GRE® Math Simplified with Video Solutions

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Dear Soon-to-Be Grad Student:

Welcome to **GRE Math Simplified with Video Solutions**!

I wrote this book to reach prospective graduate students like you and the ones I've been tutoring over the last 18 years— bright students and professionals who are looking for rigorous, affordable GRE preparation. The majority of my students have either forgotten math concepts or are in need of learning them for the first time. Some are short on time, money, or both. Almost all need a review of the basics, familiarity with "GRE speak," and lessons delivered in a way that resonate with them. If this sounds familiar, this book is for you.

Over the better part of the last two decades, I've successfully explained standardized math concepts—including GRE math— to hundreds of students, learning what barriers have prevented them from understanding certain concepts and what it takes for them to "get it." I've incorporated this knowledge and these experiences into this book, explaining math basics, identifying how concepts are tested and how to determine what is being asked, and explaining nuances across problems.

In addition, some of my students have asked to record our sessions so that they could re-play them when they got stuck, giving me the idea to record the video solutions. I know it can be hard to learn math from a textbook alone, and sometimes an oral explanation makes all the difference.

GRE Math Simplified problems are of three types: check-ins, examples, and problems for practice sets. Check-ins are problems that provide practice of the fundamental or prerequisite skill. Examples are GRE-style questions embedded within the chapter and require application of the concept most recently explained. Practice set problems are also GRE-style questions. Located at the end of each chapter, practice sets rely on a mix of the concepts presented in the chapter and correspond to a free solutions video, which can be found by going to the AndrewsTutoring YouTube channel.

GRE Math Simplified will help you: 1) eliminate holes in your math foundation, 2) learn GRE math concepts in a straightforward, comprehensive way, and 3) build confidence and know-how for the exam. It is intended to be used with the video solutions, and is indexed to **The Official Guide to the Revised General Test (3rd Edition)** for extra practice.

Good luck!

Julia Andrews
Andrews Tutoring

Suggested Study Plan for GRE Math Simplified

Assignment	GRE Math Simplified	GRE Math Simplified Video	ETS' Revised GRE (3rd Edition)
Online Practice Test 1			ets.org website
Familiarize yourself with the GRE	Read pages 15-17		Read pages: 1-10 (General), 11-42 (Essay), 43-52 (Verbal), 107-114 & 134-136 (Math)
Number Properties	Chapter 1	Chapter 1 Solutions	p. 115- #1 p. 119- #7 p. 122- #2 p. 126- #1-2 p. 128- #4 p. 232- #1, 3-8, 14, & 15 p. 261- #5
Fractions, Decimals, and Percents	Chapter 2	Chapter 2 Solutions	p. 116- #3 p.131- #4 p. 232-233- #2, 9-12 p. 262- #11
Ratios	Chapter 3	Chapter 3 Solutions	p. 232- #13 p. 262- #10
Averages	Chapter 4	Chapter 4 Solutions	p. 127- #3 p. 320-322 #1-3, 16
Expressions	Chapter 5	Chapter 5 Solutions	p. 118- #6 p. 119-120- #8-9 p. 261- #1-2
Equations and Inequalities	Chapter 6	Chapter 6 Solutions	p. 117- #5 p. 122- #1 p. 261-262- #6-8
Word Problems	Chapter 7	Chapter 7 Solutions	p. 115- #2 p. 124- #4 p. 129- #1 p. 262- #9, 12-16
Lines and Angles	Chapter 8	Chapter 8 Solutions	p. 280- #1

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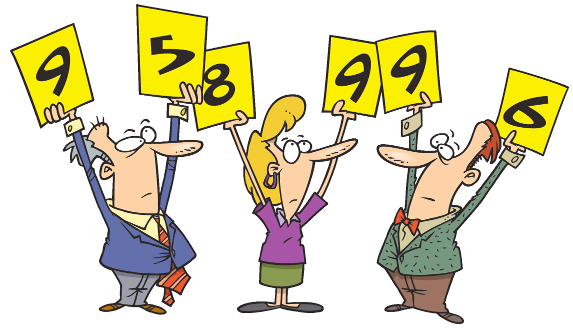
Chapter 1: Your Number's Up
(Number Properties)



