2.03 *Factoring the Common Factor*

In the last section, you learned to factor numbers into prime factors. In that section, you were asked to **factor** a number. If, for example, you were asked to factor 15, you would quite naturally write **"3 times 5"** or **"5 times 3"**! The key word was **"times,"** which means multiplication. When asked to factor a given number, you naturally answer with a product of two numbers. The following is a working definition of the verb **"factor.**"

WORKING DEFINITION:

TO FACTOR: to EXPRESS AS A PRODUCT (MULTIPLICATION!)

While there are actually many different types of factoring, especially in higher mathematics courses, most of this chapter will focus on two main types of factoring. For this reason, we will call this the AFactoring Two Step.≅ Because so many future topics in algebra require you to be able to factor expressions, it is probably safe to say that these next sections and indeed the rest of this chapter will be some of the most important lessons in the entire book! **Please spend an appropriate amount of time, even more than usual, on these sections!**

The first step in *any* factoring problem is to **"factor the common factor."** Factoring the common factor is simply using the distributive property in reverse. Study the examples below.

EXAMPLES of DISTRIBUTIVE PROPERTY	**EXAMPLES of FACTORING:**
$6(x + 7) = 6x + 42$	$6x + 42 = 6(x + 7)$
$7(2x + 3) = 14x + 21$	$14x + 21 = 7(2x + 3)$
$9(x - 4) = 9x - 36$	$9x - 36 = 9(x - 4)$
$12(2x + 1) = 24x + 12$	$24x + 12 = 12(2x + 1)$
$5(3x - 2y + 4) = 15x - 10y + 20$	$15x - 10y + 20 = 5(3x - 2y + 4)$
$5x(x + 4) = 5x^2 + 20x$	$5x^2 + 20x = 5x(x + 4)$

When factoring the common factor, look for a number or variable that divides into both (or all) terms. If there is more than one common factor, be sure to get the largest common factor you can find. First write down the common factor. Then, open parentheses, and put down all the other factors that are left over.

EXAMPLE 1. $12x + 36$

SOLUTION: There are several numbers that divide evenly into both **12** and **36: 1,2,3,4,6, and 12.** Take the **largest** common factor, which is **12**. Write down the **12**, then open parentheses: **12 (+)**. In the parentheses you put the remaining factors, ***x*** and **3**, like this:

$$12x + 36 = 12(\quad + \quad)$$
$$= 12(x + 3)$$

EXERCISES: **Factor completely.**

1. $3x + 15$
   = 3(____ + ____)

2. $8x + 24$
   = 8(____ + ____)

3. $7x - 28$
   = _____(____ − ____)

4. $5x - 25$
   = _____(−)

5. $34x + 17y$
   = 17 ( )

6. $14x + 28y$
   = _____( )

7. $12x + 36y$
   = _____( )

8. $15x + 60y$
   = ____( )

9. $7x + 7$
   = _____(+ 1)

10. $3x^2 + 3$
    = 3 (____ + ____)

11. $42x^2 + 21$
    = 21 ( )

12. $30x^2 + 15$
    = _____( )

13. $5x^2 + 15x$
    = $5x$ (x +)

14. $7x^2 + 21x$
    = $7x$ ( )

15. $7x^2 + 14x$
    = _____( )

16. $7x^2 - 14x$
    = _____( )

