1.06 *Combining Like Terms;*
The Distributive Property

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

In the last section, the concept of variables was introduced. Variables usually represent either known or unknown numbers. If the values of the variables are given as they were in the last section, then you can substitute those values in place of the variables and obtain a numerical answer. When the values of the variables are not known, the expressions may sometimes be simplified by a process called **combining like terms**. Before beginning this, however, a few preliminary remarks and definitions may be helpful.

1. **TERMS** are those quantities which are added (or subtracted) together. **FACTORS** are quantities that are multiplied. In the expression "*x* **+ 5**," the **x** and the **5** are terms, since they are added. In the expression "**5x**," the **x** and the **5** are factors, since they are multiplied. In the expression "**5x + 3**," the **5x** and the **3** are terms, but the **5** and the **x** could be considered to be factors of the **5x** term.

2. **LIKE TERMS**, or **similar terms**, are terms which have the same letters (variables) raised to the same power. Like terms and only like terms may be combined. For example, the expression $3x + 4x$ contains x terms. In the same way that 3 **apples** plus 4 **apples** would be 7 **apples**, the x terms can be combined: $3\mathbf{x} + 4\mathbf{x} = 7\mathbf{x}$. As another example, what about the expression $3x^2 + 4x^2$? When you add **apples** to **apples**, you get **apples**. When you add **x's** to **x's**, you get **x's**. So, it should be no surprise to learn that when you add **x²'s** to **x²'s**, you get **x²'s**. Therefore, $3x^2 + 4x^2 = 7x^2$.

3. **UNLIKE TERMS** are those terms which do **not** have the same letters and powers. Unlike terms cannot be combined. Examples: a) $3x + 4y$; b) $3x^2 + 4x$; c) $3x^2 + 4y^2$ These examples cannot be simplified, since in each example there are no like terms.

4. A term may contain more than one variable: $7xy + 3xy =$ _____ (Answer: 10xy)

5. A term may have no variable. We call this a **number term** or a **constant**. Number terms can be combined only with other number terms. $4x + 3 + 7y + 6 =$ _____
    (Answer: 4x + 7y + 9)

6. It makes no difference in what order the terms are given. "$4x + 7y + 9$" could also be written as "$9 + 4x + 7y$" or "$7y + 9 + 4x$" or any other order. Usually we put the terms in alphabetical order. When powers are used, such as $x^2 + 3x + 5$, again, order does not

matter, but usually the **highest powers** of the variable are placed first and the number term given last. This is called **descending powers of the variable**.

7.   A variable alone, such as x, really means $1x$ or $1\ x$. So, $3y + y$ means the same as $3y + 1y$, which is $4y$. Likewise, $4y - 3y$ would be $1y$, but it is simpler to write $4y - 3y = y$.

8.   Simplifying the expression $4y - 4y$, the result is "no y" or "0 y." However, since zero times any number is zero, the answer is simply "0."

       Notice that: $4x + 3y - 4x$ simplifies to $3y$ The 0x is not necessary!
                       $8x - 5x - 3x$ simplifies to 0   The 0x (which is 0) is necessary!

9.   Exactly what is meant by $3x$? Answer: $3x$ means **3 times** x, or three x's added together $(x + x + x)$. In this case, the number **3** is called the **coefficient of** x.

10.   What is meant by $3x^2$, and is it the same as $(3x)^2$?   Because there are no parentheses in the expression $3x^2$, according to the **order of operations agreement**, the x is squared, then the result is multiplied by 3. Therefore, $3x^2$ means 3 x x. However, because of the parentheses in $(3x)^2$, the quantity to be squared (that is, multiplied times itself!) is 3x. Therefore, $(3x)^2$ means (3x) (3x) or 3 x 3 x. When multiplying (or adding!) numbers, it can be done in any order. It is convenient to multiply the 3's together, then the x's.

     So, $(3x)^2$ means $(3x) \bullet (3x)$
                           $3x \bullet 3x$
                           $3 \bullet 3 \bullet x \bullet x$
                             $9x^2$

       Additional example: $5x^2 \bullet 6x$ equals **$30\ x^3$**,
               whereas: $(5x)^2 \bullet 6x$ means **(5x) (5x) $\bullet$ 6x or $5 \bullet 5 \bullet 6 \bullet x \bullet x \bullet x$ or 150 x^3.**

EXAMPLES: Simplify by combining all like terms. Remember, answers may be in any order.

