CHAPTER TWO
THE FRACTION KIT

Overview

Students make their own fraction kits by cutting different-colored strips of construction paper into halves, fourths, eighths, and sixteenths. Having students cut and label each of the pieces is an effective way to introduce them to fractions as parts of a whole and to the meaning of standard fractional notation. Students learn, for example, that eight eighths make one whole and that it makes sense to label each of the eight pieces as $\frac{1}{8}$ because each is one of the eight equal-sized pieces they cut. After making their fraction kits, students learn several games to play with them and then are engaged in activities that connect their experience to representing and comparing fractions. Later, students use the kits for additional activities and also extend them by cutting additional strips into thirds, sixths, and twelfths.

Materials

- 5 colors of 12-by-18-inch construction paper cut into 3-by-18-inch strips, 3 sets of strips per student, 1 set for you, and several extra sets for possible mishaps (A suggestion: Arrange pieces of 12-by-18-inch construction paper in alternating colors before cutting. Cut 5 sheets at the same time on the paper cutter and you'll have strips in sets of 5 colors.)
- fraction dice made from labeling the faces of cubes $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{16}$, 1 per pair of students plus 1 per student to take home
- 1-gallon zip-top plastic bags, 2 per student
- rules for the game of *Cover Up* (see Blackline Masters)
- rules for the game of *Uncover, Version 1* (see Blackline Masters)
- rules for the game of *Uncover, Version 2* (see Blackline Masters)
- *Comparing Pairs* worksheet, 1 per student (see Blackline Masters)
- *What's Missing?* worksheet, 1 per student (see Blackline Masters)
- to extend the activity: 3 additional colors of 12-by-18-inch construction paper cut into 3-by-18-inch strips, 1 set of strips per student, 1 set for you, and several extra sets for possible mishaps. Instead of giving students dice to play *Cover Up* and *Uncover,* write fractions on 1-inch square tiles, put the tiles in small bags, and have students draw one at a time for a move, replacing the tile after each move. Use a different color tile and a different bag for each of the three sets.

  Set 1: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{12}$
  Set 2: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{12}$
  Set 3: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{16}$

Time

- at least five class periods

Teaching Directions

1. Distribute five 3-by-18-inch strips of construction paper in different colors to each student. Ask students to take a strip of a particular color (that you choose), fold it in half, and cut it into two pieces. Have them label each piece $\frac{1}{2}$, explaining that because the pieces are the same size, each is one of the two pieces, which we represent as $\frac{1}{2}$. Then choose a color for a second strip and have the students fold and cut it into four equal pieces. Instruct students to label each piece $\frac{1}{4}$. Then have them fold, cut, and label a third strip in eighths and a fourth strip in sixteenths. Students leave the fifth strip whole, and label it *1* or $\frac{1}{1}$.

2. Teach the students how to use their fraction kits to play *Cover Up.* (See Blackline Masters for the rules.) Model the game with two students, then distribute a fraction die to each pair of students and have them play the game. Leave time at the end of class for students each to make another fraction kit to take home with a fraction die. Distribute zip-top bags for students to store their fraction kits and an additional set of uncut strips. Give them the homework assignment of cutting the additional set of strips with someone at home and playing the game.

3. On Day 2, have children report their experiences playing the game at home. Then teach the class how to play *Uncover, Version 1,* and have them play during class. (See Blackline Masters for the rules.) Give them the homework assignment of playing the game at home.

4. On Day 3, after children again report their experiences at home, discuss their strategies for exchanging pieces when playing *Uncover.* List their strategies on the board as they report them. Then teach *Version 2* of the game. (See Blackline Masters for the rules.) Give students time to play the game for the rest of class.

5. On Day 4, after children report their experiences at home, introduce *Cover the Whole.* Cover a whole strip with a train of fraction pieces, using three one-fourth pieces and two one-eighth pieces. Then model how to record a sentence that describes the train. Finally, show students how to shorten the sentence by combining fractions with like denominators.

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=1$$

$$\frac{3}{4}+\frac{2}{8}=1$$

6. On Day 5, introduce *Comparing Pairs* and *What's Missing?* (see Blackline Masters). Model examples for students, including how to explain their reasoning. Tell them: "You have to solve the problems and explain your reasoning for three of them. Then you make up two of your own problems and explain your reasoning for one of them."

7. On subsequent days, take a few minutes and pose one of the challenge problems for a class discussion:

How many eighths are there in one half?
How many eighths are there in one whole?
How many eighths are there in three fourths?
How many sixteenths are there in one half?
How many sixteenths are there in three eighths?
How many eighths would be needed to cover two wholes?
How many fourths are in one half?

8. As an extension a few weeks later, return to the fraction kits and have students cut three additional strips of different colors into thirds, sixths, and twelfths. Provide time for them to play *Cover Up* and *Uncover* with the sets of tiles described in the "Materials" section on page 11. Also, see Chapter 15, "Only One," for activities that use the kits to introduce combining fractions.

Teaching Notes

The fraction kit has been a long-standing favorite of mine as a jumping-off platform for introducing fractions to students and providing a concrete reference for them to use as they study fractions. The kit initially introduces children to halves, fourths, eighths, and sixteenths and helps them see how these fractions relate to one another. They learn, for example, that halves are larger than fourths, eighths, and sixteenths and that one-eighth of a whole is less than one-fourth of the same whole. They also learn that three-fourths of a whole can be represented by three of the one-fourth pieces or by one one-fourth piece and one one-half piece, that two of the one-eighth pieces takes up the same amount of space as does one of the one-fourth pieces, and that there are sixteen sixteenths in a whole and eight sixteenths in a half. These ideas emerge naturally as students use the kits.

After they cut the kits and learn to play the games, students experience follow-up activities that help them understand equivalence and learn to represent and compare

fractions. Eventually, of course, students have to deal with fractions without referring to pieces of paper or any other concrete materials, and these follow-up activities help move students in that direction while still allowing them the support of their kits. Additional activities with the kits are also useful for introducing students to combining fractions (see Chapter 15, "Only One").

For students just beginning to learn about fractions, the fraction kit is the most effective introduction that I've found, even though it initially focuses only on a small set of fractions—halves, fourths, eighths, and sixteenths. Later, extending the kit to include thirds, sixths, and twelfths gives students the opportunity to investigate how these fractions relate to halves, fourths, eighths, and sixteenths and further builds a foundation of understanding that can then be extended to other fractions. My advice is not to skip this chapter under any circumstances and to teach it in its entirety, even if you think that students already understand these ideas. For more advanced students, the experience will cement their understanding and also extend it. Trust me on this one.

The Lesson

To prepare, I cut 12-by-18-inch construction paper lengthwise into four 3-by-18-inch strips. I used five colors and cut enough strips so that every student would have two of each color, allowing them to make a kit to take home. I also made extra strips for me to use to demonstrate and for extras in case of cutting mishaps. So that I could distribute the strips more efficiently, I organized them into sets of five that included one of each color. I made sure that there were enough scissors available for each student. Also, I made fraction dice from cubes, one for every two students, labeling the faces $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{16}$.

DAY 1

I distributed the sets of five colored strips I had prepared and said to the students, "You'll each make a fraction kit, following along with me so that we all wind up with the same fraction pieces. Then we'll use the kit for games and activities that will help you learn about fractions. First, put the dark blue strip aside. We won't cut that one at all. Let's start with the red strip."

I've learned from experience that it's most successful to lead the students step-by-step through cutting their kits and to have all students use the same color for each fraction. This avoids the kinds of mistakes that students make when they work on their own. I have the students cut and label one color strip before I give instructions for the next one.

I modeled for the class how to fold the red strip and cut it into two halves. Most of the students were familiar with the notation for one-half, but I explained anyway. "After you cut your strip into halves, label each piece. I label them like this." I wrote $\frac{1}{2}$ on one of the strips and explained, "There are two red pieces that make the whole, and this is one of them, so I write one-half on each to show that it's one of two pieces of the whole. Cut and label your two red strips as I did mine."

$\frac{1}{2}$	$\frac{1}{2}$

I waited for all of the students to do this, going over to make sure Tom understood

the directions and then prodding Josh not to dawdle.

I then explained to the students what they were to do next. "Watch as I fold, cut, and label my light blue strip into fourths." I folded it in half and then in half again and opened the strip to show it divided into four parts. Before cutting, I labeled each part $\frac{1}{4}$.

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

"Who can explain why it makes sense to label each part as I did?" I asked. I called on Maggie.

"You have four parts, and each part is one of them," she said.

I nodded and asked, "Does anyone have a different way to explain?" No one volunteered. I then gave them time to fold, cut, and label their light blue strips into fourths.

