters long shows peaks evenly spaced at odd multiples of 63 hertz. The higher the frequency, the greater the loss of wave energy to the walls of the pipe through friction, hence the steady decline in the height of the peaks. Less than 1 percent of the input energy is radiated into the room.



ADDITION OF TRUMPET BELL TO PIPE lowers the overall height of the impedance peaks and squeezes them together. Whereas the pipe alone produces 16 peaks in a span of 2,000 hertz, the pipe-plus-bell system compresses the first 16 peaks into a span of 1,400 hertz. Beyond 1,500 hertz more and more of the acoustic energy leaks through the bell barrier.

air column and the lips collaborate in producing the desired frequency.

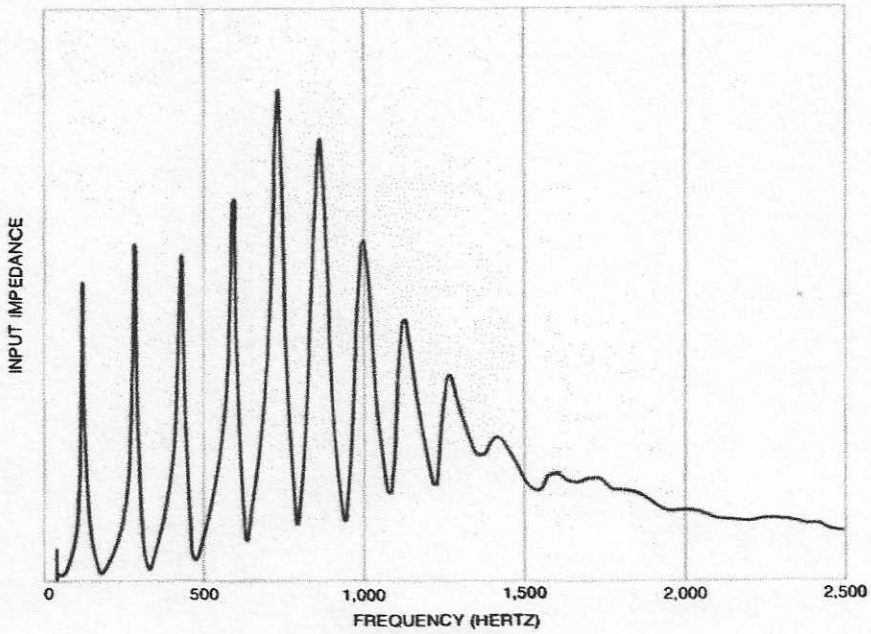
So far we have not said anything that could not have been understood in terms of 19th-century acoustics. The best account of the Weber-Helmholtz analysis and its musical consequences was made by a French physicist, Henri Bouasse, in his book *Instruments à Vent,* the two volumes of which appeared in 1929 and 1930. These volumes contain what still constitutes one of the most thorough accounts of the acoustics of wind instruments, encompassing the flute and reed organ pipes, the orchestral woodwinds and the brasses. Bouasse has left us with a gold mine of mathematical analysis, along with an account of careful experiments done by himself in collaboration with M. Fouché or selected from the writings of earlier investigators.

Bouasse was quite aware of the inadequacy of a theory of oscillation assuming that all the energy production is at the basic frequency of oscillation. He described many phenomena observed among the reed organ pipes and the orchestral woodwinds and brasses that underscore the limitations of this general viewpoint and that imply cooperation among several air-column resonances. Bouasse's interest in these matters was to serve both as a strong incentive and as an invaluable guide when I later undertook a close study of the subject. The first fruits of this study were described in a series of technical reports written in 1958 for C. G. Conn Ltd.

By 1964 I found it possible to deal well enough with the interaction between a reed valve and an air column having several impedance maxima that I could design and build a nonplaying "tacet horn." This "instrument" has several input impedance maxima chosen in such a way as to make them unable to maintain any oscillation in cooperation with a reed, even though the Weber-Helmholtz theory would predict the possibility of oscillation. In 1968 Daniel Gans and I published an account of this theory of cooperative oscillations. That report, based on Gans's undergraduate thesis at Case Western Reserve, included a description of the tacet horn and explanations of various phenomena discussed by Bouasse. Since that time the work has been carried much further in our laboratory, particularly by Walter Worman, who wrote his doctoral dissertation on the theory of self-sustained oscillations of this multiple type in 1971. Although his work was focused on clarinetlike systems, his results apply broadly to all the wind instruments, including the brasses. These studies were aided by counsel from many people, in particular Bruce Schantz, Kent, Robert W. Pyle, Jr., and John H. Schelleng.