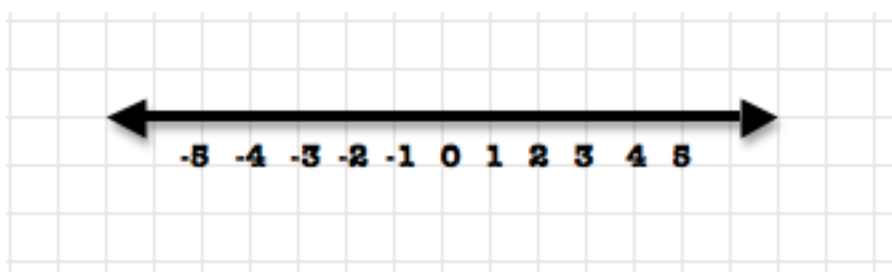
You might be thinking, "Why is there a section on NUMBERS?" (Fair question.)

"I know what numbers are. What clown wrote this?!" (Ok, now you're taking it too far.)

Well, there are different types of numbers, and they are categorized in multiple ways according to their properties—like integers and decimals, prime and composite, even and odd, factors and multiples, and consecutive and evenly spaced numbers. The writers of the GRE take it as a given that you can both recognize and draw connections between numbers and their properties.

1.1 Integers and Non-Integers

Consider the following number line:



All the numbers written on the number line above are integers. **Integers** are the counting numbers (1, 2, 3, ...), their opposites (-1, -2, -3,...), and zero (0).

There are infinitely many non-integers between each of your integers. For example, between the integers one (1) and two (2) are an infinite number of numbers, including: 1.278, $\sqrt{2}$, 3/2, and 1.9. So, when the GRE asks an abstract question that requires you to think of "a number," if it doesn't specify that the number is an integer, or state which type of integer it is, remember to consider different types of numbers, including: negative, positive, decimals, fractions, primes, and composites, because different types of numbers can yield different types of results.

Example 1.1

A number, when squared, becomes: a) bigger, b) smaller, c) the same, d) cannot be determined.

Solution:

d

Depending on what type of number you are squaring, the result can be bigger, smaller, or the same as the original number. Therefore, we cannot determine the result without additional information. For example, squaring 5 yields a larger number (25), squaring .60 yields a smaller number (.36), and squaring 1 yields the same number (1).

1.2 Order of Operations

Has this stuck with you since grade school, "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally?"

The first letters of each word spell **PEMDAS**, which is a handy mnemonic device for remembering your order of operations. You need to know it to accurately simplify mathematical expressions like:

$$(3-7)^2 + 2 \bullet 8 - 6$$

(And no, you can't just rely on your calculator to get the correct value. Nice try! If you don't input the numbers into the calculator in the proper order, you may not get an accurate result.)

PEMDAS stands for:

P- parentheses (meaning, simplify everything in parentheses)
E- exponents
M-multiplication/**D-**division
A-addition/**S-**subtraction

Quick! Tell me how many steps are in the order of operations!

Six, right? Wrong! Multiplication and division actually merge into one step, as do addition and subtraction. So there are actually only 4 steps.

That means, as you simplify an expression, when you reach the multiplication and division step, do whichever comes first in the problem—multiplication or division—reading from left to right. The same holds for addition and subtraction.

Check-in:

Evaluate the following expressions. (That means, "What's the answer?"):

1) $5-2^2+7$ 2) $(3-9)+3^4-8+2$ 3) $4-(11-4)+6$

Solutions:

1) 8	2) 69	3) 3
$5-2^2+7$	$(3-9)+3^4-8+2$	$4-(11-4)+6$
$5-4+7$	$-6+3^4-8+2$	$4-(7)+6$
$1+7$	$-6+81-8+2$	$-3+6$
8	69	3

1.3 Distributive Property

The **Distributive Property** can also be used to simplify expressions. It is a method of multiplying a number by a sum or difference of numbers.

Distributive Property: $a(b+c) = a \bullet b + a \bullet c$. The value "a" is being "distributed" to each term in parentheses.

