1.05 *Variables and Substitution*

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The study of algebra usually begins with systems of numbers and operations with those numbers. True to form, in this book you began with number systems, signed numbers, and order of operations. But algebra is much more than just the study of numbers and computations. As you progress in algebra, you will notice that the concepts and problem solving methods become more complicated and also more abstract. You will go from concrete computations of numbers that you can see with your eyes and do with a calculator, to abstract ideas that you "see" and understand with your mind.

The first step towards abstract thinking in algebra is the introduction of **variables**. Up to this point, all problems have been completely numerical. **Variables** are simply letters that take the place of numbers. Just as a **pronoun** is a word that takes the place of a **noun**, a variable could be thought of as a "**pronumber**," a letter that takes the place of a **number**.   You can find the value of an algebraic expression by substitution if you are given the numbers to be substituted.

Example 1:   Find the value of $3x + 4$,

a) if $x = 2$;   b) if $x = -2$;   c) if $x = -12$

Solution: Substitute the given values for x (plug it in, plug it in!)

a) $3(2) + 4$ b) $3(-2) + 4$ c) $3(-12) + 4$

  $6 + 4$ $-6 + 4$ $-36 + 4$

   10 -2 -32

At this point you may be wondering, "Why x?" Well, why not x! As the next examples illustrate, any letter(s) can and will be used. Historically, the favorite letter used as a variable in math is the letter x. In fact, usually we use the letters x, y, **and** z to represent variables (quantities that vary or change) and we use the letters **a, b, and c** to represent constants (quantities that remain the same.)

Example 2:   Find the value of $3x + 4y - 8$,

a) if $x = 2$ and $y = 3$;   b) if $x = -2$ and $y = -6$;   c) if $x = -12$ and $y = 4$

Solution: Substitute the given values for x and y

a) $3(2) + 4(3) - 8$ b) $3(-2) + 4(-6) - 8$ c) $3(-12) + 4(4) - 8$

   $6 + 12 - 8$ $-6 - 24 - 8$ $-36 + 16 - 8$

    10 -38 -28

Example 3 : Find the value of $x^2 + 4xy - z^2$,

  a) if $x = 2$, $y = 3$, and $z = 4$;   b) if $x = -2$, $y = -6$, and $z = -5$

Solution: Substitute the given values for x, y, and z.

  a) $2^2 + 4(2)(3) - 4^2$ b) $(-2)^2 + 4(-2)(-6) - (-5)^2$

  $4 + 24 - 16$ $4 + 48 - 25$

12   27

EXERCISES: Find the values of the following expressions for the given values of the variables.

1. Find the value of $5x + 12$

  a) if $x = 2$;   b) if $x = -2$;   c) if $x = -12$

  5 (   ) + 12   5 (   ) + _____   _____ + _____

  _____ + 12   _____ + _____   _____ + _____

  _____   _____   _____

2. Find the value of $7x - 4y - 10$,

  a) if $x = 3$ and $y = 2$;   b) if $x = -4$ and $y = -7$;   c) if $x = -3$ and $y = 5$

  7(   ) - 4(   ) - 10   7(   ) - 4(   ) - _____

3. Find the value of $x^2 - 3yz - z^2$, if $x = 5$, $y = 2$, and $z = 3$.

  $(   )^2 - 3(   )(   ) - (   )^2$

4. Find the value of $x^2 - 3yz - z^2$, if $x = -5$, $y = -2$, and $z = -3$

  $(   )^2 - 3(   )(   ) - (   )^2$

Example 4: Given that $x = -2$, $y = -3$, $z = 4$, and $w = 1$, find the value of $\dfrac{y^2 + xy - w^2}{wx + yz}$.

Solution:

$$\frac{(\;\;)^2 + (\;\;)(\;\;) - (\;\;)^2}{(\;\;)(\;\;) + (\;\;)(\;\;)}$$

$$\frac{(-3)^2 + (-2(-3)) - (1)^2}{(1)(-2) + (-3)(4)}$$

$$\frac{9 \;+\; 6 \;-\; 1}{-2 \;+\; \text{-12}}$$

$$\frac{14}{-14} \quad \text{or} \quad \text{-1}$$

In each of the following exercises, substitute the given values and find the value of the expressions, given that $x = -2$, $y = -3$, $z = 4$, and $w = -1$.

5. $xy \; + \; 5z$

$= (\quad)(\quad) + 5(\quad)$

$=$ _____ + _____

$=$ _____

6. $xy \; - \; 5z$

$= (\quad)(\quad) - 5(\quad)$

$=$ _____ - _____

$=$ _____

7. $xz \; - \; 5y$

8. $xz \; + \; 5y$

9. $xz \; - \; 5w$

10. $xz \; + \; 5w$

11. $x^2 + y^2$

$= (\quad)^2 + (\quad)^2$

$=$ _____ + _____

$=$ _____

12. $z^2 + w^2$

13. $x^2 - y^2$

14. $w^2 - z^2$

15. $x^2 + y^2 + z^2$

16. $w^2 - y^2 - z^2$

Find the values, given that $x = -2$, $y = -3$, $z = 4$, and $w = -1$.

17. $x^3 + y^3$

18. $x^3 - y^3$

19. $x^3 - z^3$

20. $-z^2 + w^2$

21. $x^2 + 3xz + z^2$

22. $x^2 + yzw - w^2$

23. $\dfrac{xy - wz}{xw + yz}$

24. $\dfrac{y^2 + w^2}{xy - z}$

25. $\dfrac{y^2 + z^2}{yz - x}$

26. $\dfrac{x^3 + y^3 + w^3}{y^2}$

27. $\dfrac{wx + yz}{y^2 + xz - w^2}$

28. $\dfrac{y^2 + xz - w^2}{wx + yz}$

ANSWERS 1.05

p. 28 - 30:

1a) 22; **b)** 2; **c)** -48; **2a)** 3; **b)** -10; **c)** -51; **3.** -2; **4.** -2; **5.** 26; **6.** -14; **7.** 7; **8.** -23; **9.** -3; **10.** -13; **11.** 13; **12.** 17; **13.** -5; **14.** -15; **15.** 29; **16.** -24; **17.** -35; **18.** 19; **19.** -72; **20.** -15; **21.** -4; **22.** 15; **23.** -1; **24.** 5; **25.** -2; **26.** -4; **27.** Undefined; **28.** 0.