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IV. COIN PROBLEMS

When solving coin problems (and mixture problems of various types later on!), it is helpful to form a three-column chart. This chart is simply a convenient way to organize the information you need to write the equation. While the number of rows in the chart varies from one problem to another, there will always be three columns. For coin problems, the first column is the number of coins (of each type!). The second column is the value of each type of coin (in dollars or in cents--cents is recommended!). The third column is the "values" column. The third column is obtained by taking the product of the quantities in the first two columns. The equation will always be in the third column. The examples that follow illustrate this method.

EXAMPLE 12. **A certain number of quarters and four times as many pennies are worth $1.45. How many of each coin are there?**

Solution: **Let x = number of quarters**
$4x$ = number of pennies

Remember to fill in the first column of the chart, then the second column of the chart. Finally, multiply the quantities in the first two columns to obtain the third column.

	No. Coins	× Each (¢)	= Values
Q	x	25	$25(x)$
P	$4x$	1	$1(4x)$
			145¢

Write the equation: (The equation will always be found in the third column.)
$$25(x) + 1(4x) = 145$$

Solve the equation:
$$25x + 4x = 145$$
$$29x = 145$$
$$x = 5$$

Answer the question: x = 5 Quarters
$4x$ = 20 Pennies

Check:
5 Quarters = $ 1.25
20 Pennies = .20
$ 1.45

EXERCISES:

50. **A certain number of dimes and four times as many pennies are worth $.98. How many of each are there?**

Solution: **Let *x* = number of _____**
_____ = number of _____

No. Coins	×	Each (¢)	=	Values

51. **A certain number of quarters and three more dimes than quarters are worth $7.30. How many of each are there?**

Solution: **Let x = number of _____**
_____ = number of _____

No. Coins	×	Each (¢)	=	Values

52. **A certain number of dimes and three less pennies than dimes are worth $7.67. How many of each are there?**

Solution: **Let x = number of _____**

_____ = number of _____

No. Coins	×	Each (¢)	=	Values

53. **A certain number of nickels and some dimes are worth $7.20. The number of dimes is three less than twice the number of nickels. How many of each are there?**

Solution: **Let x = number of _____**

_____ = number of _____

No. Coins	×	Each (¢)	=	Values

EXAMPLE 13. **A box contains 30 coins, in nickels and dimes, worth $2.40. How many of each coin are there?**

Solution: **Let x = number of nickels**
30 – x = number of dimes

	No. Coins	×	Each (¢)	=	Values
N	x		5		$5(x)$
D	$30 - x$		10		$10(30 - x)$
					240

Write the equation: $5(x) + 10(30 - x) = 240$

Solve the equation:

$$5(x) + 10(30 - x) = 240$$
$$5x + 300 - 10x = 240$$
$$-5x + 300 = 240$$
$$-5x = -60$$
$$x = 12 \text{ Nickels}$$

Answer the question: $x = 12$ Nickels
$30-x = 18$ Dimes

Check: 12(.05) = $ .60
18(.10) = 1.80
$2.40

[NOTE: You may also let x=number of dimes, and 30-x= number of nickels. See Example 14.]

EXAMPLE 14. **A box contains 30 coins, in nickels and dimes, worth $2.40. How many of each coin are there?**
[Note: This time, let x=no. dimes, 30-x=no. nickels.]

Solution: **Let x = number of dimes and $30 - x$ = number of nickels**

	No. Coins	×	Each (¢)	=	Values
D	x		10		$10(x)$
N	$30 - x$		5		$5(30 - x)$
					240

Write the equation: $10(x) + 5(30 - x) = 240$

Solve the equation:

$$10(x) + 5(30 - x) = 240$$
$$10x + 150 - 5x = 240$$
$$5x + 150 = 240$$
$$5x = 90$$
$$x = 18 \text{ Dimes}$$

Answer the question: $x = 18$ Dimes
$30-x = 12$ Nickels

Check: 18(.10) = $ 1.80
12(.05) = .60
$2.40

EXERCISES.

