# 1.10 *Word Problems*

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In this section, the following "categories" of word problems will be considered:

> **I. Number problems**
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> **II. Consecutive number problems**
>
> **III. Perimeter problems**
>
> **IV. Coin problems**
>
> **V. Mixture problems (Optional)**

Before describing these categories of word problems, it will be helpful to identify five steps in setting up and solving word problems:

*STEP 1:* ***IDENTIFY THE VARIABLE.*** State exactly what it is that the variable represents. For example, "Let $x$ = the number" or "Let $x$ = the smaller of two numbers" or "Let $x$ = the width of a rectangle" or "Let $x$ = the number of dimes." Then express all other quantities to be used in the problem in terms of $x$. This is the most important, often the most difficult, and usually the most overlooked step of the problem.

*STEP 2:* ***WRITE THE EQUATION.*** Having completed Step 1, use this step in writing the equation. This is often no more than translating a sentence of the problem into an equation. Read the problem carefully, paraphrasing as necessary.

*STEP 3:* ***SOLVE THE EQUATION.*** This is usually the easy part!

*STEP 4:* ***ANSWER THE QUESTION.*** After solving for $x$, there may be other quantities to be determined. Be sure you have answered the question before going on to the next exercise.

*STEP 5:* ***CHECK.*** Check the answers in the worded problem itself and make sure the solution actually works. Reject any extraneous (i.e., inappropriate) answers.

# I.   NUMBER PROBLEMS.

Probably the simplest type of application is the number problem.  If only one number involved, then begin by writing, **"Let $x$ = the number."**  If more than one number is involved, then begin "Let $x$ = one of the numbers," "Let $x$ = first number," "Let $x$ = the smaller number,"  "Let $x$ = the larger number," or something like this.  How to decide what to let $x$ equal will be explained later.

## EXAMPLE  1.

**Four times a number plus 12 is equal to 40.  Find the number.**

**Solution:**

**Step 1  Let $x$= the number (since there is only one number)**

**Step 2  Write the equation, (translate the sentence into math!)**

$$4x + 12 = 40$$

**Step 3  Solve the equation, (as in the previous section!).**

$$\begin{array}{r} 4x + 12 = 40 \\ - 12 \quad -12 \\ \hline 4x \quad = 28 \\ x = 7 \end{array}$$

**Step 4  Answer the question: The number is 7.**

**Step 5  Check:**
**Substitute 7 back into the original wording of the problem to see if it works: $4(7) + 12 = 40$.**

## EXAMPLE  2.

**Six less than twice a number is ten more than the number.  Find the number.**

**Solution:**
**Step 1  "Let $x$ = the number"**

**Step 2  Write the equation:  $2x - 6 = x + 10$**

**Step 3  Solve the equation:  $2x - 6 = x + 10$**
$$\begin{array}{r} -x + 6 \quad -x + 6 \\ \hline x \quad = \quad 16 \end{array}$$

**Step 4  Answer the question: The number is 16.**

**Step 5  Check**       $2(16) - 6 = 16 + 10$

$32 - 6 = 26$  **(This checks!)**

**EXERCISES.** Write equations for and solve each of the following word problems.

1.      Five times a number is 20 more than the number.  Find the number.

   Step 1    "Let x = _____"

   Step 2    Write the equation:

   Step 3    Solve the equation:


   Step 4    Answer the question:

   Step 5    Check


2.      Twice a number minus ten is equal to 22 more than the number.

   Step 1    "Let x = _____"

   Step 2    Write the equation:

   Step 3    Solve the equation:


   Step 4    Answer the question:

   Step 5    Check

3.      Three times a number plus 8 is equal to five times the number.  Find the number.

   Step 1     "Let x = _____"

   Step 2      Write the equation:

   Step 3       Solve the equation:



   Step 4      Answer the question:

   Step 5      Check

4.	Six more than four times a number is equal to 18 less than the number.  Find the number.

	Step 1

	Step 2

	Step 3


	Step 4

	Step 5


5.	Three less than four times a number is equal to 27 more than the number.  Find the number.








6.	Five more than four times a number is equal to 35 less than twice the number.  Find the number.

7.    Five more than four times a number is equal to 35 less twice the number.  Find the number. (NOTE: this looks like #6, but read the problem carefully!!)