Next I picked up a purple strip. "Watch as I fold it," I said. I folded it in half, half again, and then half again.

"How many sections do you think there will be when I unfold it?" I asked. Some thought six and some thought eight. I unfolded the strip and we counted to verify that there were eight sections.

"Raise your hand if you think you know how I should label each of the parts," I said. I waited until practically everyone had raised a hand. I called on Shannon.

"They're one-eighth because there are eight parts," she said.

"So what do I write?" I asked.

"A one over an eight," she said. I wrote $\frac{1}{8}$ in the first section and asked, as I had done with the fourths, "Who can explain why it makes sense to label this part one-eighth?" Hands shot up and I called on Davey.

"There are eight parts altogether and that's just one of them," he said. I wrote $\frac{1}{8}$ in each section.

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Then, before having the students fold, cut, and label their purple strips into eighths, I introduced some vocabulary. "The top number in the fraction is called the numerator," I said.

"Oooh, I know," Claudia said. "The bottom number is the denominator."

"Yes," I said, writing those words on the board.

"What's the line called?" Libby wanted to know.

I stopped for a moment. "I'm not sure it has a fancy name like *numerator* or *denominator*," I said. "I've always heard it referred to as the fraction bar."

"Well, how come they use it to write fractions?" Libby continued. These are the sorts of questions that Libby asked, and I had come to understand that she really wanted to know.

"Sometimes they use a slanty line," Dan said.

"Yes," I said. "You'll see fractions written like this as well." I wrote *1/8* on the board.

I took a stab at explaining about the fraction bar. "You know about the symbol for division," I said to the class and wrote ÷ on the board. They nodded.

"One way we can think about fractions is by thinking about dividing. When we make eighths for our fraction kit, we're dividing a whole into eight equal parts. That's like dividing one by eight." I wrote on the board: *1 ÷ 8*. "You can think about writing one divided by eight in the form of a fraction by replacing the dots in the division sign with the numbers one and eight."

It's important for students to understand how fractions relate to division and this was a chance to introduce the idea. I intended to return to it at a later time.

"Any other questions?" I asked. Sean raised his hand.

"Should we write the fractions before we cut the pieces or cut them first?" he asked. In contrast to Libby's questions, Sean's questions more often focus on what he needs to do than on ideas he's considering.

"Either way is fine," I said. "Just be sure when you're done that you have eight pieces."

When they had all finished, I asked them to count their purple eighths to make sure they had eight of them. Then I modeled for them how to cut their brown strips into sixteenths. The paper gets awfully bulky when the students try to fold the strip four times, so I suggested that they first fold and cut eighths and then fold each eighth in half to make sixteenths. This makes sense to students and helps reinforce that one-eighth is equivalent to two-sixteenths. Some students groaned as they labeled the sixteenths because there are so many of them. The experience of labeling is good, I think, because it helps children understand why sixteenths are smaller pieces even though the fraction has a larger denominator.

After they had cut their pieces, I held up a dark blue uncut strip. "How should we label this one?" I asked. "We're not going to cut it."

"How about one whole?" Robert suggested.

Janie said, "I think one over one, like this." She traced $\frac{1}{1}$ in the air.

"Maybe just plain one," Jennifer said.

"Any and all of those are okay," I said. "After you label your whole, put your initials on the back of every one of your pieces so that we can identify pieces that may get lost."

1															
$\frac{1}{2}$								$\frac{1}{2}$							
$\frac{1}{4}$				$\frac{1}{4}$				$\frac{1}{4}$				$\frac{1}{4}$			
$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

"There's a lot of pieces," Josh complained.

"How many?" I asked, to shift the energy from complaining to thinking mathematically. (There are thirty-one pieces in all.) "If you're interested, try figuring that out in your head first and then count to check." Some of the students were interested while others just got to the task of writing their initials.

Teaching the Rules for *Cover Up*

After the students had written their initials on each of their pieces, I taught them how to play *Cover Up*. I had them gather around one table to watch as I taught Sam and Andrew how to play. (See page 145 for rules.)

"You each start with your dark blue whole in front of you," I said. "The idea is to be the first to cover your whole completely and exactly, with nothing extra hanging over."

I showed them one of the fraction dice I had made. "The die has one-half and one-fourth each written once and one-eighth and one-sixteenth each written twice," I said. "On your turn, you roll the die, see what fraction comes up, and put a piece that matches that fraction on your whole strip."

I then had Sam and Andrew play. "Who will go first?" I asked. They boys shrugged. Then Andrew said that he would and he rolled the die. It came up $\frac{1}{8}$, and Andrew took one of his purple pieces and put it on his dark blue whole strip. While he was doing this, Sam reached for the die. I stopped him, however. "It's important that you watch each other. Sam, you need to watch to be sure you agree with what Andrew does. Andrew, when you've played, say 'Done' to signal Sam that he can roll. Until Andrew says 'Done,' Sam shouldn't even reach for the die." Sam nodded and waited.

"Say 'Done,'" Davey said to Andrew.

"Oh yeah," Andrew said. "Done."

Sam reached for the die and rolled. He got $\frac{1}{2}$. "Way to go!" he said, and put a red half on his whole. Then he said, "Done."

Andrew rolled next and got $\frac{1}{16}$. "Yuk," he said. "This is getting me nowhere." Sam waited a moment and then prompted Andrew to say "Done." Andrew did so and Sam reached for the die. This time he rolled $\frac{1}{4}$, put a light blue piece on the whole, and said, "Done."

Dan said, "Sam only needs another fourth, but Andrew needs more."

"Lots more!" Andrew added, reaching for the die.

"Before you roll, can you tell how much more you need to cover the whole?" I asked.

"Even a half isn't enough," Claudia commented.

"I think I need a half and a fourth and something more," Andrew said, studying his board.

"He needs a half, a fourth, another sixteenth, and then another fourth," Jennifer said. Jennifer wasn't correct, since a half and two fourths would cover the entire whole, but I didn't correct her or take the time to have her check her prediction with her pieces. Most of the students were more interested in the game than my question, so I kept the focus on the rules for play and didn't further interrupt the game. I decided that it was enough to plant the idea that they might think about how much there was left to cover.

Andrew rolled $\frac{1}{4}$. "Now that's better," he said, putting a light blue one-fourth piece on his whole. "Done," he said, passing the die to Sam.

Sam rolled $\frac{1}{2}$. Some of the students let out a cheer.

"Not so fast," I said. "Remember what I said about covering the whole completely but exactly. The one-half piece is too big."

"So what happens?" Lily asked.

"Sam can't put anything on. He has to say 'Done' and pass the die to Andrew."

"Things are looking up," Andrew said, getting ready to roll the die. I let the boys play out the game. Then I had the students return to their seats to play with partners.

Observing the Students

All of the students were interested in the game and stayed engaged for the rest of the period. I circulated as they played, watching to make sure they were following the rules and listening to hear what they were saying.

I noticed that Joey and Robert were racing through a game, rolling and putting pieces on their whole strips but not paying attention to what each other was doing. Also, they weren't following the rule of saying "Done." I stopped the boys and talked with them. "Not only is it good to check on what your partner does, but I'm trying to slow the game down enough so you also have time to think about the mathematics involved," I explained.

"What do you mean?" Robert asked.

"You can be thinking about what you hope you'll roll next, or how many more rolls you think you'll need to cover your whole, or how much ahead one of you is than the other. Remember the game is to help you learn about fractions. I want you to enjoy playing, but I also want you to do some thinking." The boys nodded. I made one last comment.

"So, remember, no grabbing for the die. Also, be sure to say 'Done' after you play, and pay attention to each other's pieces." Again the boys nodded. I stayed for a bit while they resumed play to make sure they were following my directions. I had to talk with several other pairs in the same way.

I then noticed that Carol and Sarah had stopped playing and Carol was trimming slivers from her fraction pieces. I've seen other students do this and I talked to Carol about why I thought it wasn't necessary. "Measurement is never exact, so it's likely that your pieces aren't exactly the right sizes. Trimming can't really solve that problem. What's important is that you think about the fractional sizes the pieces are supposed to be, rather than just relying on the paper pieces you've cut."

"I want them to be even," Carol countered. I wasn't able to convince her of the futility of trimming. Her sense of order seemed somehow violated by the inconsistencies. After a few more snips, she gave it up and she and Sarah returned to the game.

When the class time was nearly over, I called the students to attention and asked them what they thought about the game. Many of the students raised their hands. I called on Tom first and got a typical answer.

"It's fun," he said.

"What makes it fun?" I asked.