54. **A box contains 20 coins in quarters and dimes worth $2.90. How many of each coin are there? (Hint: See previous example and also page 62!)**

Solution: **Let *x* = number of _____**
_____ = number of _____

No. Coins	×	Each (¢)	=	Values

55. **A box contains 20 coins in quarters and dimes worth $3.80. How many of each coin are there?**

56. **A box contains 35 coins in quarters and nickels worth $3.15. How many of each coin are there?**

EXAMPLE 15. **A box contains nickels, dimes, and quarters worth a total of $2.10. There are twice as many dimes as quarters, and the number of nickels is two less than the number of dimes. How many of each coin are there?**

Solution:

	No. Coins	Each (¢)	Values
Q	x	25	$25(x)$
D	$2x$	10	$10(2x)$
N	$2x - 2$	5	$5(2x - 2)$
			210¢

Equation:

$$25x + 20x + 10x - 10 = 210$$
$$55x - 10 = 210$$
$$55x = 220$$
$$x = 4 \text{ Quarters}$$

Answer the question:

$$2x = 8 \text{ Dimes}$$
$$2x - 2 = 6 \text{ Nickels}$$

Check:

4(.25) = $1.00 Quarters
8(.10) = .80 Dimes
6(.05) = .30 Nickels
$2.10 Total

EXERCISES:

57. **A certain number of quarters, four times as many pennies as quarters, and 6 more dimes than pennies are worth $3.36. How many of each coin are there?**

Solution: Let _____ = no quarters
_____ = no pennies
_____ = no dimes

	No. Coins	Each (¢)	Values
Q	x		
P	$4x$		
D	( )		
			¢

58. **A certain number of dimes, twice as many pennies as dimes, and 6 less quarters than pennies are worth $6.56. How many of each coin are there?**

Solution: **Let _____ = no _____**

_____ = no _____

_____ = no _____

No. Coins ×	Each (¢) =	Values

59. **A box contains $6.60 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?**

60. **A box contains $8.00 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?**

61. **A certain number of pennies, four times as many dimes as pennies, and a number of quarters which is 16 less than twice the number of dimes, are worth $24.92. How many of each coin are there?**

62. **A sum of money consists of nickels, dimes, and quarters amounting to $1.90. If there are twice as many nickels as quarters, and three less dimes than nickels, how many of each coin are there?**

63. **A box contains nickels, dimes, and quarters worth $12.60. The number of dimes is 2 less than three times the number of nickels, and the number of quarters is 4 less than twice the number of dimes. How many of each coin are there?**

64. **A box contains nickels, dimes, and quarters worth $69.50. The number of nickels is 10 more than twice the number of dimes. There are as many quarters as nickels and dimes combined. How many of each coin are there?**

V. MIXTURE and INTEREST PROBLEMS

EXAMPLE 16. **Twenty kilograms of nuts consisting of cashews worth $6.00 per kg, pecans worth $2.50 per kg, and peanuts worth $1.50 per kg are mixed. There are twice as many pecans as cashews, and there are as many kilograms of peanuts as pecans. The total value of the nuts is $56. How many of each are there?**

Solution: Let x = kilograms cashews
$2x$ = kilograms pecans
$2x$ = kilograms peanuts

Type Nut	No. Kilograms	× Each ($)	= Values
Cashews	x	6.00	$6.00(x)$
Pecans	$2x$	2.50	$2.50(2x)$
Peanuts	$2x$	1.50	$1.50(2x)$
Total	20		$56.00

Write the equation: $6.00(x) + 2.50(2x) + 1.50(2x) = 56.00$

Solve the equation:
$$6.00(x) + 2.50(2x) + 1.50(2x) = 56.00$$
$$6.00x + 5.00x + 3.00x = 56.00$$
$$14x = 56.00$$
$$x = 4 \text{ kg cashews}$$

Answer the question:
x = 4 kg cashews
$2x$ = 8 kg pecans
$2x$ = 8 kg peanuts

Check:
4($6.00) = $24.00
8($2.50) = 20.00
8($1.50) = 12.00
$56.00

EXERCISES.