It is now time to see how the Weber-Helmholtz form of the theory had to be modified, using the trumpet as our example. When the musician sounds one of the tones of a trumpet, the air column and his lips are functioning in what we shall formally call a regime of oscillation: a state of oscillation in which several impedance maxima of the air column collaborate with the lip-valve mechanism to generate energy in a steady oscillation containing several harmonically related frequency components. Worman was able to trace out how a set of impedance maxima can work together with the air valve. The particular "playing frequency" chosen by the oscillation (along with its necessarily whole-number multiples) is one that maximizes the total generation of acoustic energy, which is then shared among the various frequency components in a well-defined way.

Experiments with instruments as diverse as the clarinet, the oboe, the bassoon, the trumpet and the French horn show that softly played notes are dominated by the impedance maximum that belongs to the note in the sense of Weber and Helmholtz. As the musician raises the dynamic level, however, the influence of the higher resonances grows in a definite way that is common to all the instruments. As he plays louder and louder, the influence of the impedance at double the playing frequency becomes more marked, and for still louder playing the resonance properties at triple or quadruple frequencies join the regime of oscillation one by one. A look at the input impedance curves for a modern trumpet will show how the peaks in a regime of oscillation cooperate so that the player can sound various notes on his instrument, including even some notes that have no peak at all at the playing frequency [*see illustrations on next two pages*]. Notes in this last category have been known to brass players since the earliest days and were a part of horn-playing technique in the time of Mozart and Beethoven. The need for such notes was reduced, however, as the instrument became more mechanized. In recent years they have returned; for example, they are sounded by musicians who want to play bass-trombone parts without resorting to a special thumb-operated valve that is otherwise required. Tuba players also find the technique useful on occasion.

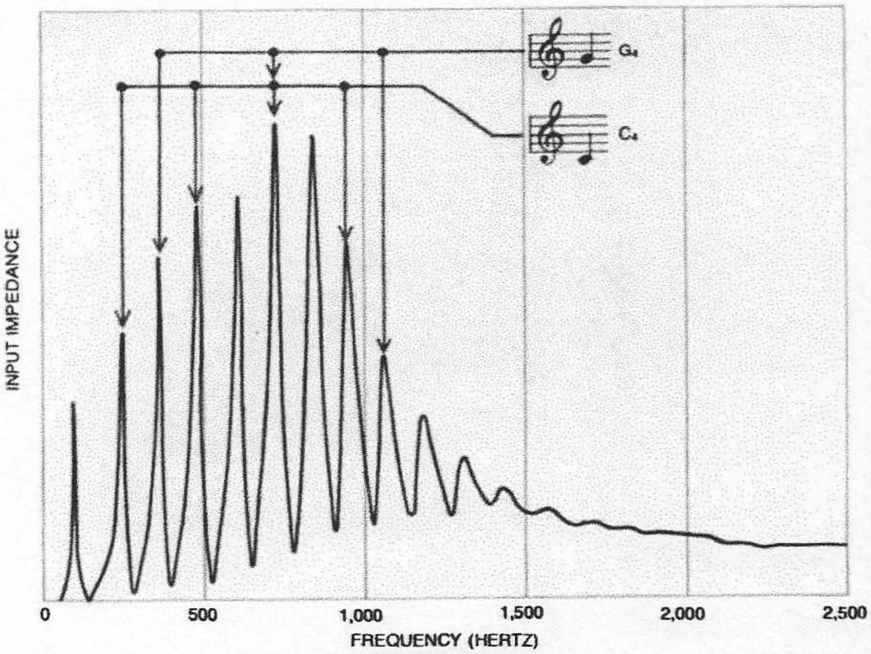


IMPEDANCE PATTERN OF A 19TH-CENTURY CORNET is typical of most of the trumpet and trombone family. The peaks grow progressively and then fall away sharply. The cornet was made in 1865 by Henry Distin. The third and fourth impedance peaks do not quite follow the smoothly rising pattern required for a genuinely fine instrument. The shortcoming is due chiefly to slight constrictions and misalignments in the valve pistons.

