# 1.01 *Number Systems, Real Numbers, Operations*

from ***Basic Algebra: One Step at a Time*** © 2002-2011

**P. 1-6**

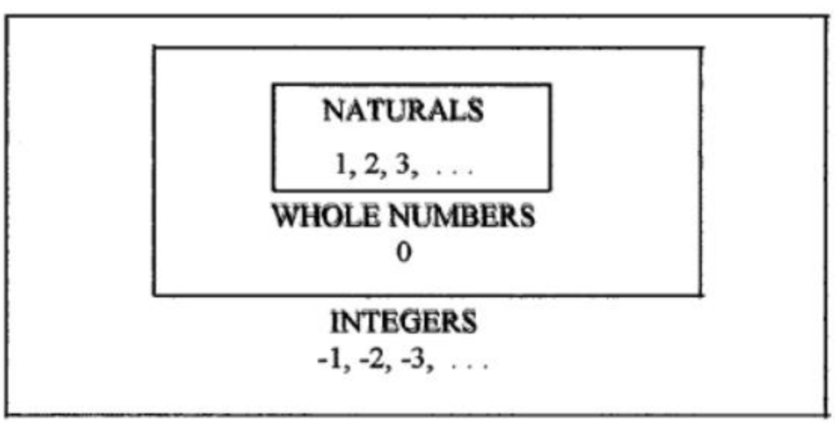
**Dr. Robert J. Rapalje**
**Seminole State College of Florida**

**ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE**

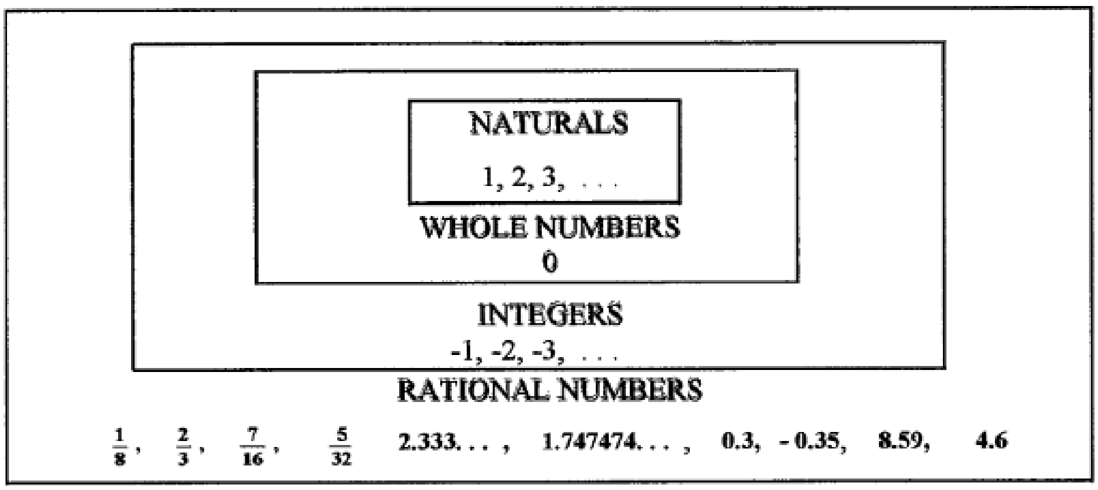
**More FREE help available from my website at www.mathinlivingcolor.com**

In any study of mathematics, there must be **number systems** and **operations** (such as addition, subtraction, multiplication, and division). It seems only "natural" to begin this study with the natural number system. The **natural numbers**, also known as the **counting numbers**, is the set of numbers that would be used for counting {1, 2, 3, 4 . . .} (three dots mean "and so on"). When the number 0 is included, this set {0, 1, 2, 3, 4 . . .} is the set of **whole numbers**. With the advent of credit, it became necessary to have negative numbers. The set of **integers** is defined to be the set of all whole numbers and their negatives: {. . . -4, -3, -2, -1, 0, 1, 2, 3, 4 . . .} .

| SYSTEMS OF NUMBERS | |
|---|---|
| NATURAL NUMBERS: | 1, 2, 3, 4, . . . |
| WHOLE NUMBERS: | 0, 1, 2, 3, 4, . . . |
| INTEGERS: | . . . -4, -3, -2, -1, 0, 1, 2, 3, 4, . . . |



You probably noticed that fractional and decimal numbers are not included in any of the sets mentioned thus far. The set of **rational numbers**, from the root word **ratio**, is the set of all numbers that can be expressed as a ratio of two integers (assuming of course that **division by zero is undefined**). When one integer is divided by another integer, the result can be expressed as a fraction, or it can be divided out to express it in decimal form. When two integers are divided, the result will either divide evenly (called a **terminating decimal**) or there will be a repeating pattern of numbers in the quotient (called a **repeating decimal**). The fractions, 1/2 = .5, 3/8 = .375, 9/5 = 1.8 result in terminating decimals, but 1/3 = .333..., 2/9 = .222..., 4/11 = .363636..., and 2/7= .285714285714... result in repeating decimals.