65. **A merchant mixes some candy worth $3.50 per pound with cheap stuff worth $1.00 per pound. There are 10 more pounds of cheap stuff than the more expensive candy. If the total value of the mixture is $28, how many pounds of each are there?**

No. Pounds	×	Each	=	Values in $$

66. **A man is making a nut mixture of almonds worth $4 per pound and cashews worth $6 per pound. If there are 3 pounds less cashews than almonds, and the total mixture is worth $122, how many pounds of each are there?**

67. **A merchant mixes a total of 50 pounds of candy, some worth $2 per pound, the rest worth $4 per pound. If the total value of the mixture is $160, how many pounds of each are there?**

68. **A nibble-mix recipe calls for mixing of cheerios, corn chex, and peanuts together. There are to be two more pounds of cheerios than nuts and twice as much corn chex as cheerios. The peanuts are $5 per pound, and both types of cereal cost $3 per pound. How much of each should be mixed in order to make a total mixture worth $60.**

EXAMPLE 17. **A woman invests a sum of money at 6% and $3000 more than this at 9%. If the total interest earned in one year is $4170, how much was invested at each rate?**

Solution: **Use the familiar formula from business: Principle × Rate × Time = Interest. In particular, for these problems, since time = 1 year, the equation can be written using the formula Principle × Rate = Interest.**

Identify the variable: **Let x = Principle invested at 6% (0.06)**
$x + 3000$ = Principle invested at 9% (0.09)

	Principle	× Rate	= Interest
6%	x	0.06	$0.06(x)$
9%	$x + \$3000$	0.09	$0.09(x+3000)$
			$4170.00

Write the equation: $0.06x + 0.09(x + 3000) = \$4170$

Solve the equation:
$$0.06x + 0.09(x + 3000) = \$4170$$
$$0.06x + 0.09x + 270 = \$4170$$
$$0.15x + 270 = \$4170$$
$$0.15x = \$3900$$
$$x = \$3900/0.15$$
$$x = \$26000 \text{ @ } 6\%$$

Answer the question: $x + 3000 = \$29000$ @ 9%

Check:
$$26000(0.06) = \$1560.00$$
$$29000(0.09) = \underline{2610.00}$$
$$\$ 4170.00 \text{ Total}$$

EXERCISES:

69. **A sum of money was invested at 8% simple interest, and three times as much at 10%. The total interest earned for the year was $190. How much was invested at each rate?**

Solution: **Principle** × **Rate** = **Interest**

8%	x	0.08	
10%	$3x$	0.10	

70. **A sum of money was invested at 12% simple interest, and $1000 less than this at 10%. The total interest earned for the year was $1000. How much was invested at each rate?**

71. **A sum of money was invested at 5% annual interest, and $500 less than twice this amount was invested at 12%. If the total interest earned for the year was $375, how much was invested at each rate?**

72. **A total of $1,000 was invested, some at 8% and the rest at 6% simple interest. The total interest earned for the year was $76. How much was invested at each rate?**

73. **A total of $10,000 was invested, some at 12% and the rest at 10% simple interest. The total interest earned for the year was $1060. How much was invested at each rate?**

74. **A man has $10,000 to invest, some in a relatively safe account earning 5% interest per year, and the rest in more speculative investments earning 12% per year. If the total interest earned for the year was $955, how much was invested at each rate?**

EXAMPLE 18. **How much water must be added to 60 liters of 20% acid solution in order to dilute the solution to 8%?**

Solution: **Use the formula for mixtures: Amt of Solution × Strength = Amt Pure Stuff**

Let x = number of liters of water added

	Amt. Sol.	Strength	Pure Stuff
20%	60	0.20	0.20(60)
Water (0%)	x	0.00	0.00(x)
8%	$x + 60$	0.08	0.08($x + 60$)

Write/Solve the Equation:

$$.20(60) + .00(x) = 0.08(x + 60)$$
$$12.0 + 0 = 0.08x + 4.8$$
$$-4.8 \qquad\qquad -4.8$$
$$7.2 = 0.08x$$
$$0.08x = 7.2$$
$$x = 7.2/(0.08)$$
$$x = 90 \text{ liters of water}$$

Check: 60(0.20) = 12 liters acid
+ 90 Water = No acid
150(0.08) = 12 liters acid

EXERCISES.

