2.02 *Factoring Numbers*

It is frequently useful to write numbers in a **factored form.** For example, if you were asked to factor the number 15, what would you say? You probably would instinctively reply 3 times 5 (or 5 times 3). The numbers 3 and 5 are factors of 15, since 3 and 5 divide evenly into 15. That is to say, if you divide 15 by 3 or by 5, there is no remainder. Notice that 1 and 15 are also factors of 15, since 1 and 15 also divide evenly into 15. Likewise, the number 26 can be written as $2 \bullet 13$, so 2 and 13 (also 1 and 26) are factors of 26. Since the number 91 can be written as $7 \bullet 13$, 7 and 13 (also 1 and 91) are factors of 91.

Question: How would you factor 18? In this case, there is more than one possibility. You could say 18 is 6 times 3, or you could say 2 times 9. If you continue to break the numbers down, you will get:

$$\begin{aligned} 18 &= 6 \bullet 3 \\ &= 2 \bullet 3 \bullet 3 \\ &= 2 \bullet 3^2 \end{aligned} \qquad\qquad \begin{aligned} 18 &= 2 \bullet 9 \\ &= 2 \bullet 3 \bullet 3 \\ &= 2 \bullet 3^2 \end{aligned}$$

The factors of 18 are 1, 2, 3, 6, 9, and 18.

Each of the numbers 15, 18, 26, and 91 could be written as the product of smaller numbers. What if you were asked to factor 17? What about 37? You will not be able to find smaller numbers that can be used to break down the 17 or the 37, as you did with 15, 18, 26, and 91. Since the numbers 17 and 37 cannot be expressed as the product of smaller numbers, they are called **prime numbers**. **A *prime number* is any number larger than 1 that has exactly two factors: 1 and itself. A *composite number* is any number that has more than two factors.** Composite numbers may be broken down into the product of smaller numbers. The number 1 is a special number in that it is neither prime nor composite.

DEFINITIONS

A *prime number* is a number larger than 1 that cannot be expressed as the product of two smaller numbers.

A *composite number* is a number that can be expressed as the product of two smaller numbers.

NOTE: The number one (1) is neither prime nor composite.

Complete the following list of prime numbers from 2 to 101.

2, 3, ___, ___, ___, ___, ___, 19, ___, ___, ___, ___, 41,

___, ___, ___, 59, ___, ___, 71, ___, ___, ___, 89, ___, 101.

Perhaps you noticed that the larger the numbers, the harder it is to tell whether or not the numbers are prime. At this point, it will be helpful to learn a few shortcuts to determine divisibility by certain numbers. With or without these shortcuts, a calculator will be very helpful.

1. **Divisibility using a calculator.** To determine if a number is divisible by a second number with a calculator, just divide the first number by the second number to see if it comes out a whole number on the calculator. For example, to see if 5289 is divisible by 41, type 5289/41, then [=] or [ENTER]. You will see that the answer is the whole number 129. Is 5289 divisible by 73? When you divide, the answer is NOT a whole number, but rather it comes out to a decimal (like 72.45205 and more!). Thus you can see that 5289 is divisible by 41, but NOT divisible by 73.

2. **Divisibility by 2.** What numbers are divisible by 2? You probably already know the answer--**even numbers,** numbers whose last digit is even.

3. **Divisibility by 5.** What numbers are divisible by 5? Again, you probably already know the answer--**numbers that end in a 5 or a 0**.

4. **Divisibility by 10.** What numbers are divisible by 10? Again, you already know this--**numbers that end in a 0**.

5. **Divisibility by 3.** What numbers are divisible by 3? Look at some numbers that you know are divisible by 3, like 15, 18, 33, 42, 66, 72, and 75 to name a few. Notice that in every case, the sum of the digits is also divisible by 3. On the other hand, try some numbers like 13, 26, 41, 43, 44, etc. that are *not divisible by 3*. Notice that the sum of the digits also is *not divisible by 3*.

6. **Divisibility by 9.** Look at some numbers that you know are divisible by 9: 18, 27, 36, 45, 54, 63, 72, 81, and 90. Notice that in every case, the sum of the digits is also divisible by 9. On the other hand, try some numbers like 12, 26, 48, 53, 84, etc. that are *not divisible by 9.* Notice that the sum of the digits also is *not divisible by 9*.

DIVISIBILITY BY 3 AND 9

If a number is divisible by 3 or by 9, then the sum of its digits is also divisible by 3 or by 9, and vice-versa.

If the sum of the digits is NOT divisible by 3 or by 9, then the number is *not* divisible by 3 or by 9, and vice-versa.

NOTICE THAT THIS RULE ONLY WORKS FOR 3 and 9.

7. **Divisibility by 4.** If the last two digits are divisible by 4, then the number is divisible by 4.

8. **Divisibility by 6.** To be divisible by 6, a number must be divisible by 2 and also by 3. In other words, it must be an even number that is divisible by 3.

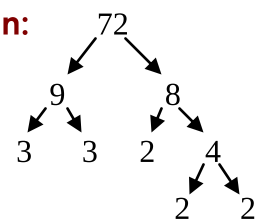
You probably noticed that for larger numbers it becomes more difficult to determine if the number is prime. For very large numbers it becomes very difficult. As a general rule, to determine if a number is prime, you must check to see if any prime numbers up to and including the square root of the number divide into it evenly. For example, to determine if 97 is a prime number, first use your calculator to find the square root of 97, which is 9.8. Then check to see if 97 is divisible by any prime number up to 9.8--that is, 2,3,5, or 7.

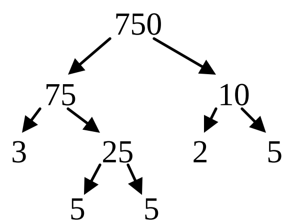
The following examples and exercises show how to give the prime factorization of a number. These are called **factor trees.** The idea is to find *any* number that divides into the number, then use the calculator to divide and find the other number. Keep breaking down the numbers until you get prime numbers. Circle the prime numbers. The product of these circled numbers is the prime factorization of the given number. The examples show the process.

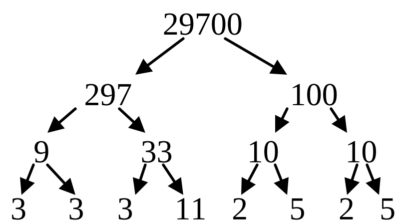
EXERCISES: Express the numbers as a product of prime numbers.

EXAMPLE 1. 72 **EXAMPLE 2. 750** **EXAMPLE 3. 29700**

Solution:







ANSWERS: $72 = 2^3 \cdot 3^2$ $750 = 2 \cdot 3 \cdot 5^3$ $29700 = 2^2 \cdot 3^3 \cdot 5^2 \cdot 11$

EXERCISES. In each of the following, express the numbers as the product of prime numbers.

1. 14	2. 33	3. 35
4. 77	5. 19	6. 37
7. 25	8. 49	9. 93
10. 87	11. 51	12. 94

13. 70	14. 28	15. 45
16. 54	17. 99	18. 105
19. 60	20. 80	21. 135
22. 700	23. 760	24. 790
25. 128	26. 450	27. 2000
28. 8400	29. 3060	30. 13,320

31. 10,000

32. 100,000

33. 52,200

34. 20,025

35. 55,800

36. 25,600

EXTRA CHALLENGE: **Factor each of the following into primes (a calculator may help).**

37. 1,000,000

38. 1,000,000,000

39. 111,111

40. 111,111,111
[Hint: 333667 is a prime number!]