"Rolling the dice. Moving the pieces. Watching your partner get stuck." The others laughed.

I called on Sarah. "I rolled one-half twice in a row once and won," she said.

"Me, too," Lily chimed in.

"So you like winning," I said. They nodded.

"Did anyone else roll one-half twice in a row?" I asked. No one else had.

"I rolled one-sixteenth a lot," Martin said.

"Yeah, me too," Josh added.

"It's hard to get one-half," Joey said. "There's only one of them on the cube and there are two eighths and two sixteenths."

Some nodded, indicating that they had noticed the same thing. Claire, however, carefully inspected one of the fraction cubes. She looked at the fraction on each face.

"Oh yeah," she said. "That explains it."

"That explains what?" I asked.

"Why I didn't get one-half at all," she said.

I then collected the fraction dice and gave each student a plastic 1-gallon zip-top baggie for their pieces. By folding the whole in half, all of the pieces fit in nicely.

For homework that night, I had the students each take home another set of strips to cut into a fraction kit and to teach someone at home to play *Cover Up*. I wanted them to have kits at home, but I didn't want to risk having them take home their class sets and not remember to bring them back. I duplicated and distributed the rules and also gave each student a cube to make a fraction die. I knew that some students would forget to return their cubes, but I had extras and it wouldn't be disastrous if I lost a few.

I planned to start the next day's class by having the students report what happened when they played the game at home. Then I would have them play again at the beginning of the next day's class. While they played, I would circulate and have conversations with as many of them as possible, asking how much more someone needed to cover the board, or how much farther ahead was the person who was winning so far. This would be valuable assessment time, but I also wanted to be sure that I didn't interfere with their enjoyment of the game. I planned to use the second half of the next day's lesson for a brief class discussion about *Cover Up* and then teach them how to play *Uncover,* another game with the fraction kit.

DAY 2

I began class as I had planned, asking the students about their experiences playing *Cover Up* at home. They seemed to have enjoyed the experience.

"I beat my dad three times," Tom said.

"My mom said the game was fun," Delia said.

"I was really lucky. I kept rolling one-half," Sarah said.

I then told the students that I'd like them to play for just a short while. "I want to be sure everyone knows how to play the game. Then I'll teach you another game to play with the same pieces." The students were happy to return to the game.

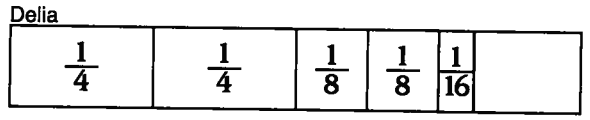
As I observed them play, I tried to assess their understanding as much as possible. But I waited to interrupt until I was sure that they were clear about how to play the game. I tried to be sensitive not to interfere with their enjoyment of the activity.

Jeremy had two fourths and one sixteenth on his whole strip. I asked him how much more he needed to cover the strip. He shrugged, completely disinterested in my

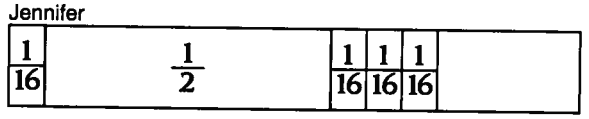
question and eager to take his turn and roll the die. I didn't push it at that time.

But when I interrupted Jennifer and Delia, they were more interested in my question than the game. "The game is just luck," Jennifer said. She likes to think about problems, and I saw Delia benefit from Jennifer's interest. The girls each had five pieces on their whole strips. Delia had two fourths, two eighths, and one sixteenth on hers; Jennifer had one sixteenth, one half, then three more sixteenths on hers.

Delia



Jennifer



Instead of asking them how much more they needed to cover their whole strips, I asked, "Who's ahead?"

"I think I am," Delia said. They pushed their whole strips side by side. "See, I have one-sixteenth more on mine."

"How much more do you need to cover your whole?" I asked them.

"I need one fourth," Jennifer said.

"I need three more sixteenths," Delia said.

"So I have a better chance of winning," Jennifer announced.

"No, you don't," Delia said. "I'm ahead."

"Yes, I do," Jennifer answered. "I can win in one turn if I'm lucky, but you have to take three turns." She turned to me and asked, "If I don't like what I roll, can I skip my turn?"

I thought for a moment. "I don't see why not," I said.

"Why would you skip your turn?" Delia asked.

"Well, I wouldn't really skip it. I'd roll first and then decide. Because if I roll one-sixteenth, then I'll need three more. But if I roll one-fourth, I'm through in one roll, and if I roll one-eighth, I could be through in one more roll with another eighth."

"Well, roll then," Delia said. She didn't seem to grasp Jennifer's strategy and was more interested in continuing with the play.

But I was still curious about Jeremy. I returned to him when he and Josh had just finished a game. "Say, Jeremy, if you had put just one red one-half piece on your whole strip, do you know how much more you would need to cover it entirely?" I had the strips there and put a red on the dark blue.

"Easy," he said. "Another half."

"What if you had one half and one fourth on it?" I asked, putting down a light blue fourth.

Jeremy looked for a moment. "You need another light blue one," he said, adding a one-fourth piece to the strip. I didn't insist that Jeremy identify the light blue piece with its fractional name. Instead I posed another question.

"Suppose you had this on your strip," I said. I removed both light blue pieces, left the red one-half piece on, and placed a purple one-eighth piece next to it.

"That's hard," he said. "Can I use the pieces?"

"Yes, but tell me what you're thinking," I said.

"A light blue one fits," he said, putting on a one-fourth piece. "And there's still room, maybe for a brown or a purple. I'm not sure." He started to fit pieces on, relying only on what he could do with the pieces, not thinking about the fractions.

"Thanks, Jeremy," I said. "I think this might be a good problem to talk about with the whole class."

One or two encounters with a student don't give me conclusive information about what he or she understands. But over time, with more conversations, I build deeper pictures of how students think, what they know well, and where the soft spots are in their understanding.

Teaching the Rules for *Uncover*

About fifteen minutes into the class period, I called the class to attention. I said, "I'd like to teach you the rules for playing another game. It's called *Uncover*." This time I wrote the rules on the board so that students could refer to them as I introduced the game and when they played. This game is a little more complicated than *Cover Up*. I had also duplicated copies of the rules for their reference. (See page 145 for rules.)

I had the class gather around Martin and Lily to watch them play a game. I read the rules on the board and had Martin and Lily follow them. I had to remind both of them to watch what each other did to be sure they agreed. "You can't take the die to roll it until the other player says 'Done,'" I said several times. "It's important that you watch each other's moves to make sure you agree with the exchanges or what's being removed."

After a few turns, Martin and Lily got into the swing of saying "Done." The others watching chimed in as well, so they all got into the habit of saying it. After this demonstration game, the students returned to their seats to play in pairs.

Observing the Students

From playing *Cover Up,* some students were familiar enough with the pieces so that they could make exchanges easily. Joey, for example, couldn't remove a piece after rolling $\frac{1}{8}$ on his first roll. "I'll exchange," he said to his partner, Josh, removing one of the one-half pieces. "I'll put down four eighths." Joey reached for the eighths and laid them in the vacant space.

Sarah had the same experience, rolling $\frac{1}{8}$ on her first roll. She deftly exchanged one of the one-half pieces for two one-eighth pieces and four one-sixteenth pieces.

At another table a bit later in a game, Mariah was left with two one-fourth pieces. She rolled $\frac{1}{8}$. "Not again!" she said. "I'll get ready for that." She removed one of the one-fourth pieces and efficiently reached for a one-eighth piece and two one-sixteenth pieces. "Now I'm ready," she said.

Other students, however, needed to compare pieces in order to be sure how to make a fair exchange. Claire, for example, rolled $\frac{1}{16}$ on her first roll. Her partner, Sarah, had gone first and made her exchange. Claire, however, seemed unsure how to exchange. She removed one of the one-half pieces and mused aloud, "I think I'll take three of these and three of these." She took three of the one-eighth pieces and three of the one-sixteenth pieces. She began placing them carefully on the vacant half of her whole strip. I watched to see what would happen. Sarah was waiting patiently but not paying close attention.

"Uh oh," Claire said. "I think that's too many. Is it, Sarah?"

Sarah had been watching Joey and Josh but turned back to look at Claire's pieces. "Take off one of the sixteenths," she said. "Then it works."

Claire did that and announced, "Done."

I didn't intervene. At this point, Claire was focusing on the pieces and not thinking about their fractional names and the relationships between them. I decided to allow Claire time to play and see what she would notice for herself with additional experience.

