1.02 *Order of Operations*

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In the first section, different number systems were defined, and symbols relating to addition (subtraction) and multiplication (division) of numbers within those systems were identified. In order to execute these basic operations it is necessary to establish a few traditions (agreements!) as to how these operations will be carried out. The signs of the numbers must also be addressed. In this section it is preferred that you use the calculator "in your head." In the next section, you will be encouraged to use the calculator "in your hand." If you do not already have one, be looking for a good sale on inexpensive calculators!

ORDER OF OPERATIONS

In your head, perform the following calculations:

1. $4 + 6 \bullet 2 =$ _____

2. $16 \div 8 \bullet 2 =$ _____

3. $5 \bullet 2^2 =$ _____

4. $(3 + 4)^2 =$ _____

Solutions:

1. If addition is performed first, the answer is 20.
If multiplication is performed first, the answer is 16.

2. If division is performed first, the answer is 4.
If multiplication is performed first, the answer is 1.

3. If multiplication is performed first, the answer is 100.
If the power is performed first, the answer is 20.

4. If you add first, the answer is 49.
If you square the 3 and then square the 4 (which the problem does <u>not</u> say to do!),
then the answer would be 25.

(SEE NEXT PAGE !!)

A mathematics system must have unique solutions to basic operations such as these. In order to perform these operations and always arrive at the same conclusion, we must agree to certain traditions which are called the **order of operations agreement**. This agreement is generally accepted across the world and in most arenas of life. If you obey these traditions, you will obtain consistent results in agreement with everyone else.

ORDER OF OPERATIONS

LEVEL 1:   Simplify within symbols of grouping: Parentheses ( ), brackets [ ], set braces { }, above or below fraction bars, and absolute value bars | | (see next section).

LEVEL 2:   Raise to the power (i.e. exponents).

LEVEL 3:   Multiply or divide in order from left to right.

LEVEL 4:   Add or subtract in order from left to right.

Now, we can say of the examples on the previous page, the "correct answers" are:

1. 16; **2.** 4; **3.** 20; **4.** 49 .

EXERCISES: Simplify each of the following according to the order of operations agreement.

1. $2 + 8 \bullet 4$

= _____

= _____

2. $20 \div 4 \bullet 5$

= _____

= _____

3. $2 \bullet 5^2$

= _____

= _____

4. $32 \div 4 \bullet 8$

= _____

= _____

5. $100 \div 5^2$

= _____

= _____

6. $(2 \bullet 5)^2$

= _____

= _____

7. $12 + 8 \div 2$

= _____

= _____

8. $12 \div (3 + 3)$

= _____

= _____

9. $5 + 9 \bullet 2$

= _____

= _____

10. $(100 \div 5)^2$

= _____

= _____

11. $12 \div 3 + 3$

= _____

= _____

12. $(5 + 9) \bullet 2$

= _____

= _____

13. $20 - 5 \bullet 0$

= _____

= _____

14. $12 + 3 \div 3$

= _____

= _____

15. $12 \div (3 \div 3)$

= _____

= _____

16. $5 + 5^2$

= _____

= _____

17. $(12 + 3) \div 3$

= _____

= _____

18. $18 + 2 \bullet 0$

= _____

= _____

19. $12 - 8 \div 2$

= _____

= _____

20. $12 \div (3 \bullet 2)$

= _____

= _____

21. $12 \div 3 \bullet 2$

= _____

= _____

22. $(12 - 8) \div 2$

= _____

= _____

23. $(12 - 8)^2 \div 2$

= _____

= _____

24. $20 - (8 \div 2)^2$

= _____

= _____

25. $(12 - 8 \div 2)^2$

= _____

= _____

26. $(12 - 8) \div 2^2$

= _____

= _____

27. $20 + (8 \div 2)^2$

= _____

= _____

28. $5 \bullet 5^2$

= _____

= _____

29. $(5 \bullet 5)^2$

= _____

= _____

30. $18 \div 2 \bullet 3$

= _____

= _____

31. $8 + 3 \bullet 2^2$

= _____

= _____

= _____

32. $5 \bullet 3^2 + 11 - 2^2$

= _____

= _____

= _____

33. $24 - 3 \bullet 4 + 6 \div 2$

= _____

= _____

= _____

34. $12 + 18 \div 3^2 + 6$

= _____

= _____

= _____

35. $12 + (18 \div 3)^2 + 6$

= _____

= _____

= _____

36. $6 + 6^2 \div 3 \bullet 2$

= _____

= _____

= _____

= _____

37. $(16 + 2^2) \div 2 \bullet 2$

= _____

= _____

= _____

= _____

38. $8 + 2 \bullet 3^2 - 5$

= _____

= _____

= _____

= _____

39. $5^2 + 4^2 \div 2^2 + 3^2$

= _____

= _____

= _____

= _____

40. $(16 + 2^2) \div (2 + 2)$

= _____

= _____

= _____

= _____

EXTRA CHALLENGE:

41. $3 \bullet 4^2 + 7 - 2 \bullet 3^2 \div 6 + 3 \bullet 5^2$

42. $24 - 12 \div 2 \bullet 3 + 6 \bullet 2^3$

43. $(5 \bullet 2)^2 - 20 \div 2 \bullet (8 - 3) + 10 - 3 + 7$

44. $35 - 20 \div 5 + 7^2 \bullet 2 - 6 \bullet 3 + 9 + 10 \div 2$

45. $\dfrac{(11-5)^2 + 3 \bullet 2^2}{(5-3)^2 + 4(7-2)}$

46. $\dfrac{(11-5)^2 - 3 \bullet 2^2}{(5-3)^2 + 4(7-2)}$

47. $\dfrac{3(5+3) + 3(9-5)}{6 \bullet 2^2 - 2 \bullet 3^2} - \dfrac{2 \bullet 3^3 - 2 \bullet 5^2}{4^2 - (7+5)}$

48. $\dfrac{(6 \div 2 \bullet 3)^2 + 2 \bullet 3^2}{(5+2)^2 - 4 \bullet 2^2} + \dfrac{(20+5) \bullet 2^2}{(20-5 \bullet 2)^2}$

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ANSWERS 1.02

1. 34; **2.** 25; **3.** 50; **4.** 64; **5.** 4; **6.** 100; **7.** 16; **8.** 2; **9.** 23; **10.** 400; **11.** 7; **12.** 28; **13.** 20; **14.** 13; **15.** 12; **16.** 30; **17.** 5; **18.** 18; **19.** 8; **20.** 2; **21.** 8; **22.** 2; **23.** 8; **24.** 4; **25.** 64; **26.** 1; **27.** 36; **28.** 125; **29.** 625; **30.** 27; **31.** 20; **32.** 52; **33.** 15; **34.** 20; **35.** 54; **36.** 30; **37.** 20; **38.** 21; **39.** 38; **40.** 5; **41.** 127; **42.** 54; **43.** 64; **44.** 125; **45.** 2; **46.** 1; **47.** 5; **48.** 4.