8.    Seven times a number, minus 5, is equal to three times the number plus 19.  Find the number.

9.    Four less than three times a number is equal to 28 less than the negative of the number.  Find the number.

10.   Twice a number less 10 is equal to the negative of the number plus 14.  Find the number.

## EXAMPLE 3.

The larger of two numbers is twice the smaller number.  Their sum is 243.  Find the numbers. [Hint: The first sentence tells about the larger number, but it assumes you already know the smaller.  Therefore, let $x$ = the smaller number.]

**Solution:**

**Step 1**  "Let $x$ = the smaller number
$2x$ = the larger number

**Step 2**  Write the equation:  $x + 2x = 243$

**Step 3**  Solve the equation:  $x + 2x = 243$
$3x = 243$
$x = 81$

**Step 4**  Answer the question: The smaller number is $x = 81$.
The larger number is $2x = 162$.

**Step 5**  Check (optional!)  The sum of the numbers $(81 + 162)$ is 243.   (This checks!)

## EXERCISES.

**11.**    Two numbers are such that the larger number is three times the smaller number.  The sum of the numbers is 60.  Find the numbers.

**Step 1**  "Let $x$  = the _____ number
_____= the _____ number

**Step 2**  Write the equation:

**Step 3**  Solve the equation:

**Step 4**  Answer the question:

**Step 5**  Check (optional!)

12.   Two numbers are such that the larger is 6 more than twice the smaller number.  The
      sum of the numbers is 60. Find the numbers.

13.   Two numbers are such that the larger is 6 less than twice the smaller number.  The
      sum of the numbers is 60. Find the numbers.

14.   The smaller of two numbers is 12 less than the larger.  The sum of the numbers is 60.
      Find the numbers.        [Hint: **Let x = the larger number** (see end of first sentence!)]

15.   The smaller of two numbers is 60 less than twice the larger.  The sum of the numbers
      is 60. Find the numbers.

**16.** The smaller of two numbers is 30 less than three times the larger. The sum of the numbers is 22. Find the numbers.

## EXAMPLE 4.

Three numbers are such that the second is five more than the first, and the third is four less than three times the first. The sum of the numbers is 36. Find the numbers.

Solution: Let $x$ = first number
$x + 5$ = second number
$3x - 4$ = third number

Equation: $x + (x + 5) + (3x - 4) = 36$
$$5x + 1 = 36$$
$$5x = 35$$
$$x = 7$$

Answer the question:
$x = 7$   first number
$x + 5 = 12$   second number
$3x - 4 = 17$   third number

Check: $7 + 12 + 17 = 36$

## EXERCISES.

**17.** Three numbers are such that the second number is 5 more than the first, and the third number is 4 more than three times the second. The sum of the three numbers is 134. Find the numbers.

Solution: Let $x$ = first number
_____ = second number
_____ = third number

Equation:

Answer the question:   x = _____   first number
_____ = _____   second number
_____ = _____   third number

Check:

18. Three numbers are such that the second number is 3 less than the first, and the third is twice the second number. The sum of the numbers is 91. Find the numbers.

19. Three numbers are such that the second number is 4 more than the first, and the third number is equal to the sum of the first two numbers. The sum of the three numbers is 256. Find the numbers.

20. Three numbers are such that the second number is 6 less than twice the first, and the third number is five more than the sum of the first two. The sum of the numbers is 293. Find the numbers.

## II.  CONSECUTIVE INTEGER PROBLEMS

Word problems frequently refer to **consecutive numbers** or **consecutive integers**.  As examples, **5, 6, 7,** and **23, 24, 25** are two sets of consecutive integers.  Whatever the first integer may be, you must add 1 to the first integer to get the second integer, and you must add 2 to the first integer to get the third integer.  In general, if $x$ represents the first integer, then the second integer is $x + 1$, and the third integer is $x + 2$.