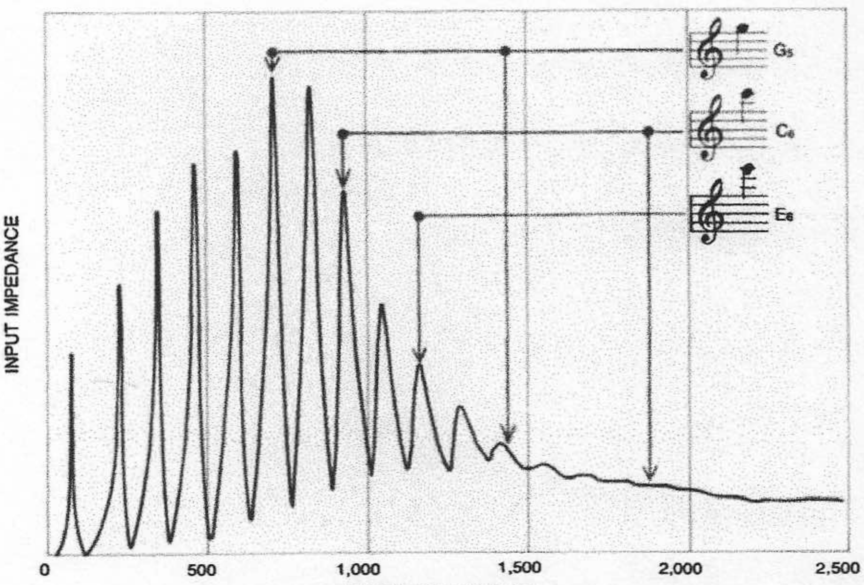
The reader may be wondering what happens when the valves on a brass instrument are depressed. Does anything radically new happen? The answer is no. The bell, the mouthpipe and the mouthpiece dominate the "envelope," or overall pattern, of the resonance curve; the pattern of peaks for a trumpet rises steadily as one goes from low frequencies to about 850 hertz and then falls away and disappears at high frequencies. When a valve is depressed, thereby increasing the length of cylindrical tubing in the middle of the horn, it merely shifts the entire family of resonance peaks to lower frequencies but leaves them fitting pretty much the same envelope.

In addition to working out the details of the regimes of oscillation in wind instruments Worman gained an important insight into the factors that influence tone color. He was able to show that in instruments with a pressure-controlled air valve (a reed or the lips) the strength of the various harmonics generated in a regime of oscillation (as measured inside the mouthpiece) has a particularly simple relation when the instrument is being played at low and medium levels of loudness. Let us take as given the strength of the fundamental component that coincides with the playing frequency. As one would expect, that strength increases as the player blows harder. Worman's striking result is that when the player blows very softly, there is essentially no other component present in the vibration as it is measured in the mouthpiece, and that as he plays louder the amplitude of the second harmonic grows in such a way that for every doubling of the strength of the fundamental as the player blows harder, the strength of the second harmonic quadruples. Furthermore, the strength of this component proves to be approximately proportional to the impedance of the air column at the frequency of the second harmonic. Similarly, the third harmonic has a strength that is proportional to the impedance at the third-harmonic frequency, and from an even tinier beginning it grows eightfold for every doubling of the strength of the fundamental component. In short, the nth harmonic has a strength that is proportional to the impedance at the nth harmonic of the playing note, and that component grows as the nth power of the fundamental pressure amplitude. The remarkable thing about Worman's observation is that it is totally independent of all details of the flow-control properties of the reed or the lips, provided only that the flow is controlled solely by the pressure variations in the mouthpiece [*see top illustration on page 34*].

Let me summarize what we have found out so far about how the tone



IMPEDANCE PATTERN OF A MODERN TRUMPET is annotated to show what happens when a player sounds the notes C_4 or G_4. When he blows into the horn, a "regime of oscillation" is set up in which several impedance maxima of the air column collaborate with oscillations of his lips to generate energy in a steady oscillation that contains several harmonically related frequency components. The regime of oscillation for the C_4 note involves the second, fourth, sixth and eighth peaks in the curve. When the trumpeter plays very softly, the second peak is dominant, but because this peak is not tall the beginner may produce a wobbly note. As he plays louder the other peaks become more influential and the oscillation becomes stabilized. The dominant oscillation for the G_4 note corresponds to the third impedance peak; since it is taller than the second peak, G_4 is easier than C_4 to play pianissimo. As the trumpeter plays louder the tall sixth peak comes in and greatly stabilizes the regime of oscillation, making the G_4 one of the easiest notes of all to play.