Jeremy was also unsure about how the fractional pieces related to one another or whether they did at all. His partner, Claudia, watched him closely. "You can't do that," she said when Jeremy tried to exchange a one-fourth piece for a one-eighth piece and a one-sixteenth piece. "They don't match up."

"What do you mean?" Jeremy said.

"Look," Claudia said. "There's a space left over. You have to put on another sixteenth." Claudia did this for Jeremy.

"Oh, I get it," he said.

No matter how clear I think my directions are, there's typically someone for whom they don't make sense.

The spirit in the class was high during the game, with the sound of animated involvement.

A Class Discussion

About fifteen minutes before the end of class, I told the children that I would interrupt them

in one minute for a class discussion. I've found that when I give the children a warning first, it's easier for them to stop their activity when I ask them to give me their attention.

I began the discussion by asking, "How does this game compare to *Cover Up?*"

There was an outburst of comments.

"Cool."

"It's better."

"I won twice."

"I like this one."

"Wait, wait," I said. "Raise your hand if you want to tell how you think the games compare." I called on Andrew.

"I think it's better," he said. "You get to decide what to do, and I like that."

I called on Delia next. "You can play a different way each time."

"Explain more about that," I said.

"Well, the first time I just exchanged a one-half piece for two fourths, but then I kept rolling eighths and sixteenths," she said. "So the next game I did a different exchange."

Other students were also interested in explaining the exchanges they made and I kept the discussion going until the end of class. For homework, the students were to take home the rules and play *Uncover* with someone in their families.

DAY 3

At the beginning of class I had the students report what happened when they played *Uncover* at home. Many were eager to report who won and how many times. Some had developed theories about rolling the cube to get the desired outcome, by dropping it from on high to keeping it close to the tabletop. Most agreed that *Uncover* was a better game than *Cover Up.*

"There isn't much strategy with *Cover Up,*" Maggie said.

"There isn't *any* strategy with *Cover Up,*" Andrew countered.

"It's just luck," Lily added.

"What strategies did you use for your first roll when playing *Uncover?*" I asked. Conversation burst out. Students were eager to report their ideas. I quieted them and asked them to raise their hands if they wanted to tell their ideas. Robert went first.

"If I don't roll one-half, then I exchange one of the one-half pieces so I have one fourth, one eighth, one sixteenth, and one sixteenth," he said. "Then I know I can take off something on my next roll."

"And if you roll one-half?" I asked.

"Then you just take it off," he said.

I recorded Robert's strategy on the board:

Strategy for Roll #1

If you don't roll $\frac{1}{2}$, *exchange one of the* $\frac{1}{2}$ *pieces for* $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, *and* $\frac{1}{16}$.

Some other students commented that they used the same strategy, but Claudia had a different approach. "I exchanged for just eighths and sixteenths," she said.

"How many of each?" I asked.

"Two eighths and four sixteenths," Claudia answered. I recorded Claudia's suggestion underneath Robert's:

$\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$, $\frac{1}{16}$, $\frac{1}{16}$

"That doesn't work," Dan said.

"Yes, it does," Janie said. "Look. Two sixteenths make an eighth, so it's the same as four eighths, and that's a half."

Dan thought for a moment. "Oh yeah," he said.

"I don't agree that Claudia's idea is a good one," Davey said. "She has too many pieces to take off."

"So you think Robert's idea is better?" I asked.

"No," Davey said. "He has too many pieces, too. I exchanged for one fourth and two eighths." I recorded Davey's idea:

$\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$

"What do you think about Davey's idea?" I asked the class.

Claudia was insistent. "He may have only three pieces, but one fourth is just as hard to get off as one half. You may as well leave the one half."

"But it's easier to get eighths off," Davey said, defending his choice.

I took the opportunity to talk about the probabilities of each of the fractions coming up when the die was rolled. "There's only one chance out of six to roll one-half when you roll the cube," I said. "Who can explain why that makes sense?"

I waited to see who would raise a hand. About half of the students did, and I called on Libby.

"There are six sides, and one-half is only on one of them," she said.

"Yes," I said. "A cube has six faces and each face has the same chance of landing up. Since one-half is written on only one face, it has a one in six chance." I used the correct terminology of *face* instead of *side.*

"And one-fourth has the same chance. It's written only once," Joey said.

"I agree," I said. "The probability of rolling one-half is one out of six. And there's also a one-sixth chance that you will roll one-fourth." I wrote on the board:

$P(\frac{1}{2}) = \frac{1}{6}$

$P(\frac{1}{4}) = \frac{1}{6}$

$P(\frac{1}{8}) =$

$P(\frac{1}{16}) =$

"What do you think the probability is of rolling one-eighth?" I asked. Hands shot up. I called on Sarah.

"It's two-sixths," she said.

"Why do you think so?" I prompted.

"Because one-eighth is on two sides," she said.

"It's the same for one-sixteenth," Sam said. "It's on there twice, too."

"Sometimes you should exchange one way and sometimes the other," Libby said. "You can get different rolls at different times."

I then introduced a variation on the rules for *Uncover.* When I had taught the game to a different class, one of the students, Jose, had raised a question. He had asked, "If I don't have a one-fourth piece on my board but I have two one-eighth pieces, and I roll one-fourth, can I take off the two one-eighth pieces?"

I hadn't addressed that possibility in the rules I had written, but in thinking about his question, I decided Jose's idea was mathematically sound. Also, looking for combinations of pieces would certainly support children's mathematical thinking. However, the suggestion hadn't come up in this class, so I explained Jose's idea.

"Any combination of pieces is fine as long as they add up to the fraction that comes up on the die," I told them.

"That's the way we played last night," Delia said. "It was my mom's idea."

"Ohhhh," Claudia said, always quick to understand a new situation. "Maybe you don't need to hold on to the one-half piece at all."

For the rest of the class period, I had them play *Uncover* with this new variation. For homework that night, I told the children they were to play either version of *Uncover* with someone at home and also to think more about what might be a good strategy for exchanging on the first roll.

DAY 4

I began class, as I had done on the two previous days, by having the students discuss their experiences from the night before. Instead of having a class discussion, however, I asked students to talk in small groups. In this class, the same students typically volunteered in whole-class discussions, and having students talk in small groups encouraged more of them to share their ideas. After the groups had the chance to talk, I called the class to attention.

Sam raised his hand, eager to report what his group had decided. He said, "We all liked the new way of playing better. It lets you think more." There was agreement from others in the class.

Lily had something different to report from her group. "We tried those strategies from yesterday and agree that they're all about the same," she said. There was an outburst of protest from others. I quieted the class and returned to Lily.

"Can you explain your group's reasoning?" I asked.

"Well, you don't know what you're going to roll and it's all luck, so it doesn't matter which way you go."

Josh waved his hand to disagree. "We know for sure that Davey's is better than Robert's because Davey only has three and Robert has four, and the chance of rolling one-eighth and one-sixteenth is the same, so Davey has a better chance."

"But it's still luck," Dan, a member of Sam's group, chimed in. I reminded Dan not to blurt out and then called on Jennifer.

"We agree with Josh's group, and we think that Claudia's idea was okay because her fractions are all more possible, but we still think that Davey will win. At least that's what happened when we played."

Joey had a suggestion to offer. He and Sam had played the game after school and added another rule. "It's like a fourth choice when you role," Joey explained. "If you want, you can put one of your partner's pieces back on his whole."

Sam added, "As long as there's room, you can do it. It's good in an emergency."

"It makes the game last longer, but it's better," Joey said.

Sam laughed. "Yeah, we finally made a rule that you can do it only ten times in a game. It's like having only so many time-outs."

The rest of the class was very interested in playing this version, but I had other plans for the rest of the period. When the other students played it later, however, it was a big hit and became *Uncover, Version 3.*

Introducing *Cover the Whole*

"You'll have a chance to try out Joey and Sam's game and also to test your theories," I said. "But now I'd like to introduce two other activities that you'll do with your fraction kits. They're not games but explorations that I think will help you learn more about fractions."

I wrote the names of the activities on the board:

Cover the Whole

Comparing Pairs

"Can we work with partners?" Claire wanted to know.

I replied, "You can talk with others, but you each have to do your own work and hand in your own papers." In learning situations, I want students to have as much support as possible, and talking with classmates is a way to provide that support. But I also want all students to have the experience of recording fractions, explaining their reasoning, and being responsible for their own assignments.

I began by explaining *Cover the Whole.* "In this activity, you cover your whole strip as you do when playing *Cover Up,* but you don't have to roll the die or play the game. Just make a train of pieces that covers the whole strip exactly." I took a moment and had each student cover his or her whole strip.