For example, the expression 3(x +2) can be simplified to:

$3(x+2) = 3 \bullet x + 3 \bullet 2 = 3x + 6$

Check-in:

Simplify the following expressions:

1) $5(2+6)$ 2) $8(x+4)$ 3) $-3(x-7)$

4) $4-(x-9)$ 5) $4x-5(2-x)$ 6) $2x-9[3-(7+x-3)]$

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Examples of Even Integers:

a) -62 b) 32 c) 1,111,110 d) $2 \bullet x$, if x is an integer

Examples of Odd Integers:

a) -3 b) 51 c) 2,227 d) $2 \bullet x + 1$, if x is an integer

The relationships of adding, subtracting and multiplying even and odd integers are as follows:

Sum	Difference	Product
Even + Even = Even	Even - Even = Even	Even • Even= Even
Odd + Odd = Even	Odd - Odd = Even	Even • Odd = Even
Even + Odd = Odd	Even - Odd = Odd	Odd • Odd = Odd

There are no such rules for division, because when you find the quotient of two integers (meaning, the result of dividing two integers), you don't always end up with another integer. For example: $5 \div 3 = 1.66666...$

Note: Don't stress if you can't easily memorize rules like these. Since these rules are absolute—meaning they always hold true— just test any even number (say, 4) and any odd number (say, 3) when a question on even and odd numbers arises.

Check-in:

1) When possible, identify each of the following as even or odd:

a) An even number cubed plus one

b) The product of an integer x and the number 4

c) Twice the sum of 3 and an integer

d) An integer x multiplied by the product of integers y and z

Solution:

1)

a) **Odd:** An even number raised to any positive integer is even. Raising a number to an integer power greater than 1 is repeated multiplication. In this case, we are taking the cube of an even number, say, the number 2, which is equivalent to $2 \bullet 2 \bullet 2 = 8$, which is even. Add one (or any odd number, for that matter) to any even number, and the result is odd.

b) **Even:** The product of any integer (even or odd) times any even number – in this case, the number 4– is always even.

c) **Even:** Doubling an integer, whether even or odd, will always result in an even integer.

d) **Can't be determined:** This example is asking for the product of integers x, y, and z. If all three numbers are odd, the product is odd. If at least one number is even, the entire product is even.

1.6 Zero

Much ado about… zero.

Zero is a funny little number. It gets a lot of attention given that it's worth absolutely nothing.

Here are some things to remember about the number zero:

1) It is neither negative NOR positive. In fact, it separates negative numbers from positive ones.

2) Zero is an EVEN integer. (Yes, I know I mentioned it before– it's just that important to know.)

3) You cannot divide a number by zero, like 8/0 or 100/0. (Didn't your teacher ever tell you that the world would explode if you tried?)

4) You can, however, divide zero by any number—the result is zero. (Think, if you split nothing up among a group of 8 people, each person gets nothing: 0/8=0.)

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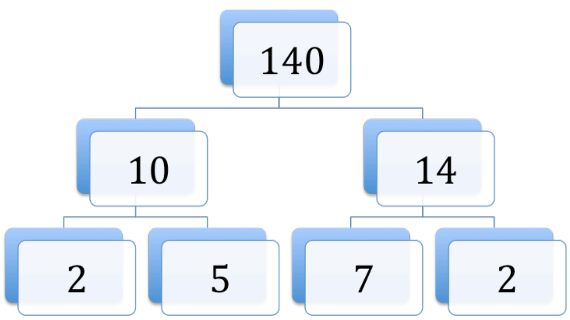
Show 'Em What You're Made Of!: Factor Tree

A **factor tree** enables you to break down a composite number into the product of its prime factors. This allows you to simplify expressions or to find commonality among numbers.

To create a factor tree, start with a composite number and create two "limbs" by writing **any** two factors that multiply together to equal that composite number. Continue this process—adding limbs to each factor that can be broken down— until every factor is a prime number.

This is a simple process, as illustrated in the following example.

Ex.: Find the prime factorization of 140.



The prime factorization of 140, then, is: $2 \bullet 2 \bullet 5 \bullet 7$, or $2^2 \bullet 5 \bullet 7$.