Examples of **consecutive odd integers** are **7, 9, 11** or **29, 31, 33**.  Notice that beginning with the first integer, you must add 2 to obtain the second integer, and you must add 4 to the first integer to obtain the third integer.  In general, if $x$ represents the first integer, then the second integer is $x + 2$, and the third integer is $x + 4$.

Examples of **consecutive even integers** are **6, 8, 10** or **28, 30, 32**.  Notice that, like with the consecutive odd integers, beginning with the first integer, you must add 2 to obtain the second integer, and you must add 4 to the first integer to obtain the third integer.  In general, if $x$ represents the first integer, then the second integer is $x + 2$, and the third integer is $x + 4$.

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Consecutive integers: $x, x+1, x+2$

Consecutive odd integers: $x, x+2, x+4$

Consecutive even integers: $x, x+2, x+4$

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## EXAMPLE 5.   Find two consecutive integers whose sum is 75.

**Solution:**  Let $x$ = the first integer
$x + 1$ = the second integer

**Equation:**  $x + x + 1 = 75$

**Solve:**  $2x + 1 = 75$
$2x = 74$
$x = 37$

**Answer question:**  $x + 1 = 38$

**Check:**  $37 + 38 = 75$

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**EXAMPLE 6.**  Find three consecutive even integers whose sum is 78.

Solution:  Let $x$ = the first integer
$x + 2$ = the second integer
$x + 4$ = the third integer

Equation:  $x + x + 2 + x + 4 = 78$
Solve:  $3x + 6 = 78$
$3x = 72$
$x = 24$
Answer question:  $x + 2 = 26$
$x + 4 = 28$

Check:  $24 + 26 + 28 = 78$

**EXAMPLE 7.**  Find three consecutive odd integers whose sum is 123.

Solution:  Let $x$ = the first integer
$x + 2$ = the second integer
$x + 4$ = the third integer

Equation:  $x + x + 2 + x + 4 = 123$
Solve:  $3x + 6 = 123$
$3x = 117$
$x = 39$
Answer:  $x + 2 = 41$
$x + 4 = 43$

Check:  $39 + 41 + 43 = 123$

## EXERCISES.

21.  Find two consecutive integers whose sum is 71.

**22.**     Find two **consecutive even integers** whose sum is 86.

**23.**     Find three **consecutive even integers** whose sum is 396.

**24.**     Find two **consecutive odd integers** whose sum is 100.

**25.**     Find three **consecutive odd integers** whose sum is 399.

**26.**     Find three **consecutive integers** whose sum is 369.

**EXAMPLE 8.** Three **consecutive even integers** are such that twice the first, plus four times the second, plus the third is equal to 96. Find the numbers.

**Solution:** Let $x$ = the first integer
$x + 2$ = the second integer
$x + 4$ = the third integer

**Equation:** $2(x) + 4(x+2) + (x+4) = 96$
**Solve:** $2x + 4x + 8 + x + 4 = 96$
$7x + 12 = 96$
$7x = 84$
$x = 12$
**Answer:** $x + 2 = 14$
$x + 4 = 16$

**Check:** $2(12) + 4(14) + 16 = 96$
$24 + 56 + 16 = 96$

## EXERCISES.

**27.** Two **consecutive integers** are such that twice the first, plus three times the second, is equal to 78. Find the numbers.

Let $x$ = first
$x + 1$ = second

**Equation:** $2(\quad) + 3(\quad) = 78$

**Solve:**

**28.** Two **consecutive odd integers** are such that twice the second, plus three times the first, is equal to 29. Find the numbers.

29.  Two **consecutive even integers** are such that twice the first, plus three times the second, is equal to 156. Find the numbers.

30.  Three **consecutive even integers** are such that the first, plus twice the second, plus three times the third, is equal to 100. Find the numbers.

31.  Three **consecutive odd integers** are such that the sum of the integers is 7 less than four times the smallest number. Find the numbers.