I then asked Libby what she had done. "Tell me the pieces you used, reading from left to right across your strip, and I'll record what you tell me." As Libby reported, I recorded the fractions on the board:

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{8}$

Then I added plus signs and = *1* to write a complete mathematical sentence:

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 1$

"Who can explain why it makes sense to put in plus signs and write 'equals one' at the end?" I asked.

"Because plus is adding and you're adding more pieces to the board," Mariah said.

"If you add them all up you get one," Davey added.

"Yeah," Joey said. "The two eighths make one fourth, and four fourths make a whole."

"Is this what we do?" Lily wanted to know. "It's easy."

"Yes," I answered. "You'll each cover your whole strip with pieces and record what you did in a complete mathematical sentence as I did for Libby's. You need to do this for at least five different combinations of pieces. Then there's one more step you need to do for each of your sentences. You have to shorten them, if it's possible. Watch as I shorten what Libby reported." I wrote:

$\frac{3}{4} + \frac{2}{8} = 1$

"Who can explain why this shorter sentence is equivalent to the longer one?" I asked.

"What does equivalent mean?" Sean asked.

"It means the same," Jennifer answered. "They're not exact, but they mean the same thing."

I added, "If I saw just the shortened sentence, I would still be able to figure out which pieces Libby had used." Sean seemed satisfied and there were no other questions. Because the students had experience playing *Cover Up* and *Uncover*, talking about fractions like three-fourths and two-eighths wasn't new, and the use of the correct notation was an easy connection for them to make.

The students worked for the rest of the period on *Cover the Whole.* (Figures 2–1 and 2–2 show what two students did with *Cover the Whole.*) A few who finished early returned to playing *Uncover.*

DAY 5

To begin class, I said, "I'm going to introduce two new activities for you to do with your fraction kits." I gathered the students so that they could see me use the fraction

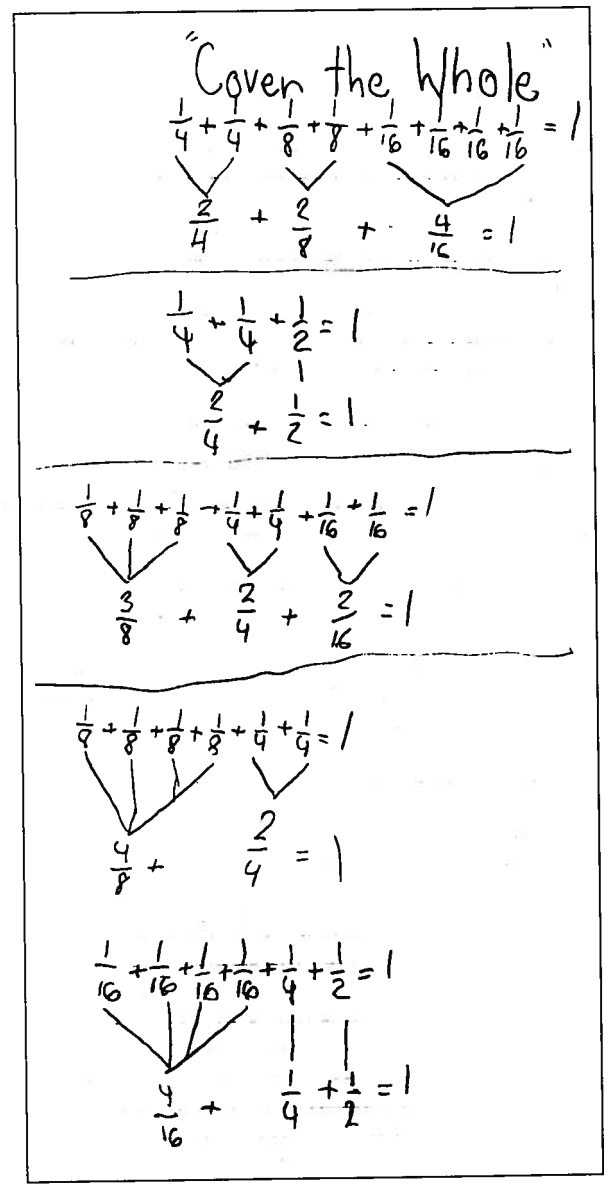


Figure 2–1 ***Delia had a unique recording system for keeping track when she shortened her sentences.***

pieces and also see the board. I wrote the names of the activities on the board:

Comparing Pairs
What's Missing?

To explain *Comparing Pairs,* I wrote an example on the board that was like those on the *Comparing Pairs* worksheet:

$\frac{3}{8}$ $\frac{9}{16}$

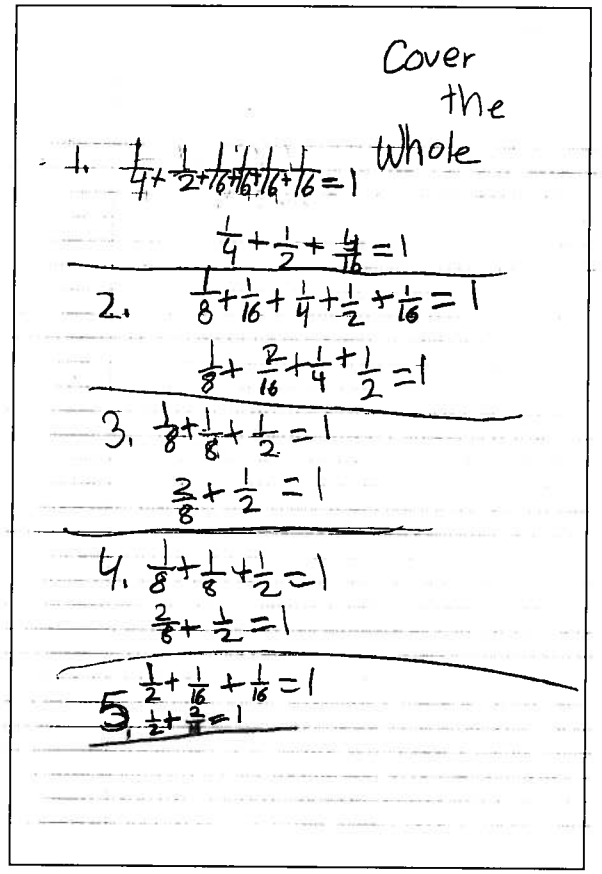


Figure 2–2 *Carol completed the assignment quickly and correctly.*

I began, "For each problem like this, you write the symbol for less than, greater than, or equal to in between the two fractions to make it a true statement. You can use your fraction kit pieces to figure out which sign to write." I wrote the three symbols on the board and reviewed them with the class:

= *is equal to*

> *is greater than*

< *is less than*

I then used a fraction kit to model for the class what to do, explaining as I arranged the pieces. "First I make a train for three-eighths using three of the one-eighth pieces. Below it I make a train that is nine-sixteenths long." I lined up nine of the one-sixteenth pieces. "I can see that the nine-sixteenths train is longer, so I know that nine-sixteenths is greater than three-eighths."

1/8	1/8	1/8						
1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16

"Who would like to come to the board and write in the correct sign?" I asked. I waited to see who would volunteer. When about half the students had raised their hands, I called on Carol. She came up and carefully wrote the "less than" sign. "I remember it's this one because less than goes like the letter *L*," she commented.

I then showed the class the worksheet on which I had written ten pairs of fractions for them to compare. "You'll see that there are spaces for two more problems," I said. "Make those up with any fractions you choose and then solve them." The problems students would make up would give me some information about their comfort with fractions.

"There's one more part to this assignment," I said. "When you've solved all of the problems, you have to explain three of them. Who can explain why nine-sixteenths is more than three-eighths?" Only a few students raised their hands. As I usually do when students seem hesitant, I asked the students to talk with their neighbors. I repeated the question before letting them begin talking. When I called the class back to attention, more hands were raised and I called on several students to reply.

Sam explained, "Well, it takes two sixteenths to make one eighth. So that means that you need four sixteenths to make two eighths, and four more to make three eighths." Sam seemed to understand the relationship between eighths and sixteenths, but he made an error in his thinking. After using four sixteenths to make two eighths, it would take only two more to make three eighths.

"How many sixteenths altogether would make three eighths?" I asked him.

Sam replied, "Two and two are four and four more . . . no, no, I mean two more. Wait, I'm mixed up."

Several students' hands went up. "Let's give Sam a chance to think," I said. "He has a good idea and just needs to look at it again. Take your time, Sam, and try again."

"Okay," he said. "It's six."

"Six what?" I probed.

"You need two, four, six of the sixteenths to make three of the eighths," he said, now with certainty.