1.11 Greatest Common Factor (GCF)

The **greatest common factor (GCF)** between two or more numbers is the largest factor that the numbers share— the product of their common prime factors. Identifying the GCF helps simplify expressions and aids in problem solving.

One way to determine the greatest common factor is to make a factor tree for both (or all) of your numbers. Then, circle the prime factors they have in common and multiply one set of these factors together.

1.13 Exponents and Square Roots

Exponents are used to represent repeated multiplication. For example, to multiply 8 by itself 7 times, instead of writing $8 \bullet 8 \bullet 8 \bullet 8 \bullet 8 \bullet 8 \bullet 8$, simply write 8^7.

Laws of Exponents:

1) $(x^y \bullet x^z) = x^{(y+z)}$

To multiply exponential terms with identical bases, add the exponents.

Examples:

i. $x^4 \bullet x^7 = x^{11}$

ii. $2y^6 \bullet 6y^3 = 12y^9$ Multiply 2 and 6 as normal; add exponents

iii. $x^2y^6 \bullet xy^2 = x^3y^8$

As in iii, note that if there is no visible power of the variable, it is being raised to the power of 1. For example, in the term xy^2, the variable x is being raised to the first power.

2) $(x^y)^z = x^{(y \bullet z)}$ and $(x \bullet y \bullet z)^b = x^b \bullet y^b \bullet z^b$

To raise an exponent to another power, multiply the exponents and keep the base.

Examples:

i. $(x^4)^5 = x^{20}$

ii. $(x^7y^4z^2)^3 = x^{21}y^{12}z^6$

iii.) $(3xy)^2 = 3^2 \bullet x^2 \bullet y^2 = 9x^2y^2$

iv.) $3(xy)^2 = 3 \bullet x^2 \bullet y^2 = 3x^2y^2$

Note the difference between iii and iv. In iii, the "3" is inside the parentheses and in iv, the "3" is outside the parentheses. In the former, the 3 is raised to the second power and in the latter, the 3 is NOT raised to the second power.

3) $x^0=1$

Any nonzero number or variable raised to an exponent of zero is 1.

Examples:

i. $3^0 = 1$

ii. $(xyz)^0 = x^0 \bullet y^0 \bullet z^0 = 1$

iii. $xyz^0 = x \bullet y \bullet z^0 = x \bullet y \bullet 1 = xy$

Note that, in ii, the zero exponent applies to x, y, and z, because all three variables are in parentheses. In iii, the exponent only applies to z, the variable which is directly in front of it.

4) $\mathbf{ax^y + bx^y = (a+b)x^y}$

$\mathbf{ax^y - bx^y = (a-b)x^y}$

In order to add or subtract terms with the same base and power, add or subtract the **coefficients** (the numbers multiplied to the variables— a and b in this example) and leave the base and exponent unchanged.

Examples:

i. $3x^4 + 9x^4 = 12x^4$

ii. $-7y^8 - 6y^8 = -13y^8$

iii. $2x^7 + 9x^5$ This expression is already simplified because the two variables are raised to different exponents.

* * * Head's Up: The remaining rules of exponents include working with fractions. If you find that you are having trouble with this section, you may want to review pages 60-65 before preceding. * * *

5) $\mathbf{\frac{x^y}{x^z} = x^{(y-z)}}$

When dividing numbers with the same base, subtract the exponents.

Examples:

i. $\dfrac{x^{12}}{x^5} = x^7$

ii. $\dfrac{6x^3}{2x} = 3x^2$

6) $(x)^{-y} = \dfrac{1}{x^y}$

To raise a number to a negative exponent, take the reciprocal of that number and raise it to the absolute value of the exponent.

Examples:

i. $x^{-4} = \dfrac{1}{x^4}$

ii. $6x^{-2} = \dfrac{6}{x^2}$

iii. $(6x)^{-2} = \dfrac{1}{(6x)^2} = \dfrac{1}{36x^2}$

7) $\left(\dfrac{x}{y}\right)^{-z} = \left(\dfrac{y}{x}\right)^{z}$

To raise a fraction to a negative exponent, take the reciprocal of the fraction and raise the result to the absolute value of the exponent.