REGIMES OF OSCILLATION FOR HIGHER NOTES show why they become increasingly hard to play as one moves up the scale. G_5 is still quite easy to play because its regime of oscillation is dominated by the tall sixth impedance peak; the 12th peak makes only a minor contribution. C_6 is somewhat more difficult to play because the dominant peak of the note is lower than the peak for G_5. It takes an athletic trumpeter to reach the high E_6 and higher notes. The trumpet at this point has become virtually a megaphone: the energy production of the instrument is due almost completely to the interaction of the air column with the lips themselves, much as the human larynx operates in producing vocal sounds.

quality develops as measured inside the mouthpiece of the brass instruments. When one plays very softly, only the fundamental component associated with the playing frequency is present. As one plays louder the second, third, fourth and still higher harmonics grow progressively. If the oscillation is in the nature of a regime involving several cooperating resonance peaks, the harmonics grow in the simple fashion described by Worman's theorem; it is only at very loud playing levels that his theorem fails to give simple results. Furthermore, the theorem shows that the strength of the various components is proportional to the height of the various impedance maxima that are cooperating to generate the tone. In other words, when one plays rather loud, the strengths of the various harmonics have heights that correspond roughly to the heights of the impedance maxima from which they draw their chief sustenance. On the other hand, when a tone is generated on the basis of only a single resonance peak, as is the case in the upper part of the trumpet's range, we would be able to describe the strength of the components only if we could specify all the details of the flow-control characteristic.

Up to this point I have been discussing only the strength of the various harmonics as they are measured by a small probe microphone inside the brass instrument's mouthpiece cup. What one hears in the concert hall is, of course, very different. The transformation from the spectrum generated inside the mouthpiece, where the actual dynamics of the oscillation are taking place, into the spectrum found in the concert hall has to do with the transmission of sound from the mouthpiece into the main air column and thence out through the bell. There are many facets to the total transmission process, even without taking into account the complexities of room acoustics or the complications of our perceptual mechanism, which does a remarkable job of processing the great irregularity of room properties to give us clear-cut, definite impressions of the tone quality of musical instruments. I shall only remark that the transformation of the spectrum inside the mouthpiece to the external spectrum has the general nature of a treble boost. In other words, whatever sounds may be generated inside the instrument, it is the higher components that are radiated into the room [*see bottom illustration on page 34*].

The very fact that the bell of an instrument leaks energy preferentially at high frequencies has two important consequences. On the one hand the leakage enhances the relative amount of high-frequency energy that comes out of the horn; on the other it serves to reduce the height of the impedance peaks at high frequencies that lead to the weak generation of the high-frequency part of the spectrum inside the instrument. As a result measurements made outside the instrument in a room do not show nearly as much instructive detail about the dynamics of the entire system as measurements made inside the instrument do.

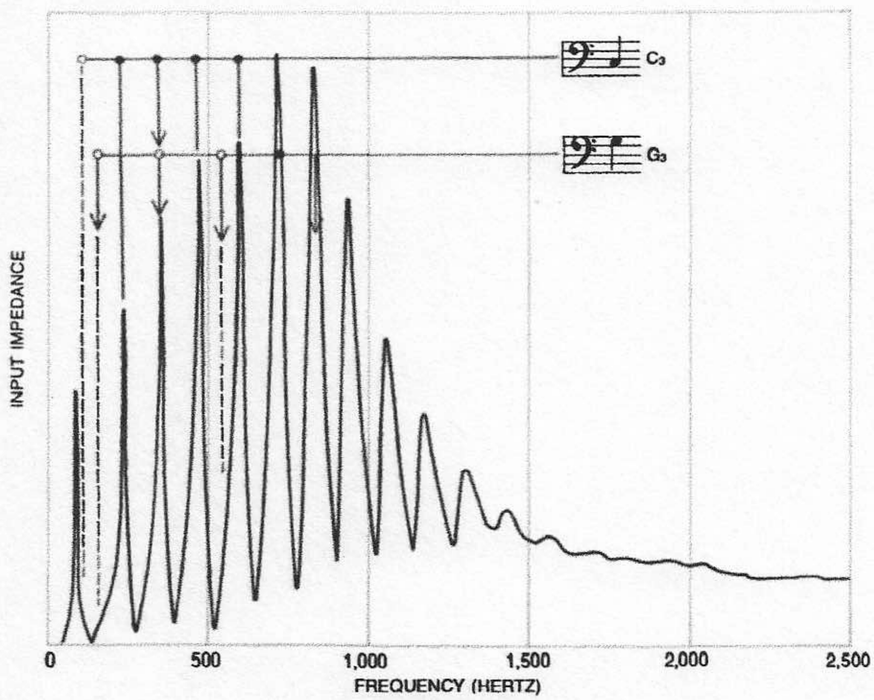
Let me conclude this discussion of the physics of brass instruments by indicating some of its implications for the musician and the instrument maker. As an illustration of the way physics can help the musician, I shall quote from an article of mine that appeared recently in the magazine *Selmer Bandwagon*. In this passage it was my intention to help French-horn players clarify and systematize their technique of placing one hand in the bell of the instrument to enhance certain frequencies.