I pointed to the problem on the board and said, "How does that help you explain this problem? Why is three-eighths less than nine-sixteenths?"

Sam said, "Because three-eighths is six-sixteenths, and nine-sixteenths is more." I nodded and recorded on the board:

$\frac{3}{8} < \frac{9}{16}$ *because* $\frac{3}{8} = \frac{6}{16}$ *and* $\frac{9}{16}$ *is more.*

"It would be helpful if you also wrote about how you knew that three-eighths is equal to six-sixteenths. Then I'll know more about how you are thinking. Sam, can you explain that part again?"

Sam said, "It takes two sixteenths to make one eighth, so you need four for two eighths and six for three eighths."

"That's good and clear to me," I said, and added to what I had written on the board:

$\frac{3}{8} < \frac{9}{16}$ *because* $\frac{3}{8} = \frac{6}{16}$ *and* $\frac{9}{16}$ *is more. It takes* $\frac{2}{16}$ *to make* $\frac{1}{8}$*, so you need* $\frac{4}{16}$ *for* $\frac{2}{8}$ *and* $\frac{6}{16}$ *for* $\frac{3}{8}$.

The struggle to get Sam to clarify his thinking was worth the effort, not only for him but for the others to hear.

"I have an easier way to explain," Jennifer said. "Nine-sixteenths is more than one-half and three-eighths is less than one-half." I nodded and wrote:

$\frac{3}{8} < \frac{9}{16}$ *because* $\frac{9}{16} > \frac{1}{2}$ *and* $\frac{3}{8} < \frac{1}{2}$.

I then said, "Both of these are fine. On your paper, you need to explain in some way that makes sense to you. If you are having difficulty, talk to a neighbor or ask me for help."

"Do we have to write for all of them?" Dan asked.

"No," I answered. "You have to write 'because' explanations for three of them, and then also for one of the problems you make up." (See Figures 2–3, 2–4, and 2–5 for examples of the work students produced for this activity.)

I then gave directions for *What's Missing?* Although the problems were different, the directions were the same—solve the problems, explain three of them, make up two of your own, and explain one of those. I did an example on the board using fractions that I knew would be easy for the students

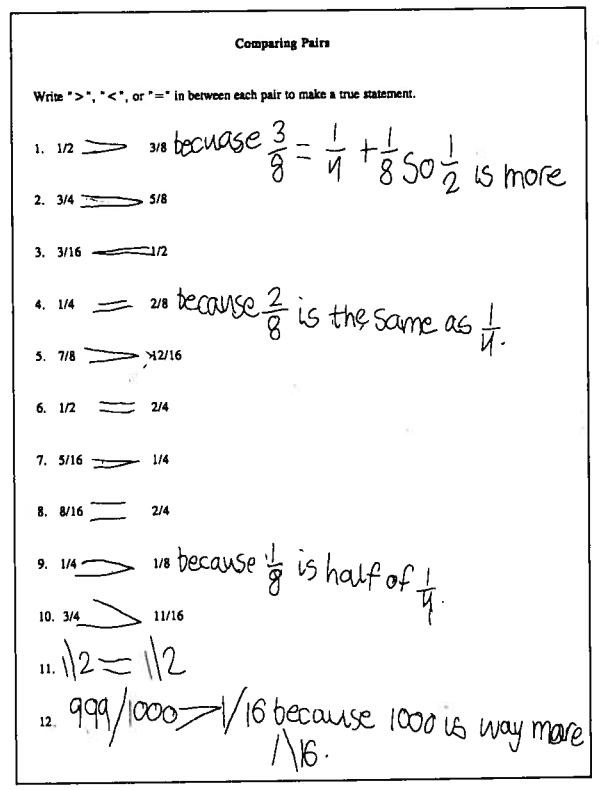
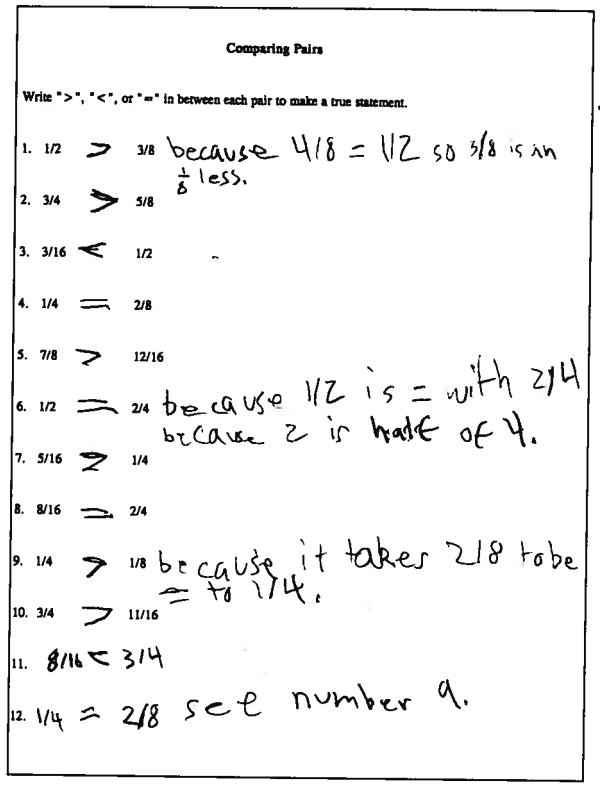
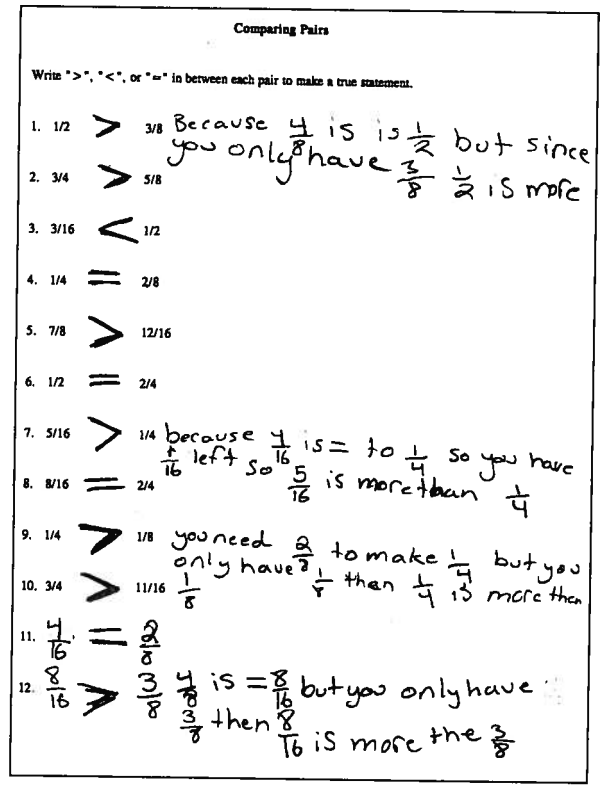


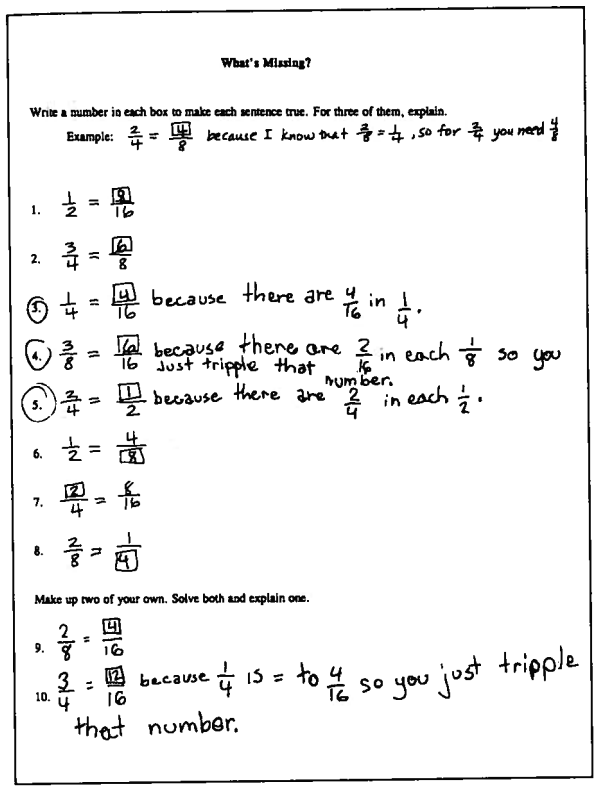
Figure 2–3 *For one of the problems he made up, Philip compared* $\frac{999}{1000}$ *to* $\frac{1}{16}$.