You probably also noticed that each set of numbers so far is developed or built upon the previous set of numbers. This makes each previous set of numbers a **subset** (i.e., a set contained within a set) of each succeeding set of numbers. These illustrations that have been used are called **Venn Diagrams,** named after the mathematician John Venn (1880).

Having built up to the set of rational numbers, there are some numbers that are not rational--that is, these numbers cannot be expressed as a ratio of integers. For example, the solution to the equation $x^2 = 2$ is $\pm\sqrt{2}$. Another example of a number that cannot be expressed as a ratio of integers is the number $\boldsymbol{\pi}$, whose value is approximately (but not exactly!) 22/7 or 3.14. It can be proven that the actual value of $\boldsymbol{\pi}$ will never terminate, and it will never repeat a pattern. The set of all numbers like $\sqrt{2}$, $-\sqrt{5}$, and $\pi$ that never terminate and never repeat a pattern form the set of **irrational numbers**.

## THE REAL NUMBER SYSTEM

| RATIONAL NUMBERS | IRRATIONAL NUMBERS |
|---|---|
| $\frac{1}{8}$, $\frac{2}{3}$, $\frac{7}{16}$, $\frac{5}{32}$ **2.333. . ., 1.747474. . .** | $\pi$ $2\pi$, $\frac{\pi}{2}$, $\frac{7\pi}{4}$, $e \approx 2.718$, |
| **INTEGERS** -1, -2, -3, . . . — 0.3, 0.35; 4.6 | $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $2\sqrt{6}$, $2 + \sqrt{6}$, $3 - \sqrt{2}$, |
| **WHOLES** 0 — 8.59 | $\frac{\sqrt{2}}{2}$, $\frac{9\sqrt{3}}{5}$, $\frac{-3 \pm \sqrt{6}}{3}$ |
| **NATURALS** 1, 2, 3, . . . — $7\frac{3}{8}$; -1.74 | |

These two sets, the rational and the irrational numbers, are said to be **disjoint sets**, which means that they have no members in common. The **intersection** of the sets (i.e., what is common to both sets) is the **empty set**, denoted $\varnothing$ or { }. Notice that {$\varnothing$} is <u>not</u> acceptable notation for the empty set--this is <u>not</u> empty because it has a "$\varnothing$" in it! The set of all rational and irrational numbers combined (the **union** of the two sets) is the **real number system R**. [Note: in math, this is the <u>real thing</u>!]

Now that you have all of these numbers, what can you do with them? Well, you can take them two at a time and **add** them, **subtract** them, **multiply** them, or **divide** them. These are called arithmetic operations. Since these operations combine two numbers at a time, they are called **binary operations**. Addition uses the "+" sign, subtraction the "-" sign. For multiplication, the "**x**," "×," or "•" sign is used. Notice that either the **cross sign** or the **dot** means **multiplication**.

**Example 1.** $6 \times 5 = 30$, or $6 \bullet 5 = 30$. Moreover, if no sign is given between the two numbers, then the operation is still multiplication, although parentheses are necessary to ensure that "(6)(5)" = 30 doesn't look like "65." All of the following mean the same:

| | | | |
|---|---|---|---|
| $6 \times 5 = 30$ | $(6)(5) = 30$ | $(6) \bullet (5) = 30$ | $(6) \times (5) = 30$ |
| $6 \bullet 5 = 30$ | $6\,(5) = 30$ | $6 \bullet (5) = 30$ | $6 \times (5) = 30$ |
| | $(6)\,5 = 30$ | $(6) \bullet 5 = 30$ | $(6) \times 5 = 30$ |

In algebra, there is a tendency to avoid use of the "×" or "x" signs for multiplication, since the letter "x" is usually reserved to be used as an unknown. (See Section 1.05 on variables.)

Another operation used in math as a shorthand notation is called "**raising to a power**."

**Example 2.** Suppose you wanted to take 2 times itself five times: that is, $2\times2\times2\times2\times2$ or $2\bullet2\bullet2\bullet2\bullet2$. Instead of writing out all those two's, you can write $2^5$, where the 5 is a **superscript.** This 5 number is called an **exponent**, and $2^5$ is read "two raised to the fifth power." The value of $2^5$ is 32.

**Example 3.** $3^4$ (read "three raised to the fourth power") means $3\bullet3\bullet3\bullet3$, which equals 81. In the case of raising to the second power, we call it "squaring," while raising to the third power is called "cubing."

