1.03 *Signed Numbers; Absolute Value*

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Frequently in algebra you are required to simplify expressions with negative as well as positive numbers. When adding numbers it is best to think in terms of **MONEY $$$$$**! A **positive number** is like **money coming in** to your possession, or income; a **negative number** is like **money going out**, like **expenditures** or **debts**. When writing a negative number, it is helpful to write the negative number in parentheses.

For example, 8 + (-5) means **you have 8 dollars** in your possession and **you spend 5 dollars**. The result is **+3** or 3, which means **you have 3 dollars**.

What if the larger magnitude is negative? For example, **5+(-12)** means you have 5 dollars and **spend 12 dollars**. The result **is (-7)** or a **debt of 7 dollars**.

What if both numbers are negative? If you have two debts, like **(-7) + (-12)**, the result is **(-19)** or a total **debt of 19 dollars**.

In summary, it is obvious that when you add positive numbers, you get a positive number. When you add positive and negative numbers, you subtract the numbers and the sign of the answer is the same as the sign of the larger magnitude. When you add negative numbers, you always add the numbers and the sign is always negative.

ADDITION RULES

RULE	SIGN OF ANSWER	WHAT TO DO	EXAMPLE
(+)+(+)	+	Add the numbers	(+8)+(+4) = +12
(−)+(−)	−	Add the numbers	(−8)+(−4) = −12
(+)+(−)	Sign of the larger	Subtract the numbers	(+12)+(−8) = +4
(−)+(+)	Sign of the larger	Subtract the numbers	(−12)+(+8) = −4

EXERCISES:

1. (-8) + 12 = ____
2. (-12) + 8 = ____
3. (-10) + 8 = ____
4. 10 + (-4) = ____
5. (-2) + (-6) = ____
6. (-14) + (-6) = ____
7. (-32) + 28 = ____
8. (-18) + 30 = ____
9. (-12) + (-14) = ____
10. (-6) + 30 = ____
11. (-20) + 8 = ____
12. (-32) + (-8) = ____
13. 26 + (-14) = ____
14. (-24) + (-16) = ____
15. (-14) + (-14) = ____
16. (-15) + 28 = ____
17. (-22) + 64 = ____
18. (-12) + (-88) = ____
19. (-12) + 20 = ____
20. (-20) + 12 = ____
21. (-12) + (-8) = ____
22. 16 + (-4) = ____
23. (-34) + (-16) = ____
24. (-14) + (-28) = ____
25. (-125) + 28 = ____
26. (-52) + 120 = ____
27. (-12) + (-87) = ____
28. (-12) + 34 + (-26) = _____
29. (-28) + (-125) + 95 = _____
30. 200 + (-120) + (-85) = _____
31. (-200) + (-135) + 75 = _____
32. (-40) + 62 + (-62) +(-73) + 73 + 726 + (-726) = _____

When subtracting negative numbers, remember that the negative of a negative is the positive of the number. For examples, -(-8) is a (+8) or just 8; -(-10) is +10 or 10; -(-x) is x; etc. Of course, the negative of a positive is a negative. As examples, -(+8) is (-8); -(+10) is -10; etc.

SUBTRACTION RULES

$$-(-x) = +x$$

$$-(+x) = -x$$

EXERCISES:

33. -(-2) = _____ **34.** -(-6) = _____ **35.** -(-10) = _____

36. -(-12) = _____ **37.** -(-18) = _____ **38.** -(-120) = _____

39. 8 - (-12) = 8 + ___
= _____

40. -8 - (-12) = -8 + ____
= _____

41. -12 -(-12) = ____ + ____
= _____

42. -8 - (-8) = ____ + ____
= _____

43. 12 - (-8) = _____
= _____

44. -12 - (-8) = _____
= _____

45. (-38) - (-12) = _____
= _____

46. 38 - (-12) = _____
= _____

When multiplying or dividing positive and negative numbers, remember that multiplying is actually a short way to add. For example, 4 x 3 means 4 threes. In this way 4 x (-3) means 4 three dollar debts or (-12). Likewise, (-3) x 4 is also (-12). In the same way that a **negative of a negative** is a positive, a **negative times a negative** is a positive. The following are rules for multiplication and division of signed numbers. Remember also, that the word **"of"** (for now at least!) means **"times"**.