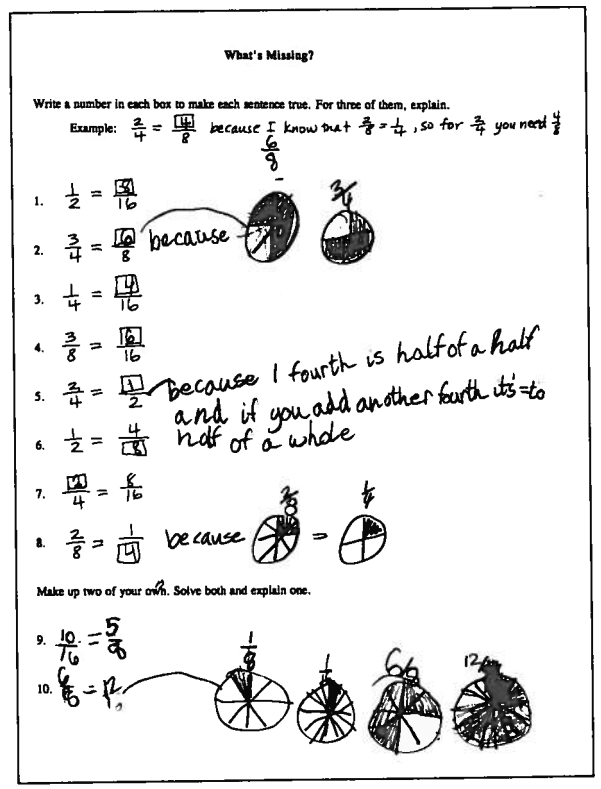
▲▲▲▲▲**Figure 2–4** *George used the same reasoning to explain two of the problems—#9 and #12.*



▲▲▲▲▲**Figure 2–5** *Elizabeth wrote clear explanations about how she compared the fractions.*



▲▲▲▲▲**Figure 2–6** *Marlo was the only student who used the idea of tripling to explain two of the statements.*



▲▲▲▲▲**Figure 2–7** *Ruthie used sketches for three of her four explanations.*

to think about without the fraction pieces. I wrote:

$\frac{1}{2} = \frac{\square}{4}$

"What number goes in the box to make this statement true?" I asked. Hands flew up and I called on Sarah.

"A two," she said. Then she added, "Should I explain?" I nodded.

"One-fourth is half of a half, so it takes two of them to make one-half." I wrote on the board:

$\frac{1}{2} = \frac{2}{4}$ *because* $\frac{1}{4}$ *is half of* $\frac{1}{2}$*, so it takes* $\frac{2}{4}$ *to make* $\frac{1}{2}$.

"Does anyone have another way to explain?" I asked. I called on Joey.

"It's kind of the same way," he said.

"Let's hear anyway," I replied.

Joey explained, "One-half is two times one-fourth, so you need two of them." I recorded his idea:

$\frac{1}{2} = \frac{2}{4}$ *because* $\frac{1}{2}$ *is two times* $\frac{1}{4}$*, so you need* $\frac{2}{4}$ *to make* $\frac{1}{2}$.

I gave one last direction. "Remember to write explanations for three of the problems I put on the worksheets, then also for one of your own." The students got to work and I circulated, observing, answering questions, and keeping students on task. (Figures 2–6 and 2–7 show how two students worked on this activity.)

Challenge Problems

At times at the beginning of class, or when a few minutes remained at the end of a period, I asked the children to close their eyes, visualize their fraction pieces, and answer questions as I posed them. The questions I asked were like the following:

How many eighths are there in one-half?
How many eighths are there in one whole?
How many eighths are there in three-fourths?
How many sixteenths are there in one-half?
How many sixteenths are there in three-eighths?
How many eighths would be needed to cover two wholes?
How many fourths are in one-half?

Sometimes I would also ask students to explain their reasoning, noting when students didn't rely only on their fraction kits but also on the relationships between numerators and denominators.

For example, one day a few days later, I began class by saying, "Raise your hand if you know how many eighths there are in one-half without having to look at your fraction kit," I said.

"What do you mean?" Robert asked.

"If you were going to cover one of your one-half strips with one-eighth pieces, how many pieces would you need?" I asked. I called on Delia.

"I think it's four," she said.

"Is that a maybe answer or are you sure?" I asked.

"No, I'm sure," Delia said.

"How do you know?" I asked.

"Well, I know that two-eighths make one-fourth, and you need two one-fourths to make one-half, so you need four of the eighths," she said. I wrote on the board:

$\frac{2}{8} = \frac{1}{4}$
$\frac{2}{4} = \frac{1}{2}$
So $\frac{4}{8} = \frac{1}{2}$

Delia nodded to indicate she agreed with what I wrote. Davey had a different way to explain. "Eight of the eighths cover the whole strip, so if you cover up half of the strip, then you use only four of them."

"That makes better sense to me," Lily said.

"Does anyone have another way to explain?" I asked. No one volunteered.

"Here's another question," I said. "How many eighths would cover two whole strips?"

"We don't have enough for that," Tom said.

"We could share," Jennifer said.

"How many?" I asked. "Talk about this at your tables." Noisy conversation broke out and after a few moments hands were waving. I called on Joey.

"You'd need sixteen, because you need eight for one strip and then eight more for the other, and eight plus eight is sixteen." I wrote on the board:

$\frac{8}{8} = 1$

$8 + 8 = 2$

$\frac{16}{8} = 2$

"That looks weird," Claudia said.

"What looks weird?" I asked.

"The sixteen over eight," she said. "But I kind of get it."

"Me, too," Sarah said.

"I don't," Carol said.

"Can I explain?" Sam asked. I agreed.

"If you have one whole covered with eighths, then you use all of them, and that's eight," he said. "For two wholes you need eight more. And two eights make sixteen." I wasn't sure if Carol and some others understood, but I knew that they would have other opportunities to encounter this same idea. I continued with the lesson I had planned.

Questions and Discussion

▲ ***I noticed when I had my students cut their fraction kits that some of their pieces were uneven, and some students tried to trim their pieces to make them all the same size. How would you handle these situations?***

I've had these very things happen. Regarding the unequal pieces, I tell the students, "Measurement is never exact, so it's likely that your pieces won't be exactly the right sizes. There may be some difference, for example, among the sizes of your eighths. That makes it all the more important to think about the fractions and the fractional sizes of the pieces, rather than relying only on the paper pieces you've cut." I try to discourage students who want to be exact, trimming slivers off pieces that seem "off" to them. But even though I explain the futility of trimming, some students insist on doing so, as if their sense of order is somehow being violated by the inconsistencies. Still, I keep the emphasis on talking about the ideas of the fractions, not relying on the pieces alone for proof.

▲ ***How do I deal with students who race through the games and don't pay attention to what their partners are doing?***

When I notice that happening, sometimes I talk with the particular pairs and sometimes with the entire class. I explain why they should pay attention to each other's moves. I say, "Not only is it good to check on what your partner does, but I'm trying to slow the game down enough so you also have time to think about the mathematics involved, such as what you'd like to roll next, or how many more rolls you think you'll need to cover your whole, or how much ahead one person is than the other. Remember the game is meant to help you learn about fractions. I want you to enjoy playing, but I also want you to do some thinking." I also reinforce the rules: "No grabbing for the die. Be sure to say 'Done' after you play. Pay attention to each other's pieces."

▲ ***When you're observing the students play, how do you decide when to ask them about how much more they need to win, as you did with Andrew when he and Sam were playing and the class was watching?***

Because I'm interested in assessing what students understand, I want to find time to ask all of them. But I try to balance my need to probe their thinking with their need to experience the game, so I tread lightly. As an alternative, I sometimes give a written assignment, just a quick-write, and ask students to respond to questions such as "If I had one-half and one-sixteenth on my board, what would be the fewest rolls possible to cover it completely? What would the rolls be?" A class set of papers would give me a sense of the overall progress of the class as well as information about what individual students understand.

▲ ***How do you know when it's a good time for a class discussion or when students need more time to play the game?***

Cover Up isn't very complicated or demanding. As soon as they've all grasped the rules and have had the chance to play five games or so, it's fine to have a discussion. With this class, even though *Cover Up* is a game merely of luck, the children seemed willing to play it more. I observe carefully to be sure that I introduce *Uncover*, which is a more challenging game, before their interest dissipates. *Uncover* invites more thinking, both about strategies for playing and about fractions.

