# 1.07 *Number Properties*

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There are several properties involving addition and/or multiplication, most of which are widely known and perhaps taken for granted. Rather than take these properties for granted and use them haphazardly, it is better to know and use these properties by name. Recognizing and naming these properties is the objective of this lesson.

**I.** **COMMUTATIVE PROPERTY.** When adding or multiplying two real numbers, **the order may be changed without affecting the result.** (This is like **commuting** back and forth from work or school.)

**A.** **Commutative Property for Addition: a + b = b + a.**

**Examples:**

$$4 + 6 = 6 + 4$$
$$8 \cdot (4 + 6) = 8 \cdot (6 + 4)$$
$$2 + (4 + 6) = 2 + (6 + 4)$$
$$2 + (4 + 6) = (4 + 6) + 2$$
$$(5x + 3y) \cdot 14 = (3y + 5x) \cdot 14$$

**B.** **Commutative Property for Multiplication: a · b = b · a**

**Examples:**

$$4 \cdot 6 = 6 \cdot 4$$
$$8 \cdot (4 + 6) = (4 + 6) \cdot 8$$
$$2 \cdot (4 \cdot 6) = 2 \cdot (6 \cdot 4)$$
$$2 \cdot (4 \cdot 6) = (4 \cdot 6) \cdot 2$$
$$(5x + 3y) \cdot 14 = 14 \cdot (5x + 3y)$$

Note: Subtraction and division are not commutative. Why not?

**II.** **ASSOCIATIVE PROPERTY.** The way in which real numbers are **associated** (by means of parentheses or other symbols of grouping) may be changed without affecting the result. The associative property is frequently used to "reorganize" a problem so it can be simplified. Notice that the order of the numbers does <u>not</u> change.

**A.** **Associative Property for Addition:**

$$\mathbf{a + (b + c) = (a + b) + c}$$

**Examples:**

$$2 + (4 + 6) = (2 + 4) + 6$$
$$(8 + 9) + 1 = 8 + (9 + 1)$$
$$3x + (7x + 10) = (3x + 7x) + 10$$

**B.** **Associative Property for Multiplication:**

$$\mathbf{a \cdot (b \cdot c) = (a \cdot b) \cdot c}$$

**Examples:**

$$2 \cdot (4 \cdot 6) = (2 \cdot 4) \cdot 6$$
$$(8 \cdot 9) \cdot 1 = 8 \cdot (9 \cdot 1)$$
$$3x \cdot (7x \cdot 10) = (3x \cdot 7x) \cdot 10$$

Give an example to show that subtraction and division are <u>not</u> associative.

## III. DISTRIBUTIVE PROPERTY. $a \cdot (b + c) = a \cdot b + a \cdot c$

Technically, this property is called the distributive property for multiplication over addition. Sometimes it is called the distributive property for multiplication or, even better, the distributive property. In this property, a multiplier **distributes** over a sum or difference. If this equation is written "backwards," such as the equation:

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

then this is the process of **factoring the common factor**. Factoring the common factor (to be studied in the next chapter!) is accomplished by using the **distributive property.**

**Examples:** $5 \cdot (x + 8) = 5 \cdot x + 5 \cdot 8$ or $5 \cdot (x + 8) = 5x + 40$

$8 \cdot (y - 9) = 8 \cdot y - 8 \cdot 9$ or $8 \cdot (y - 9) = 8y - 72$

$7 \cdot (3x + 5y - 6) = 21x + 35y - 42$

$8x + 24y = 8 \cdot (x + 3y)$

$48x^2 - 32xy = 16x(3x - 2y)$

## IV. IDENTITY PROPERTY.

The identity property involves keeping something **identically the same** or preserving the **identity** of a quantity when adding or multiplying.

### A. Identity Property for Addition, Additive Identity, or Zero Property for Addition:

**For every X in the set, $X + 0 = X$ and $0 + X = X$.**

When **0** is added to any number **X**, the result is identically the same as the original number **X.** This is why it is called the **identity property for addition.**

**Examples:** $5 + 0 = 5$ or $0 + 5 = 5$

$b + 0 = b$ or $0 + b = b$

$(3x - 4y) + 0 = (3x - 4y)$ or $0 + (3x - 4y) = (3x - 4y)$

### B. Identity Property for Multiplication, Multiplicative Identity, or Unity Property:

**For every X in the set, $X \cdot 1 = X$ and $1 \cdot X = X$.**

When any number **X** is multiplied by **1** the result is **identically** the same as the original number.

**Examples:** $5 \cdot 1 = 5$ or $1 \cdot 5 = 5$

$b \cdot 1 = b$ or $1 \cdot b = b$

$(3x - 4y) \cdot 1 = (3x - 4y)$ or $1 \cdot (3x - 4y) = (3x - 4y)$

## V. INVERSE PROPERTY.

The inverse properties are as simple as "wrapping" and "unwrapping" presents! When you are given a wrapped present, you must unwrap it to get to the present. Likewise, the inverse properties are used to "undo" an operation. If **4** has been added, then adding a **(-4)** will "undo" the **4**. If there has been a multiplication by **4**, then multiplication by **¼** will "undo" the operation. In general, you would **inverse** (undo) an **X** with a **(-X)** for addition, and with $\frac{1}{X}$ for multiplication.