"The player's hand in the bell is, acoustically speaking, a part of the bell. ... A properly placed hand provides... resonance peaks out to 1,500 hertz on an instrument that otherwise would lose all visible peaks at about 750 hertz [*see illustration on page* 35]. Suppose you meet a totally unfamiliar horn (perhaps during a museum visit when the curator opens the display cases) and you wish to find out quickly how well the instrument plays. Blow a mid-range note (for example concert F_3 in the bass clef) and, keeping your hand absolutely flat and straight, push it into the bell little by little until you feel a slight tingle in your fingertips. At this point (keeping the hand always perfectly straight) move the hand in and out a little until the horn sings as clearly as possible and the oscillation feels secure to your lips. Any listening bystander will agree with your final choice. Keep your hand in this slightly strained position and blow a tone an octave or a twelfth above the first one (say concert F_4 or C_5). Keeping your fingertips always in their original position, bend the palm of your hand so that its heel moves toward a position more familiar to the horn player. As you do so the tone will again fill out and get a ringing quality to it; also your lips will vibrate with a more solid feel. Your hand will now be in an excellent position for playing all notes on this horn, although an expert will be able to do even better after careful practice.

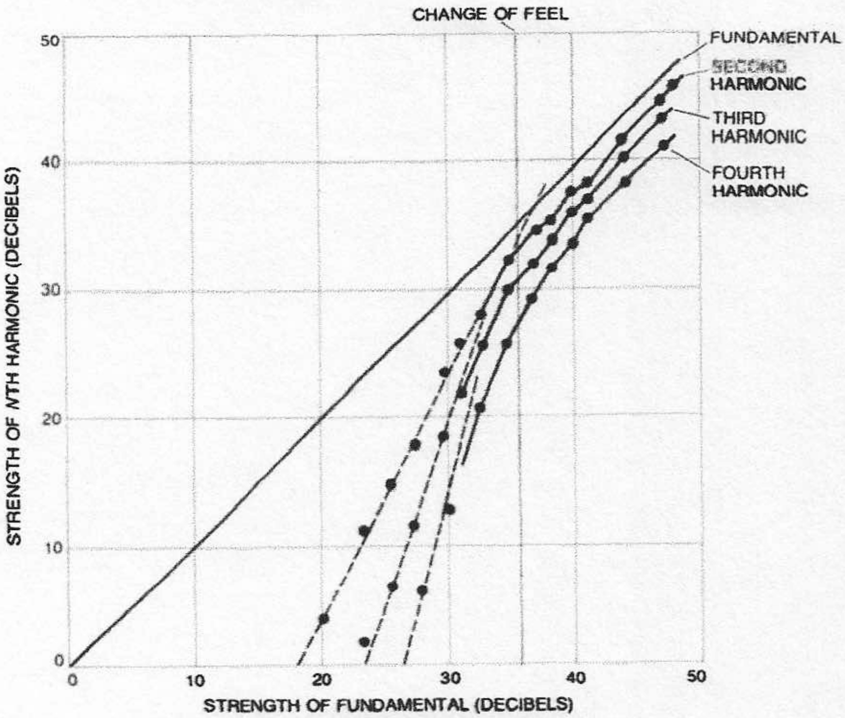
"Moving your straightened hand in and out while sounding the low *F* allows you to arrange to have an accurately located second helper for the tone. The unstopped horn works somewhat like a trumpet playing G_5 above the staff, while putting in the flattened hand serves to set up a regime that is analogous to the one which runs the trumpet's midstaff C_5. Bending the palm of one's hand while keeping the fingertips in place will leave the resonance peaks adjusted so far pretty much intact, but will make them taller (and hence more influential). This also gives rise to more peaks at the high-frequency end of things. The frequencies of these peaks move as the hand is bent more, so that once again the player has a means for tuning them for optimum cooperation with the other members of the regime. Trumpet players sometimes find it interesting and technically worthwhile to adapt the horn player's hand technique for their own purposes—especially for playing high passages on a piccolo trumpet."