▲ ***At the end of the third day, you gave a homework assignment asking students to try the strategies other students had suggested. What if none of those strategies had come up?***

This is a good question and illustrates the benefit of teaching the same lesson to different classes. In another class where none of the students came up with strategies, I told the class about the ideas that students in other classes had had. I wrote on the board:

Possible Strategies for Roll #1

If you don't roll $\frac{1}{2}$, exchange one of the $\frac{1}{2}$ pieces for one of these sets of pieces:

1. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, *and* $\frac{1}{16}$
2. $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$, $\frac{1}{16}$, *and* $\frac{1}{16}$
3. $\frac{1}{4}$, $\frac{1}{8}$, *and* $\frac{1}{8}$

Then I gave the assignment of trying these strategies at home and seeing which they thought was best and why.

Cover Up

You need:

your fraction kit
a fraction die with faces marked $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$
a partner

Rules

1. Take turns rolling the fraction die.
2. On your turn, the fraction that comes up on the die tells what size piece to place on the whole strip.
3. Check with your partner to be sure he or she agrees with what you did.
4. After finishing your turn, say "Done" and pass the die to your partner.
5. The first player to cover his or her whole strip *exactly* wins. If you need only a small piece—$\frac{1}{8}$ or $\frac{1}{16}$, for example—and you roll a larger fraction—$\frac{1}{2}$ or $\frac{1}{4}$, for example—you can't play. You must roll a fraction smaller than or exactly what you need.

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Uncover, Version 1

You need:

your fraction kit
a fraction die with faces marked $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$
a partner

Rules

1. Each player covers his or her whole strip with the two $\frac{1}{2}$ pieces.
2. Take turns rolling the fraction die.
3. On your turn, take one of three options:
   - remove a piece (only if you have a piece the size indicated by the fraction facing up on the die);
   - exchange any of the pieces on your whole strip for equivalent pieces;
   - do nothing.
4. Check with your partner to be sure he or she agrees with what you did.
5. After finishing your turn, say "Done" and pass the die to your partner.
6. The first player who removes all pieces from the whole strip wins.

NOTE 1: You may not remove a piece and exchange on the same turn; you can do only one or the other.

NOTE 2: You have to go out exactly. That means if you have only one piece left and roll a fraction that's larger, you may not remove the piece.

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Uncover, Version 2

The rules are the same as for The Game of Uncover *except for the first option of rule 3.*

You need:

your fraction kit
a fraction die with faces marked $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$
a partner

Rules

1. Each player covers his or her whole strip with the two $\frac{1}{2}$ pieces.
2. Take turns rolling the fraction die.
3. On your turn, take one of three options:
   - **New Rule:** remove one or more pieces from your board as long as they add up to the fraction facing up on the die;
   - exchange any of the pieces on your whole strip for equivalent pieces;
   - do nothing.
4. Check with your partner to be sure he or she agrees with what you did.
5. After finishing your turn, say "Done" and pass the die to your partner.
6. The first player who removes all pieces from the whole strip wins.

NOTE 1: You may not remove pieces and exchange on the same turn; you can do only one or the other.

NOTE 2: You have to go out exactly. That means if you have only one piece left and roll a fraction that's larger, you may not remove the piece.

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Wipeout

You need:

pattern blocks

a fraction die with faces marked $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{1}{6}$

a partner

Rules:

1. Decide if you each will start with one, two, or three hexagons.
2. Take turns rolling the fraction die.
3. On your turn, take one of three options:
   - remove a block if it's the fractional part of the hexagon indicated by the fraction die;
   - exchange any of your remaining blocks for equivalent blocks;
   - do nothing.
4. Check with your partner to be sure he or she agrees with what you did.
5. After finishing your turn, say "Done" and pass the die to your partner.
6. The first person to discard all of his or her blocks wins.

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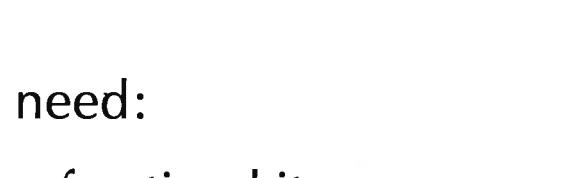
Pick Two

You need:

your fraction kit

1. Make a train of two pieces on your whole strip using pieces that are *not* the same color.
2. Record.
3. Build another train the same length using pieces that are all the same color.
4. Record only one fraction.
5. Try to build other one-color trains the same length. For each, record.

Example:



$\frac{1}{2} + \frac{1}{16} = \frac{9}{16}$

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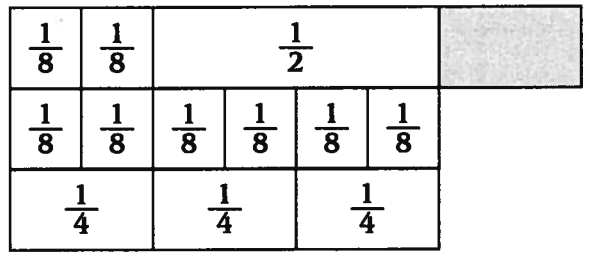
Pick Three

You need:

your fraction kit

1. Make a train of three pieces on your whole strip using pieces that are *not* all the same color. (It's okay to use two of one color and one of another color.)
2. Record and also shorten, if possible.
3. Build another train the same length using pieces that are all the same color.
4. Record only one fraction.
5. Try to build other one-color trains the same length. For each, record.

Example:



$\frac{1}{8} + \frac{1}{8} + \frac{1}{2} = \frac{6}{8} = \frac{3}{4}$

$\frac{2}{8} + \frac{1}{2}$

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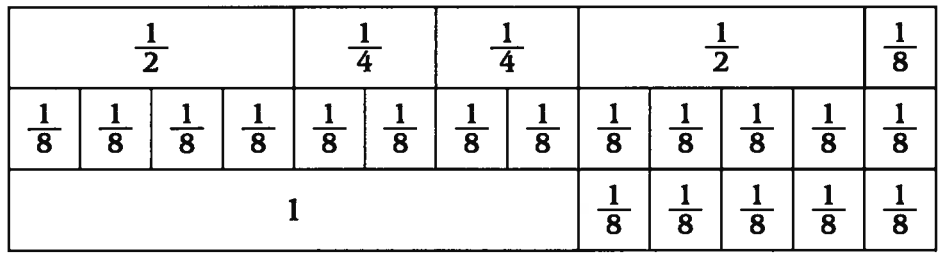
Roll Five

You need:

your fraction kit
fraction die

1. Roll the fraction die five times and build a train with the pieces that match the fractions that come up.
2. Record and also shorten, if possible.
3. Build another train the same length using pieces that are all the same color.
4. Record only one fraction.
5. Try to build other one-color trains the same length. For each, record.

Example:



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \frac{1}{8} = \frac{13}{8} = 1\frac{5}{8}$$

$$\frac{2}{2} + \frac{2}{4} + \frac{1}{8}$$

Can you figure out two more ways to cover this train?

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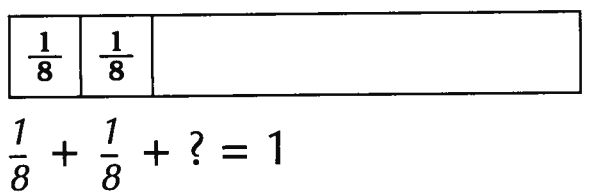
Make a Whole

You need:

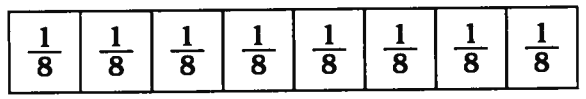
your fraction kit
fraction die

1. Roll the fraction die twice and record the fractions that come up.
2. Figure out what one fraction you would add on to these two to make one whole. Use your fraction kit to help by putting on the pieces from your two rolls and then figuring out what pieces, all of the same color, you need to add to cover the whole strip exactly.
3. Record.
4. Try to find other fractions that would also work. For each, record.

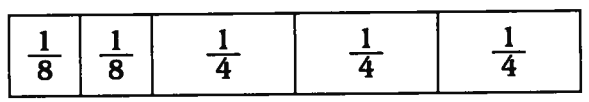
Examples:



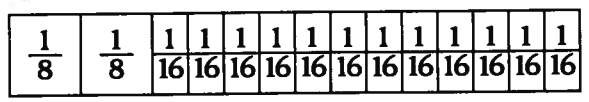
$$\frac{1}{8} + \frac{1}{8} + ? = 1$$



$$\frac{1}{8} + \frac{1}{8} + \frac{6}{8} = 1$$



$$\frac{1}{8} + \frac{1}{8} + \frac{3}{4} = 1$$



$$\frac{1}{8} + \frac{1}{8} + \frac{12}{16} = 1$$

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