Examples:

i. $\left(\dfrac{x^4}{y^5}\right)^{-3} = \left(\dfrac{y^5}{x^4}\right)^{3} = \dfrac{y^{15}}{x^{12}}$

ii. $6\left(\dfrac{3x^2}{y}\right)^{-2} = 6\left(\dfrac{y}{3x^2}\right)^{2} = 6 \bullet \dfrac{y^2}{9x^4} = \dfrac{6y^2}{9x^4} = \dfrac{2y^2}{3x^4}$

Check-in:

Simplify the following expressions using positive exponents:

a) $5x^2 \bullet 4x^5$ b) $3x^7 \bullet x^{-8}$ c) $(-5x)^{-2}$ d) $5x^{-2}$

e) $\frac{40m^7n^2}{5m^3}$ f) $5(x^3)^5$ g) $(5x^3)^5$ h) $(\frac{x^2}{y^5})^{-1}$

Solution:

a) $20x^7$ b) $\frac{3}{x}$ c) $\frac{1}{25x^2}$ d) $\frac{5}{x^2}$

e) $8m^4n^2$ f) $5x^{15}$ g) $3{,}125x^{15}$ h) $\frac{y^5}{x^2}$

Example 1.6

Given integers m and n, if $m^n n^m = 72$, what could be the average of m and n?

Indicate all that apply.

a) -2.5
b) -0.5
c) 0
d) 2.5
e) 5.0

Solution:

d only

In order to find the values of m and n, find the prime factorization of 72, which is $2^3 \bullet 3^2$. Let m=2 and n=3 (or vice versa).

The average, therefore, is 2.5: $\frac{2+3}{2}$.

(Note that it does not matter which of the two values is assigned to m or to n, as the question is only asking for the average.)

Example 1.7

If $x^{-5} = \frac{1}{32}$, what is x^2?

Solution:

4

$x^{-5} = \frac{1}{x^5} = \frac{1}{32}$

$x^5 = 32$

Since $2^5 = 32$, $x = 2$. Thus, $x^2 = 4$.

Square Roots

Taking the **square root** of a number is the inverse operation of raising a number to an exponent. The square root of a number x is the number y such that the product of y and y is the number x. For example, the √9 is 3 because 3*3=9.

It is helpful to be familiar with square roots that are integers. The first 20 square roots of perfect squares are:

$\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$ $\sqrt{16} = 4$ $\sqrt{25} = 5$

$\sqrt{36} = 6$ $\sqrt{49} = 7$ $\sqrt{64} = 8$ $\sqrt{81} = 9$ $\sqrt{100} = 10$

$\sqrt{121} = 11$ $\sqrt{144} = 12$ $\sqrt{169} = 13$ $\sqrt{196} = 14$ $\sqrt{225} = 15$

$\sqrt{256} = 16$ $\sqrt{289} = 17$ $\sqrt{324} = 18$ $\sqrt{361} = 19$ $\sqrt{400} = 20$

Note: You cannot take the square root of a negative number under the real number system, because there is no real number, when multiplied by itself, that results in a negative product.

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1.14 Set Notation

A **set** is a group of items, often numbers, either listed in number or expressed by rule. The following are examples of sets:

Set A= {3, 4, 5}

Set B= {2, 3, 5, 11}

Set C={3, 4}

Set D is comprised of the multiples of 5 between 10 and 30, **inclusive** (meaning, including the endpoints).

Examples of the terms defined below reference the above mentioned sets.

The **elements** (or members) of Set A are 3, 4, and 5. The elements of Set D are 10, 15, 20, 25, 30.

The **intersection** (denoted by the symbol ∩) of sets means those elements common to each set. For example, **Set A** ∩ **Set B** is {3, 5}.

The **union** (denoted by the symbol ∪) of sets means all of the elements in each set. For example: **Set A** ∪ **Set B** is {2, 3, 4, 5, 11}.

For a set to be **contained** (denoted by the symbol ⊂) in another set, all of the elements in that set must be in the set that contains it. For example: **Set C** ⊂ **Set A,** because both of the elements in Set C (3 and 4) are in Set A.