It is only in the past few years that we have begun to have an understanding of the acoustics of mouthpieces. William Cardwell of Whittier, Calif., has provided a good theoretical basis for dealing with the relation of the mouthpiece dimensions to the tuning of the various resonance peaks. We in Cleveland, with the help of George McCracken of the King Musical Instrument Division of the Seeburg Corporation, have given attention to how the mouthpiece design controls the height of the impedance peaks. I quote again from the article for musicians to indicate the practical implications of mouthpiece acoustics.

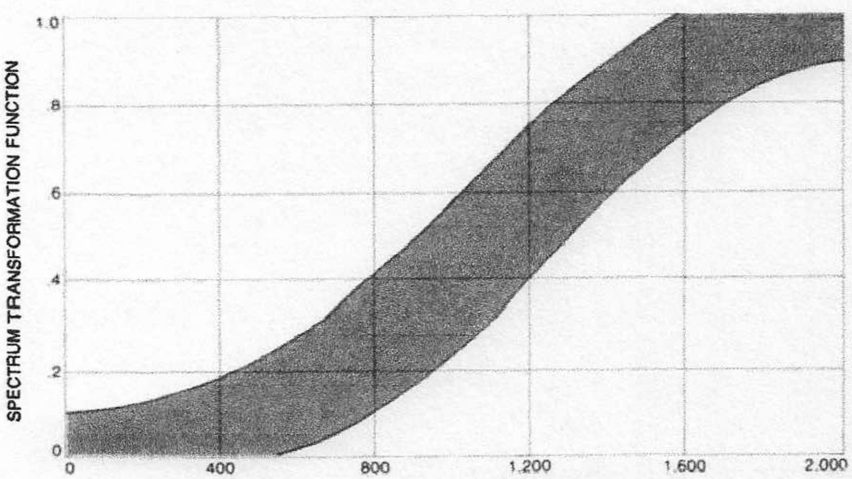
"Acoustical theory tells us that, first and foremost, a given instrument will require that the mouthpiece have a certain well-defined 'popping frequency' when its cup is slapped shut against the palm of the hand. In other words, the lowest natural frequency of the mouthpiece alone (with the cup closed) must be of the correct value. It is this requirement



UNUSUAL REGIMES OF OSCILLATION are associated with notes whose frequencies correspond to impedances that are close to minimum values. The note C_3 in the bass clef is known to musicians as the pedal tone. Its regime of oscillation is such that the second, third and fourth resonance peaks of the trumpet sustain an oscillation that lies at a frequency equal to the common difference between their own natural frequencies. Since there is actually a loss of energy at the fundamental playing frequency for this note rather than a gain, there is only a small amount of fundamental component in the sound, and even the small quantity present is converted to that frequency from the higher components by way of the nonlinearity in the flow-control characteristics of the player's lips. The situation for G_3 is even more unusual in that the second and fourth components of the tone are the chief source of oscillatory energy, whereas the fundamental component and the other odd harmonics contribute virtually nothing since the impedance is minimal at their frequencies.



TONE COLOR OF TRUMPET is related to the way harmonic frequencies make up an increasing fraction of the total sound emitted as the player blows louder. The strengths of the various harmonic components are plotted as a logarithmic scale (decibels) against the logarithm of the strength of the fundamental component. At low and medium playing levels each harmonic lies on a straight line whose slope is approximately equal to the serial number of the harmonic. As one plays pianissimo essentially no harmonics are present in the vibration as measured in the mouthpiece. For every doubling in strength of the fundamental component the second harmonic increases from an initial tiny value by a factor of four. Similarly, the third harmonic increases in strength by a factor of eight for each doubling in strength of the fundamental, and so on. This finding corresponds to a theory developed by Walter Worman at Case Western Reserve University. At the loudness where Worman's relation begins to break down the player senses a change in "feel" and listeners are aware of a change in sound. The data that are reflected in the curves were obtained with the help of Charles Schlueter, who now plays principal trumpet in the Minnesota Orchestra.



TRANSMISSION OF TRUMPET SOUND INTO ROOM is characterized by the "spectrum transformation function," which indicates what fraction of the acoustic energy at each frequency, as measured inside the mouthpiece, is emitted from the bell. Depending on the level of play and characteristics of the instrument, the energy emitted usually falls within the band plotted here. The curve has the qualitative nature of a "treble boost" because the bell leaks energy preferentially at high frequencies. Numbers on vertical scale are arbitrary.

that determines which of the peaks in the trumpet's response curve are the tallest. It also helps the peaks in this region to have the proper frequencies for good cooperation with the low-note regimes. The second most stringent requirement on the mouthpiece is that its total volume be correct (cup plus backbore). We must have this volume right in order to make the bottom two or three regimes of oscillation work properly."