32.  Three **consecutive integers** are such that the first, plus twice the second, plus three times the third is equal to 200. Find the numbers.

# III. PERIMETER PROBLEMS

The perimeter of a geometric figure (shape) is the total distance around the outside of that figure.

For a rectangle, the perimeter consists of two widths and two lengths.  For a triangle, the perimeter is just the sum of the three sides.  For a circle, the perimeter (for circles it is called the **circumference**) is $\pi$ times the diameter of the circle, where $\pi$ is approximately 3.14.

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## PERIMETER

**Triangle:   P = a + b + c**

**Rectangle:   P = 2W + 2L**

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**EXAMPLE 9.**     **The second side of a triangle is twice the first side, and the third side is 6 less than the second side.  The perimeter of the triangle is 94 feet. Find the sides of the triangle.**

**Solution:**          Let $x$ = first side
$\quad\quad 2x$ = second side
$2x$ - 6 = third side

**Equation:**          $x + 2x + 2x - 6 = 94$
$\quad\quad\quad 5x - 6 = 94$
$\quad\quad\quad\quad 5x = 100$
$\quad\quad\quad\quad\quad x = 20$ feet (first side)
$\quad\quad\quad 2x = 40$ feet (second side)
$\quad\quad 2x - 6 = 34$ feet (third side)

**Check:**          $20 + 40 + 34 = 94$

**33.** The second side of a triangle is 6 more than the first side, and the third side is 10 more than the first side. The perimeter of the triangle is 70 meters. Find the sides of the triangle.

**Let** $x$ **= first side**

**_____ = second side**

**_____ = third side**

**Equation:** **_____ + _____ + _____ = 70**

**Solve:**

**34.** The second side of a triangle is 6 more than the first side, and the third side is 10 more than the second side. The perimeter of the triangle is 70 meters. Find the sides of the triangle.

**35.** The second side of a triangle is 4 more than the first side, and the third side is 10 less than the second side. The perimeter of the triangle is 70 meters. Find the sides of the triangle.

36.     The second side of a triangle is twice the first side, and the third side is 10 more than the second side.  The perimeter of the triangle is 70 feet.  Find the sides of the triangle.

37.     The third side of a triangle is 6 less than the first side, and the second side is twice the third side.  The perimeter of the triangle is 82 feet.  Find the sides of the triangle.

38.     The third side of a triangle is twice the first side, and the second side is 10 less than the third side.  The perimeter of the triangle is 70 feet.  Find the sides of the triangle.

**EXAMPLE 10.** The length of a rectangle is 5 more than the width.  The perimeter of the rectangle is 50 meters.  Find the dimensions of the rectangle.

Solution:                 Let  $x$   =  width of rectangle
                                    $x + 5$  =  length of rectangle

Equation:                 2W  +   2L   =  Perimeter
                                    $2(x) + 2(x+5) = 50$
                                    $2x + 2x + 10 = 50$
                                         $4x + 10$    = 50
                                               $4x$    = 40
                                                 $x$    = 10 meters

Answer question:                    $x$  = 10 m. width of rectangle
                                    $x + 5 = 15$ m. length of rectangle

Check:                    2W  +  2L    = P
                                    $2(10) + 2(15) = 50$
                                     $20 + 30 = 50$


**EXAMPLE 11.** The length of a rectangle is 8 less than twice the width.  The perimeter of the rectangle is 104 centimeters.   Find the dimensions of the rectangle.

Solution:                 Let  $x$  = width of rectangle
                                    $2x$-8  = length of rectangle

Equation:                 2W  +   2L  =  Perimeter
                                    $2(x) + 2(2x$-8$) = 104$
                                    $2x + 4x - 16 = 104$
                                         $6x - 16$   = 104
                                               $6x$       = 120
                                                 $x$       =  20 centimeters

Answer question:                    $x = 20$ cm width of rectangle
                                    $2x - 8 = 32$ cm length of rectangle

Check:                    2W  +  2L  = P
                                    $2(20) + 2(32) = 104$
                                     $40 + 64 = 104$

## EXERCISES.

**39.** The length of a rectangle is 5 meters longer than the width. The perimeter is 50 meters. Find the dimensions of the rectangle.