1. $2x + 4x + 3y =$ _____   ANS: $6x + 3y$
2. $7x + 4y + 3x =$ _____   ANS: $10x + 4y$
3. $2x + 4 + 3y + x + 10y =$ _____   ANS: $3x + 13y + 4$
4. $7x^2 + 3y^2 + 3x + 2x^2 + y^2 =$ _____   ANS: $9x^2 + 4y^2 + 3x$

5. $12x - 8y + 3y - 8x + x - 7y + 7y^2 - 8$
   = _____   ANS: $5x + 7y^2 - 12y - 8$

6. $13x - 4y^2 - 9x + y^2 + 4x + 3y^2 - 7x$
   = _____   ANS: x

7. $12x^2 - 8y - 9xy + y^2 + 8x - 3xy^2 - 8$
   = _____   ANS: Same as the problem!
                              Cannot be simplified!

EXERCISES: Simplify by combining all like terms.

1. $5x - 2x =$ _____

2. $5d - 3d =$ _____

3. $8x + x =$ _____

4. $3b + 3g =$ _____

5. $h + h =$ _____

6. $9x + x =$ _____

7. $9c - 4c =$ _____

8. $9x - 8x =$ _____

9. $x + 3y + 4 =$ _____

10. $5w + 6w - 3w =$ _____

11. $4x - 3y + 12x - 2y =$ _____

12. $8k + 7v + 3k - 2v =$ _____

13. $2a + 3c - 8a - 4c =$ _____

14. $12x - 6y + 4z - 3z + 11y + x =$ _____

15. $xyz + 3xy + 2xyz + xy =$ _____

16. $2LH + 2WH + 2LW - 6WH + 6LH =$ _____

17. $7x^2 + 3x^2 =$ _____

18. $7y^2 + 3y^2 =$ _____

19. $7x^2 + 3y^2 =$ _____

20. $7y^2 + 3y =$ _____

21. $7x^2 - 3x^2 =$ _____

22. $-7y^2 - 3y^2 =$ _____

23. $3x^2 + 5x^2y^2 + 9y^2 =$ _____

24. $14x^2 + 6x - 8y^2 - 6xy + 3y^2 + 12xy =$ _____

The Distributive Property

Consider the following examples

EXAMPLE 8. **Find the value of   5(4 + 6)   and   5·4 + 5·6**

Solution: 5 (10)   20 + 30

                                     50   50

EXAMPLE 9. **Find the value of 10(8 + 7) and   10·8 + 10·7**

Solution: 10 (15)   80 + 70

                                     150   150

EXAMPLE 10. **Find the value of   10(8 - 7)   and   10·8 - 10·7**

Solution: 10 (1)   80 - 70

                                     10   10

These are examples of the **distributive property**, formally called the **distributive property for multiplication over addition**. The distributive property is much more than a "neat trick" that works with numbers. It is one of the most important and frequently used properties in all of math. You really need to know and be able to use this one, both now and for future reference. This property is formally stated as follows:

The Distributive Property $a (b + c) = a b + a c$

also $a b + a c = a (b + c)$

EXAMPLE 11.   **6(x + 5)**   According to the order of operations agreement, you must perform the addition within the parentheses before you multiply. However, these are unlike terms, so they cannot be combined. Therefore, in order to eliminate the parentheses, you can use the distributive property as follows:

$$6(x + 5) = 6 \cdot x + 6 \cdot 5$$

$$= 6x + 30$$

The distributive property also works when subtracting and when using longer expressions.