### A. Inverse Property for Addition or Additive Inverse Property:

$$X + (-X) = 0.$$

Every number **X** in the set has an inverse **(-X)** such that the sum of the number and its inverse is the identity element **0.**

**Examples:**

$5 + (-5) = 0$

$(-56) + 56 = 0$

$6b + (-6b) = 0$

### B. Inverse Property for Multiplication or Multiplicative Inverse Property:

$$X \bullet \frac{1}{X} = 1$$

For every number **X** in the set (except zero!) there is an inverse such that the product of the number and its inverse is the identity element **1.**

**Examples:**

$5 \bullet \frac{1}{5} = 1$ $\qquad$ $\frac{1}{4} \bullet 4 = 1$

$(-6) \bullet (\frac{-1}{6}) = 1$ $\qquad$ $(-\frac{3}{4}) \bullet (-\frac{4}{3}) = 1$

Zero has no multiplicative inverse. Why not?

Notice that the **identity properties** always involve **addition of 0** or **multiplication times 1** and the result is always **"identically the same."** For the **inverse properties**, you always end up with the **identity** number: **0** for **addition**, or **1** for **multiplication**.

## SUMMARY OF NUMBER PROPERTIES

**Commutative**--always involves a change in the order.

**Associative**--always involves a regrouping or re-association of the numbers in parentheses.

**Distributive**--always involves a product with parentheses on one side, distributed to each term on the other side of the equation.

**Identity**--always involves a 0 for addition or 1 for multiplication, ending up with identically the same thing you started with.

**Inverse**--always ends up with the identity element, 0 for addition or 1 for multiplication.

## EXERCISES.

**In each of the following, give the complete name of the property:**

**1.** $50 \cdot (2 \cdot 98) = 50 \cdot (98 \cdot 2)$ **1.** ________________________

What happened? Changed order multiplying 2 and 98 ________

**2.** $50 \cdot (2 \cdot 98) = (2 \cdot 98) \cdot 50$ **2.** ________________________

What happened? ________________________

**3.** $50 \cdot (2 \cdot 98) = (50 \cdot 2) \cdot 98$ **3.** ________________________

What happened? Re-association of numbers. Order did not change

**4.** $50 + (2 + 98) = 50 + (98 + 2)$ **4.** ________________________

What happened? ________________________

**5.** $50 + (2 + 98) = (2 + 98) + 50$ **5. ______________________**

What happened? ______________________

**6.** $50 + (2 + 98) = (50 + 2) + 98$ **6. ______________________**

What happened? ______________________

**7.** $50 \cdot (2 + 98) = 50 \cdot (98 + 2)$ **7. ______________________**

What happened? ______________________

**8.** $50 \cdot (2 + 98) = (2 + 98) \cdot 50$ **8. ______________________**

What happened? ______________________

**9.** $50 \cdot (2 + 98) = (50 \cdot 2) + (50 \cdot 98)$ **9. ______________________**

What happened? ______________________

**10.** $(50 \cdot 2) + (50 \cdot 98) = 50 \cdot (2 + 98)$ **10. ______________________**

What happened? ______________________

**11.** $50 \cdot (1 \cdot 98) = 50 \cdot 98$ **11. ______________________**

What happened? ______________________

**12.** $50 \cdot (98 \cdot 1) = 50 \cdot 98$ **12. ______________________**

What happened? ______________________

**13.** $50 \cdot [98 + (-98)] = 50 \cdot 0$ **13. ______________________**

What happened? ______________________

**14.** $50 \cdot [98+(-98)] = 50 \cdot 98 + 50 \cdot (-98)$ **14.______________________**

What happened? ______________________

**15.** **$50 \cdot [98+(-98)] = 50 \cdot [(-98) + 98]$** **15.** ____________________

What happened? ____________________

**16.** **$50 \cdot (0 + 98) = 50 \cdot (98 + 0)$** **16.** ____________________

What happened? ____________________

**17.** **$50 \cdot (0 + 98) = 50 \cdot 98$** **17.** ____________________

What happened? ____________________

**18.** **$50 \cdot (0 + 98) = 50 \cdot 0 + 50 \cdot 98$** **18.** ____________________

What happened? ____________________

**19.** **$50 \cdot (2 \cdot \frac{1}{2}) = 50 \cdot 1$** **19.** ____________________

What happened? ____________________

**20.** **$50 \cdot (2 \cdot \frac{1}{2}) = 50 \cdot (\frac{1}{2} \cdot 2)$** **20.** ____________________

What happened? ____________________

**21.** **$50 \cdot (2 \cdot \frac{1}{2}) = (50 \cdot 2) \cdot \frac{1}{2}$** **21.** ____________________

What happened? ____________________

**22.** **$50 \cdot [2 + (-2)] = 50 \cdot 0$** **22.** ____________________

What happened? ____________________

**23.** **$50 \cdot [(-2) + 2] = 50 \cdot [2 + (-2)]$** **23.** ____________________

What happened? ____________________

**24.** **$50 \cdot (\frac{1}{4} \cdot 4) = 50 \cdot 1$** **24.** ____________________

What happened? ____________________

## ANSWERS 1.07

**p. 42 - 44:**

**1.** Commutative for mult; **2.** Commutative for mult; **3.** Associative for mult;
**4.** Commutative for add; **5.** Commutative for add; **6.** Associative for add;
**7.** Commutative for add; **8.** Commutative for mult; **9.** Distributive;
**10.** Distributive; **11.** Identity for mult; **12.** Identity for mult;
**13.** Inverse for add; **14.** Distributive; **15.** Commutative for add;
**16.** Commutative for add; **17.** Identity for add; **18.** Distributive;
**19.** Inverse for mult; **20.** Commutative for mult; **21.** Associative for mult;
**22.** Inverse for add; **23.** Commutative for add; **24.** Inverse for mult.