**Solution:**     Let $x$ = width of the rectangle
                    _____ = length of the rectangle

**Equation:**      2W  +  2L      =  Perimeter
                    2(    ) + 2(        ) = _____

**Check:**

**40.** The length of a rectangle is twice the width. The perimeter is 24 meters. Find the dimensions of the rectangle.

**Solution:**     Let $x$  = width of the rectangle
                    _____= length of the rectangle

**Equation:**      2W  +   2L      =  Perimeter
                    2(   ) + 2(       )  = _____

**Check:**

**41.** The length of a rectangle is 5 meters more than twice the width. The perimeter is 130 meters. Find the dimensions of the rectangle.

**Solution:**     Let $x$  = width of the rectangle
                    _____= length of the rectangle

**Equation:**      2W   +   2L     =  Perimeter
                    2(   ) + 2(       )  = _____

**Check:**

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**42.** The length of a rectangle is 5 meters less than twice the width.  The perimeter is 50 meters.  Find the dimensions of the rectangle.

<span style="color:red">**Solution:**</span>　　　　　<span style="color:red">**Let $x$ = width of the rectangle**</span>

　　　　　　　　　　<span style="color:red">**_____ = length of the rectangle (Not 5 - 2x)**</span>

**43.** The length of a rectangle is 50 feet less than three times the width.  The perimeter is 500 feet.  Find the dimensions of the rectangle.

**44.** The length of a rectangle is 50 meters less than twice the width.  The perimeter is 1100 meters.  Find the dimensions.

**45.** The length of a rectangle is 75 meters less than twice the width.  The perimeter is 600 meters.  Find the length and width of the rectangle.

46. The width of a rectangle is 50 feet less than the length. If the perimeter is 400 feet, find the length and width of the rectangle.

47. The length of a rectangle is 3 more than twice the width. The perimeter is 56 meters. Find the dimensions of the rectangle. (Note: the length and/or width do not have to come out even! Express the answer as in fractional form or as a repeating decimal.)

48. The length of a rectangle is three less than five times the width. The perimeter is ten times the width. Find the dimensions and perimeter of the rectangle.

49. **EXTRA CHALLENGE.** The perimeter of a rectangle is 46 meters. Twice the length is 4 more than five times the width. Find the dimensions of the rectangle.

**Solution:** Let $x$ = width

_____ = two lengths

2 widths + 2 lengths = Perimeter

# ANSWERS 1.10

**1.** 5; **2.** 32; **3.** 4; **4.** -8; **5.** 10; **6.** -20; **7.** 5; **8.** 6; **9.** -6; **10.** 8; **11.** 15, 45; **12.** 18, 42; **13.** 22, 38; **14.** 36, 24; **15.** 40, 20; **16.** 13, 9; **17.** 22, 27, 85; **18.** 25, 22, 44; **19.** 62, 66, 128; **20.** 50, 94, 149; **21.** 35, 36; **22.** 42, 44; **23.** 130, 132, 134; **24.** 49, 51; **25.** 131, 133, 135; **26.** 122, 123, 124; **27.** 15, 16; **28.** 5, 7; **29.** 30, 32; **30.** 14, 16, 18; **31.** 13, 15, 17; **32.** 32, 33, 34; **33.** 18m, 24m, 28m; **34.** 16m, 22m, 32m; **35.** 24m, 28m, 18m; **36.** 12ft, 24ft, 34ft; **37.** 25ft, 38ft, 19ft; **38.** 16ft, 22ft, 32ft; **39.** W=10m, L=15m; **40.** W=4m, L=8m; **41.** W=20m,L=45m; **42.** W=10m, L=15m; **43.** W=75ft, L=175ft; **44.** W=200m, L=350m; **45.** W=125m, L=175m; **46.** W=75ft,L=125ft; **47.** W=8 1/3m, L=19 2/3m; **48.** W=3, L=12 P=30; **49.** W=6, L=17;