75. **How much 10% alcohol solution must be added to 20 liters of 50% solution to make a 20% solution?**

Solution: **Let x = amount of 10% alcohol solution**

	Amt. Sol.	Strength	Pure Stuff
10%	x	0.10	0.10(x)
50%	20	0.50	
20%	$x + 20$	0.20	

76. How much water must be added to 20 liters of 50% alcohol solution to dilute it to 10%?

Solution: Let x = amount of water

	Amt. Sol.	× Strength	= Pure Stuff
Water (0%)	x	0	
50%	20	0.50	
10%	$x + 20$	0.10	

77. How much pure alcohol must be added to 20 liters of 10% alcohol solution to create a 50% solution?

Solution: Let x = amount of pure 100% alcohol

	Amt. Sol.	× Strength	= Pure Stuff
100%	x	1.00	
10%	20	0.10	
50%	$x + 20$	0.50	

78. How much pure alcohol must be added to 100 liters of 10% alcohol solution to create an 80% solution?

EXAMPLE 19. **Some 10% alcohol solution is to be mixed with some 30% alcohol solution to make 20 liters of 16% solution. How much of each must be used?**

Solution: **Let x = amount of 10% solution.**

	Amt. Sol.	× Strength	= Pure Stuff
10%	x	0.10	$0.10(x)$
30%	$20 - x$	0.30	$0.30(20 - x)$
16%	20	0.16	$0.16(20)$

Equation:

$$0.10(x) + 0.30(20 - x) = 0.16(20)$$
$$0.10x + 6 - 0.30x = 3.20$$
$$-0.20x + 6 = 3.20$$
$$-0.20x = -2.80$$
$$x = -2.80/(-0.20)$$
$$x = 14 \text{ liters of 10\% solution}$$

Answer the question:

x = 14 liters of 10% solution
$20 - x$ = 6 liters of 30% solution

Check:

14(0.10) = 1.4 liters alcohol
6(0.30) = 1.8 liters alcohol
20(0.16) = 3.2 liters Total

79. **Some 80% acid solution is to be mixed with some 35% acid solution to make 300 liters of 50% solution. How much of each acid should be used?**

80. **How much 25% acid solution should be mixed with some 85% solution in order to obtain 300 liters of 65% solution?**

81. **How much water must be added to 50% alcohol solution to obtain 100 liters of 10% solution?**

82. **How much pure alcohol must be mixed with some 30% alcohol solution to obtain 700 liters of 75% solution?**

ANSWERS 1.10

p. 65 - 100:

50. 7D, 28P; **51.** 20Q, 23D; **52.** 70D, 67P; **53.** 30N, 57D; **54.** 6Q, 14D; **55.** 12Q, 8D;
56. 7Q, 28N; **57.** 4Q, 16P, 22D; **58.** 13D, 26P, 20Q; **59.** 10Q, 26D, 30N; **60.** 12Q, 36N, 32D;
61. 12P, 48D, 80Q; **62.** 4Q, 8N, 5D; **63.** 8N, 22D, 40Q; **64.** 70D, 150N, 220Q**;**
65. 4 lb. @ $3.50, 14 lb. @ $1.00; **66.** 14 lb. almonds, 11 lb. cashews;
67. 20 lb. @ $2, 30 lb. @ $4; **68.** 3 lb. nuts, 5 lb. cheerios, 10 lb. corn chex;
69. $ 500 @ 8%, $1500 @ 10%; **70.** $5000 @ 12%, $4000 @ 10%;
71. $1500 @ 5%, $2500 @ 12%; **72.** $ 800 @ 8%, $ 200 @ 6%;
73. $3000 @ 12%, $7000 @ 10%; **74.** $3500 @ 5%, $6500 @ 12%;
75. 60 L; **76.** 80 L; **77.** 16 L; **78.** 350 L; **79.** 100 L; **80.** 100 L; **81.** 80 L; **82.** 450 L.