RULES FOR MULTIPLICATION AND DIVISION

RULE	SIGN OF ANSWER	RULE	SIGN OF ANSWER
(+) • (+)	+	(+) ÷ (+)	+
(+) • (−)	−	(+) ÷ (−)	−
(−) • (+)	−	(−) ÷ (+)	−
(−) • (−)	+	(−) ÷ (−)	+

EXERCISES:

47. 4(-5) = _____ **48.** (-3)(-4) = _____ **49.** (-6) 4 = _____

50. (-7)(-3) = _____ **51.** (-6)(7) = _____ **52.** (-7) (-9) = ____

53. 7(-8) = _____ **54.** (-6)(-9) = _____ **55.** (-23) 4 = _____

56. (-17)(-3) = _____ **57.** (-16)(9) = _____ **58.** (-5) (-24) = ____

59. 7(-8)(-2) = _____ **60.** 5(-9)(2) = _____ **61.** (-2)(-4)(-25) = _____

62. (-7)(-8)(-2) = ____ **63.** 5(-9)(-2) = _____ **64.** (-14)(-4)(-25) = ____

When raising to a power, did you notice that a positive number raised to any power is always positive? Did you notice that a negative number raised to an even power is always positive? Perhaps you noticed that a negative number raised to an odd power is always negative.

POWER RULES

$(\text{POSITIVE})^{\text{ANY POWER}} = \text{POSITIVE}$

$(\text{NEGATIVE})^{\text{EVEN POWER}} = \text{POSITIVE}$

$(\text{NEGATIVE})^{\text{ODD POWER}} = \text{NEGATIVE}$

EXERCISES:

65. $(-2)^2 =$ _____ **66.** $(-5)^2 =$ _____ **67.** $(-4)^2 =$ _____

68. $(-5)^3 =$ _____ **69.** $(-3)^3 =$ _____ **70.** $(-4)^3 =$ _____

71. $(-1)^{12} =$ _____ **72.** $(-1)^9 =$ _____ **73.** $(-1)^{15} =$ _____

74. $(-1)^{24} =$ _____ **75.** $(-10)^3 =$ _____ **76.** $(-3)^4 =$ _____

77. $(-2)^3(-1) =$ ____ **78.** $(-3)^4(-1)^8 =$ ____ **79.** $(-2)^4(-1)^3 =$ ____

80. $(-2)^5 (-1)^2 =$ ____ **81.** $(-5)^2 (-2)^3 =$ ____ **82.** $(-3)^2 (-2)^3 =$ ____

83. $(-2)^2 (-3)^3 =$ ____ **84.** $(-2)^3 (-3)^3 =$ ____ **85.** $(-3)^2 (-2)^2 =$ ____

However, be careful when the negative number does not have parentheses. Is there a difference between $(-2)^2$ and -2^2 ? The quantity $(-2)^2$ means (-2) times (-2), which is +4. However, -2^2 means the negative of (two to the second power). By order of operations agreement, this means to raise two to the second power, and then take the negative. This result is -4. The important question to ask is this: "What is it that you are raising to the power?" In the case of $(-2)^2$, you are raising (-2) to the second power. However, with -2^2 only the 2 is squared, not the "–". Therefore,

$$(-2)^2 = 4, \text{ but } -2^2 = -4.$$

Also notice that in (-2^2), only the 2 (<u>not</u> the negative) is squared.

Therefore $(-2^2) = -4$. Notice that $(-2)^2 \neq (-2^2)$.