**Example 4.** $3^2$ means $3\bullet3 = 9$ ("three raised to the second power" or "three squared.")

**Example 5.** $3^3$ means $3\bullet3\bullet3 = 27$ ("three raised to the third power" or "three cubed.")

As there are different symbols to indicate the operation of multiplication, there are also different symbols for division.

**Example 6.** To indicate 12 divided by 4, you may write

$12 \div 4 = 3$ ; $\frac{12}{4} = 3$; $12/4 = 3$ ; $4\overline{)12} = 3$

**EXERCISES.** Perform the indicated operations.

| | | | |
|---|---|---|---|
| 1. $7 \bullet 8$ | 2. $6 \times 9$ | 3. 7(9) | 4. (8)(9) |
| 5. $(8) \bullet (6)$ | 6. $(7) \times (6)$ | 7. $4^2$ | 8. $5^2$ |
| 9. $2^3$ | 10. $4^3$ | 11. $5^3$ | 12. $6^3$ |
| 13. $2^4$ | 14. $4^4$ | 15. $5^4$ | 16. $6^4$ |
| 17. $10^2$ | 18. $10^3$ | 19. $10^4$ | 20. $10^5$ |
| 21. $36 \div 4$ | 22. $\frac{72}{9}$ | 23. 72/8 | 24. $12\overline{)48}$ |
| 25. $\frac{72}{4}$ | 26. $72 \div 6$ | 27. $12\overline{)72}$ | 28. 92/4 |

In the previous division exercises, you may have noticed that the answers all came out even. However, math, like life, doesn't always come out even. Remember that every fraction can be expressed as a decimal by simply dividing the denominator into the numerator. If you continue the division process far enough, there will either be no remainder (a **terminating decimal**) or a pattern will repeat (a **repeating decimal**)--guaranteed! When the decimal repeats a pattern, the number(s) in the repeating pattern may be indicated with a bar over the number such as 0.3333 . . . = $0.\overline{3}$ .

**Example 7.** Do you think there is a difference between the decimals $0.\overline{34}$ $0.3\overline{4}$ ?

**Answer:** Yes! There is a difference.

In the decimal $0.\overline{34}$ , the 34 repeats, so you have 0.34343434 . . .

In the decimal $0.3\overline{4}$ , only the 4 repeats, so you have 0.344444 . . .

A calculator is a good way to convert fractions to decimals, by simply dividing the numerator by the denominator. In most cases, it will be obvious whether the decimal is terminating or repeating. Consider the following examples.

**Example 8.** $\frac{5}{4}$ means $5 \div 4$ or $4\overline{)5.00}$ which is 1.25 (a terminating decimal!)

**Example 9.** $\frac{1}{3}$ means $1 \div 3$ or $3\overline{)1.0000}$ which is 0.3333. . . or $0.\overline{3}$

**Example 10.** $\frac{6}{11}$ means $6 \div 11$ or $11\overline{)6.0000}$ which is 0.545454 . . . or $0.\overline{54}$

**Example 11.** $\frac{3}{7}$ means $3 \div 7$ or $7\overline{)3.0000}$ which is 0.428571428571 . . . or $0.\overline{428571}$

**Example 12.** $\frac{1}{16}$ means $1 \div 16$ or $16\overline{)1.0000}$ which is 0.0625 (a terminating decimal!)

**EXERCISES.** Divide the fractions (by calculator and by hand) to express as terminating or repeating decimals.

29. $\frac{3}{4}$

30. $\frac{3}{5}$

31. $\frac{2}{9}$

32. $\frac{2}{11}$

33. $\frac{13}{8}$

34. $\frac{13}{3}$

35. $\frac{5}{16}$

36. $\frac{3}{20}$

37. $\frac{17}{44}$

38. $\frac{17}{40}$

39. $\frac{5}{14}$

40. $\frac{25}{21}$

## ANSWERS 1.01

**p. 4 - 6:** **1.** 56; **2.** 54; **3.** 63; **4.** 72; **5.** 48; **6.** 42; **7.** 16; **8.** 25; **9.** 8; **10.** 64; **11.** 125; **12.** 216; **13.** 16; **14.** 256; **15.** 625; **16.** 1296; **17.** 100; **18.** 1000; **19.** 10,000; **20.** 100,000; **21.** 9; **22.** 8; **23.** 9; **24.** 4; **25.** 18; **26.** 12; **27.** 6; **28.** 23; **29.** 0.75; **30.** 0.6; **31.** 0.2; **32.** 0.18; **33.** 1.625; **34.** 4.3; **35.** 0.3125; **36.** 0.15; **37.** 0.3863; **38.** 0.425; **39.** 0.3571428; **40.** 1.190476.