EXAMPLE 12. $6(x - 5) = 6x - 30$

EXAMPLE 13. $6(x + 3y - 7) = 6x + 18y - 42$

In the next examples, remember that $x \cdot x = x^2$

EXAMPLE 14. $x (x - y + 4) = x^2 - xy + 4x$

EXAMPLE 15. $7 (3x + 5y - 6) = 21x + 35y - 42$

EXAMPLE 16. $-7x (3x + 5y - 6) = -21x^2 - 35xy + 42x$

EXERCISES: **Use the distributive property to remove the parentheses.**

25. $5 (x + 3) =$ _____   26. $5 (x + 5) =$ _____

27. $6 (x - 4) =$ _____   28. $8 (y - 9) =$ _____

29. $7 (2x + 5) =$ _____   30. $5 (4x + 7) =$ _____

31. $6 (9x - 8) =$ _____   32. $9 (7y - 9) =$ _____

33. $-5 (x - 5) =$ _____   34. $-6 (x + 5) =$ _____

35. $-1 (3x + 7) =$ _____   36. $-1 (3y - 4) =$ _____

37. $- (-8x - 7) =$ _____   38. $- (-8x + 5) =$ _____

39. $5x (x - 3) =$ _____   40. $8y (y + 9) =$ _____

41. $8x (5x + 3y) =$ _____   42. $5x (7x - 5y) =$ _____

43. $5x (2y-3x+7) =$ _____   44. $4y (7x-9y-6) =$ _____

45. $-6y (2y+3x+7) =$ _____   46. $-3x (7x-5y+1) =$ _____

47. $-7x (5y-x+1) =$ _____   48. $-4y (-7x+9y-1) =$ _____

The following exercises demonstrate the simplification of problems by using the distributive property followed by combining like terms. In the process of doing these exercises, you will be learning to show the steps in a neat, well-organized manner. This will be extremely important in the next section on **equation solving**.

Math Study Skill

A very important skill necessary for success in higher math is the ability to organize the problems neatly (and to be able to read your own handwriting!)

EXAMPLE 17: $4(x - 3) + 6(4x + 5)$

         $= 4x - 12 + 24x + 30$ Distributive Property

         $= 28x + 18$ Combine like terms

EXAMPLE 18: $6(x - 3) - 4(4x + 5)$

         $= 6x - 18 - 16x - 20$ Distributive Property

         $= -10x - 38$ Combine like terms

EXERCISES: **Remove parentheses and combine like terms.**

49. $3(2x + 5) + 6(x + 2)$

= _____   Distributive Property

= _____   Combine like terms

50. $3(2x + 5) - 6(x + 2)$

= _____   Distributive Property

= _____   Combine like terms

51. $3(2x - 5) + 6(x + 2)$ 52. $3(6x - 7) + 6(5x - 8)$

= _____   = _____

= _____   = _____

53. $7(3x + 9) - 6(2x + 5)$ 54. $5(8x - 4) - 8(5x + 3)$

= _____   = _____

= _____   = _____

55. $3x(5x - 9) + 8(3x - 2)$ 56. $2x(6x - 5) + 6(5 - 8x)$

= _____   = _____

= _____   = _____

57. $4x(x + 3) - 8x(5x - 2)$ 58. $4x(x + 3) - 8x(5 - 2x)$

= _____   = _____

= _____   = _____

59.   9(4x - 5y) - 6(2y - 3x) - 3(6x - 7y)

= _____

= _____

60.   6x(4x - 5) - 5y(2y + 3) - 4(3x + 8y)

= _____

= _____

ANSWERS 1.06

1. 3x; **2.** 2d; **3.** 9x; **4.** 3b+3g; **5.** 2h; **6.** 10x; **7.** 5c; **8.** x;   **9.** x+3y+4; **10.** 8w; **11.** 16x-5y;   **12.**   11k+5v;   **13.** -6a-c;   **14.** 13x+5y+z;   **15.** 3xyz+4xy; **16.** 8LH-4WH+2LW; **17.** $10x^2$; **18.** $10y^2$;   **19.** $7x^2+3y^2$; **20.** $7y^2+3y$; **21.** $4x^2$; **22.** $-10y^2$; **23.** $3x^2$ $+5x^2y^2+9y^2$; **24.** $14x^2+6x-5y^2+6xy$; **25.** 5x+15; **26.** 5x+25; **27.** 6x-24; **28.** 8y-72; **29.** 14x+35;   **30.** 20x+35;   **31.** 54x-48; **32.** 63y-81; **33.** -5x+25; **34.** -6x-30; **35.** -3x-7; **36.** -3y+4; **37.** 8x+7; **38.** 8x-5; **39.** $5x^2$-15x; **40.** $8y^2$+72y; **41.** $40x^2$+24xy; **42.** $35x^2$-25xy; **43.** 10xy-$15x^2$+35x; **44.** 28xy-$36y^2$-24y; **45.** $-12y^2$-18xy-42y; **46.** $-21x^2$+15xy-3x;   **47.** -35xy+$7x^2$-7x; **48.** 28xy-$36y^2$+4y; **49.** 12x+27; **50.** 3; **51.** 12x-3; **52.** 48x-69; **53.** 9x+33; **54.** -44; **55.** $15x^2$-3x-16; **56.** $12x^2$-58x+30; **57.** $-36x^2$+28x; **58.** $20x^2$-28x; **59.** 36x-36y; **60.** $24x^2$-42x-$10y^2$-47y.