If sets have no elements in common, their intersection is the **empty set** (denoted by the symbol ∅). For example: **Set C** ∩ **Set D** is ∅, because they have no common elements.

Example 1.8

Set A= {all prime numbers}

Set B ={all even numbers}

How many elements are in the intersection of Set A and Set B?

a) 1
b) 2
c) 5
d) Infinitely many
e) None of the above

Solution:

a

Set A and Set B only have one element in common, which is the number 2. Recall that the number 2 is the only even number that is also a prime number.

CHAPTER 1 PRACTICE SET:

1) The sum of 5 consecutive integers is 35. What is the value of the greatest integer?

2) What is the product of consecutive integers a, b, c, d, and e, if $a<0$ and $e>0$?

3) If possible, determine which of the following is/are even, given that each variable represents an integer:

a) $2x^3y^5$

b) $x+x$

c) $5xyz^2$

d) 2^x, where $x > 0$.

4)

Quantity A	Quantity B
x	x^5

$x < 0$

5)

Quantity A	Quantity B
$-\lvert x\rvert$	x

6) $1<x<5$

Quantity A	Quantity B
The number of numbers between 0 and 1	The number of possible integer values of x

7) Leo and Larry are not morning people. They love to hit snooze. If Leo's alarm rings every 10 minutes, beginning at 7:00 a.m., and Larry's alarm rings every 9 minutes, beginning at 8:00 a.m., at what point will their alarms ring simultaneously after Leo and Larry each have hit snooze at least once?

a) 7:30 a.m.
b) 8:00 a.m.
c) 9:15 a.m.
d) 9:30 a.m.
e) 10:30 a.m.

8) What is the smallest integer greater than 2 that leaves a remainder of 2 when divided by 3, 6, and 9?

9) If $x^4 = 81$, then what is 2^{2x}?

10)

Quantity A	Quantity B
9^{14}	3^{28}

11)

Quantity A	Quantity B
27^{8}	9^{14}

SOLUTIONS: Chapter 1 Practice Set

1) 9
2) 0
3) Even, Even, Cannot Be Determined, Even
4) d
5) c
6) a
7) d
8) 20
9) 64
10) c
11) b

EXPLANATIONS: Chapter 1 Practice Set

1) **9**

$$x + x + 1 + x + 2 + x + 3 + x + 4 = 35$$

$$5x + 10 = 35$$

$$\frac{-10}{5x} = \frac{-10}{25}$$

$$\frac{5x}{5} = \frac{25}{5}$$

$$x = 5$$

Five is the lowest integer, so the integers are 5, 6, 7, 8, and 9. Nine is the highest.

2) **0**

If there are five consecutive numbers, at least one of which is negative and at least one of which is positive—then one of them MUST be zero. (For consecutive numbers, to go from negative to positive, the list must pass through—and therefore include—zero.)

3)

a) **Even**

The product of integers is always even if at least one of them is even.

b) **Even**

$x + x = 2x$. Any integer times two is even.

c) **Cannot be determined.**

The only number we know for sure is 5, which is odd. Since the product of an odd and an odd is odd, and the product of an odd and an even is even, we do not know the result.

d) **Even**

The product of 2 and any integer is even. Therefore, 2 times itself any number of times is even.

4) **d**

***Remember (as mentioned on p. 15) for these types of questions the answer choices are as follows:**

a: if Quantity A is always bigger
b: if Quantity B is always bigger
c: if Quantity A=Quantity B
d: if the relationship cannot be determined

Since we have no information about x, the two columns could be the same, or one could be bigger than the other. If x is zero, for example, the columns are equal. If x is greater than one, Column B is bigger. If x is between zero and one (exclusive), Column A is bigger.

5) **c**

Since both columns are based on x, they have the same absolute value. Since it is given that $x<0$, Column B is negative. And in Column A, the opposite of an absolute value (that isn't zero) is negative.

6) **a**

There are an infinite number of numbers between 0 and 1. Remember, "number" does not just refer to integers! There are three possible integer values between 1 and 5: {2, 3, 4}.

Pages intentionally omitted