So far I have discussed only the factors that contribute to favorable oscillation inside the horn and have said nothing about the tuning of instruments in the musician's sense: the relation between the pitches of the various tones that the instrument will generate. Fortunately the requirements for good tuning are almost identical with the requirements for favorable oscillation. It is for this reason that the traditional musical-instrument maker, focusing the major part of his attention on the tuning of the notes of the instrument, was able to develop instruments that would "speak" well and have good tone.

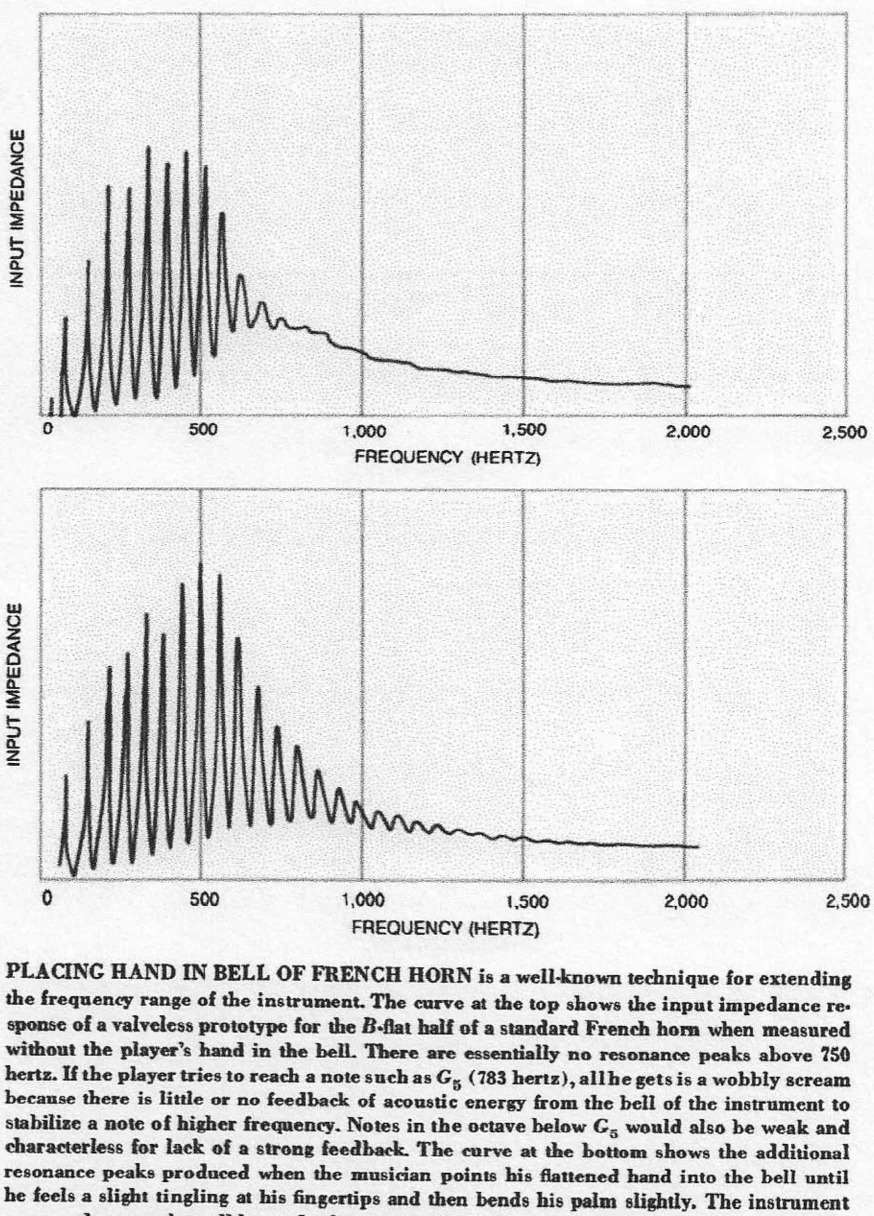
In more recent years, as our knowledge of acoustics has grown and the computer has become available, efforts have been made to design good brass instruments with the computer's help. Here the influence of loudspeaker acoustics has been great. Substantial efforts have been made to mathematically piece together a sequence of short loudspeaker-horn segments, each one intended locally to represent the shape of a workable brass instrument. This segmental approach to the problem has certain computational advantages. As we have seen, wherever the bore of a horn has a discontinuity of angle or of cross section there are anomalies in the standing-wave pattern. In spite of this fact it is always possible in principle to find suitable angles and cross sections that will place the impedance maxima of the horn with an accuracy that is acceptable by tuning standards. Although instruments built in this manner may play fairly well in tune, they can be quite disappointing in their musical value because of the neglect of the more subtle cooperative phenomena that ultimately distinguish between mediocrity and genuine excellence. Furthermore, the ability of an instrument to speak promptly and cleanly at the beginning of a tone is extremely sensitive to the presence of discontinuities, so that even though these discontinuities are arranged to offset one another in such a way as to give an excellent steady tone, it does not follow that the instrument starts well. The musician must of course have a "clean attack" as well as a clear, steady tone.