EXERCISES. Complete the following:

86. $-2^4 =$ _____ **87.** $(-2)^4 =$ _____ **88.** $(-5)^2 =$ _____

89. $-5^2 =$ _____ **90.** $(-2)^3 =$ _____ **91.** $-2^3 =$ _____

92. -3^4 = _____ **93.** $(-3)^4$ = _____ **94.** $(-5)^3$ = _____

95. -5^3 = _____ **96.** -1^{10} = _____ **97.** -1^{13} = _____

98. $-2^2\,(-3)^3$ **99.** $-2^3\,(-3)^2$ **100.** $(-3)^3\,(-2^2)$

ABSOLUTE VALUE

The **absolute value** of a number, denoted by vertical bars on each side of a number or a quantity, represents the **size** or **magnitude** of that number. Another way to think of absolute value of a number is the distance on the number line of that number from zero. As examples, |-3| is 3; |-7| is 7; |10| is 10; |0| is 0. Notice that the absolute value definition does not apply to what is outside the absolute value bars. For example, -|3| is -3; -|-3| is -3; -|7| is -7; -|-7| is -7.

EXERCISES:

101. |-4| **102.** |-6| **103.** -|4|

104. -|-6| **105.** - |-6| **106.** - |-4|

107. $|-4| + 3|-3|$

108. $|-5| - 3|-3|$

109. $-|-8| + 3|-9|$

110. $-3|-5| - 5|-6|$

111. $-4|4-6| - 8|-8+3|$

112. $9|-7+1| - 3|5-9|$

113. $|-11| - |-5|^2$

114. $|-5-3|^2 - 4|7-2|$

115. $|-8-5|^2 + |3-12|^2$

116. $|-8^2 - 5| + |3^2 - 12|$

117. $|-8^2 + 5| - |3^2 - 12|$

118. $-|-5^2 - 3^2|$

ANSWERS 1.03

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1. 4; **2.** -4; **3.** -2; **4.** 6; **5.** -8; **6.** -20; **7.** -4; **8.** 12; **9.** -26; **10.** 24; **11.** -12; **12.** -40; **13.** 12; **14.** -40; **15.** -28; **16.** 13; **17.** 42; **18.** -100; **19.** 8; **20.** -8; **21.** -20; **22.** 12; **23.** -50; **24.** -42; **25.** -97; **26.** 68; **27.** -99; **28.** -4; **29.** -58; **30.** -5; **31.** -260; **32.** -40; **33.** 2; **34.** 6; **35.** 10; **36.** 12; **37.** 18; **38.** 120; **39.** 20; **40.** 4; **41.** 0; **42.** 0; **43.** 20; **44.** -4; **45.** -26; **46.** 50; **47.** -20; **48.** 12; **49.** -24; **50.** 21; **51.** -42; **52.** 63; **53.** -56; **54.** 54; **55.** -92; **56.** 51; **57.** -144; **58.** 120; **59.** 112; **60.** -90; **61.** -200; **62.** -112; **63.** 90; **64.** -1400; **65.** 4; **66.** 25; **67.** 16; **68.** -125; **69.** -27; **70.** -64; **71.** 1; **72.** -1; **73.** -1; **74.** 1; **75.** -1000; **76.** 81; **77.** 8; **78.** 81; **79.** -16; **80.** -32; **81.** -200; **82.** -72; **83.** -108; **84.** 216; **85.** 36; **86.** -16; **87.** 16; **88.** 25; **89.** -25; **90.** -8; **91.** -8; **92.** -81; **93.** 81; **94.** -125; **95.** -125; **96.** -1; **97.** -1; **98.** 108; **99.** -72; **100.**-108; **101.** 4; **102.** 6; **103.** -4; **104.** -6; **105.** -6; **106.** -4; **107.** 13; **108.** -4; **109.** 19; **110.** -45; **111.** -48; **112.** 42; **113.** -14; **114.** 44; **115.** 250; **116.** 72; **117.** 56; **118.** -34.