The skillful instrument maker gradually acquires an almost instinctive feel for the subtleties of instruments, so that he can sometimes be astonishingly quick in the use of his empirical store of knowledge to find a correct solution to a tuning or response problem. Consider the problem that such a person must solve when he is asked to correct a trumpet that is faulty, with the sole error being the behavior of the tone corresponding to C_4. Let us suppose that the problem is caused by the fourth impedance peak (beginning from the peak of lowest frequency), which is somewhat high in its frequency. When the C_4 is played at a pianissimo level, the note will be in tune, but as the loudness increases somewhat the note will tend to run a little sharp as the second member of the regime (the mistuned fourth peak) begins to show its influence. The player will also notice that he can "lip" the tone up and down over a considerable range in pitch without appreciable change in tone color. He will complain that at this moderate dynamic level the tone "lacks center." If he plays louder, the influence of the still properly tuned third and fourth members of the regime becomes strong enough to partly overcome the defect of the second member. When this occurs, the player finds that the tone once again acquires what he calls a core, or center, at a certain playing level, which happens then to fall pretty well back in tune because all but one of the resonances in the regime agree on the desired playing pitch.

In the practical world of the instrument maker or designer one often meets instruments in which one or more notes are "bad" in this way. It has often proved quite difficult to correct such problems with only instinct and experience. Once one understands what is going on, however, it is often possible to bypass laboratory measurements and diagnose the errors with the help of carefully designed "player's experiments." One then uses acoustical perturbation theory to guide the alteration of the shape of the air column to give a desired correction. Such corrections are made by enlarging or reducing the cross section of the bore in one region or more of the air column. The problem is complicated by the need to preserve the locations of the correctly tuned resonance peaks while the faulty peak is being moved.

Whether one is a physicist, a musician or an instrument maker, one tries to make use of any tools at hand to provide an instrument that helps rather than hinders the creative effort of music making. At first it would seem that the computer is ideally suited to be one of these tools and that it could immediately be put to work designing the perfect instrument. As a practical matter one finds that although we have a reasonable understanding of the goals to be achieved, the complexity of the problem is such that it is very difficult to specify the problem for the computer in sufficient detail. I have found that it is much more efficient to start with an already existing good instrument developed by traditional methods and then apply the physical understanding and the technical facilities available to us today to guide the improvement of the instrument, whether it is for an individual player in a symphony orchestra or for the development of a prototype for large-scale production.

In all my work I have found it always important to keep in constant touch both with professional players and with instrument makers. They provide an inexhaustible supply of information about the properties of instruments. They also are a source of questions that have proved enormously fruitful in guiding my investigations. As the subject continues to develop it is becoming increasingly possible for the results of formal acoustical research to be translated into useful information for the player and the instrument maker.



PLACING HAND IN BELL OF FRENCH HORN is a well-known technique for extending the frequency range of the instrument. The curve at the top shows the input impedance response of a valveless prototype for the *B*-flat half of a standard French horn when measured without the player's hand in the bell. There are essentially no resonance peaks above 750 hertz. If the player tries to reach a note such as G_5 (783 hertz), all he gets is a wobbly scream because there is little or no feedback of acoustic energy from the bell of the instrument to stabilize a note of higher frequency. Notes in the octave below G_5 would also be weak and characterless for lack of a strong feedback. The curve at the bottom shows the additional resonance peaks produced when the musician points his flattened hand into the bell until he feels a slight tingling at his fingertips and then bends his palm slightly. The instrument now produces peaks well beyond a frequency of 1,000 hertz, making it possible for the musician to play the note G_5 quite dependably and even a few higher notes when he is pressed.