17. $3x^2 + 12x$
    = ________________

18. $21x^2 + 30x$
    = __________________

19. $16x^2 - 18x$

$= \_\_\_\_\_(\qquad\qquad)$

20. $12x^2 - 30x$

$= \_\_\_\_\_(\qquad\qquad)$

21. $x^3 + 3x^2$

$= x^2(\qquad\qquad)$

22. $x^3 + 4x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

23. $x^3 + 4x$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

24. $x^3 - 4x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

25. $4x^3 + 8x^2$

$= 4x^2(\qquad\qquad)$

26. $4x^3 + 8x$

$= 4x(\qquad\qquad)$

27. $12x^3 - 8x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

28. $16x^3 - 24x^2$

$= 8x^2(\qquad\qquad)$

29. $16x^3 + 32x^2$

$= \_\_\_\_\_(\qquad\qquad)$

30. $12x^3 + 18x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

31. $12x^3 - 18x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

32. $45x^3 + 30x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

33. $12x^2 + 12$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

34. $24x^2 + 12$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

35. $24x^2 + 12x$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

36. $144x^2 + 12x$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

37. $16x^2 + 48x^3$

$= 16x^2(\qquad\qquad)$

38. $16x^2 - 48x^3$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

39. $24x^4 + 36x^3$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

40. $24x^3 + 24x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

41. $24x^3 + 24x$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

42. $24x^4 + 16x^2$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

43. $6x + 9y - 12$

$= 3(\_\_\_\_ + \_\_\_\_ - \_\_\_\_)$

44. $3x + 6y + 12z$

$= \_\_\_\_(\qquad\qquad)$

45. $9x + 18y + 9$

$= 9(\qquad\qquad)$

46. $30x + 20y + 10$

$= 10(\_\_\_\_ + \_\_\_\_ + \_\_\_\_)$

47. $30x + 20y - 5$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

48. $35x + 28y - 14z$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

49. $30x^2 + 20xy - 10y^2$

= ______________________

50. $30x^3 + 20xy + 10x^2$

= ______________________

51. $12x^3 + 24x^2 + 24x$

= ______________________

52. $12x^3 - 24x^2 + 3x$

= ______________________

53. $19x^3 + 19x^2y + 38x^2$

= ______________________

54. $36x^3 + 24x^2y + 12x^2$

= ______________________

55. $16x^2 + 32x^3$

$= 16x^2(\qquad\qquad)$

56. $16x^3 + 32x^2$

= ______________________

57. $16x^2 - 12x^3$

= ______________________

58. $16x^3 - 12x^2$

= ______________________

59. $y^5 - 14y^3$

$= y^3(\qquad\qquad)$

60. $x^{10} + 5x^3$

$= x^3(\qquad\qquad)$

From exercises #55 – 60 above, observe the rule listed below!

RULE

When factoring powers, take out the *lowest* exponent (power) of the factor.

Then *subtract* exponents.

61. $16x^2y^3 - 12x^3y^2$

$= 4x^2y^2(\qquad\qquad)$

62. $4x^3y^3 + 8x^2y^4$

$= 4x^2y^3(\qquad\qquad)$

63. $5x^3y^3 + 10x^2y$

= ______________________

64. $8x^5y^2 - 16x^4y^3$

= ______________________

65. $8x^5y^3 + 12x^3y^4$

= ______________________

66. $8x^3y^4 + 24x^2y^6$

= ______________________

When an expression contains negative terms, it is sometimes helpful to be able to "**factor out the negative**" or to factor out a "**−1**". If you factor out a "**−1**", this changes each sign inside the parentheses that follow, as in the following example.

EXAMPLE 2. Factor out a "**−1**" from the expression $-4x - 6y + 9$.

Solution: $-4x - 6y + 9 = -1\,(4x + 6y - 9)$ **or**

$= -\,(4x + 6y - 9)$

This can be verified by using the distributive property!

EXERCISES. In each of the following, factor completely, including the "negative."

67. $-x^2 - 4x + 6$

$= -1\,(\quad + \quad - \quad)$

or $-\,(\quad + \quad - \quad)$

68. $-x^2 + 5x - 1$

$= -1\,(\qquad)$

or $-\,(\qquad)$

69. $-x^2 + 3x + 7$

= ____________________

70. $-6x^2 + 12$

$= -6\,(\qquad)$

71. $-6x - 9y + 15$

$= -3\,(\qquad)$

72. $-8x + 20y - 24$

= ____________________

73. $-x^2 - 4x$

$= -x\,(\qquad)$

74. $-x^2 - 7x$

= ____________________

75. $-x^2 + 7x$

= ____________________

76. $-4x + 12x^2$

$= -4x\,(1 -$ _______ $)$

77. $-8x - 12x^2$

= ____________________

78. $-4x^2 - 4y^2$

= ____________________

79. $-x^2 - 3x + 8xy$

= ____________________

80. $-4x^2 - 16x + 16$

= ____________________

81. $-8 + 8x - x^2$

= ____________________

82. $-8 - 8x - 8x^2$

= ____________________

83. $8 - 16x - 40x^2$

= ____________________

84. $35 + 20x - 5x^2$

= ____________________

In each of the following exercises, factor the common factors. As you do, observe how you move from the simple to the more complicated; from the concrete to the abstract.

85a) $yx + 7x$

$= x(\qquad\qquad)$

b) $ya + 7a$

$= a(\qquad\qquad)$

c) y$ + 7$

= $ (   )

d) y (Junk) + 7 (Junk)

= (Junk) (   )

e) $y(x+4) + 7(x+4)$

$= (x+4)(\qquad\qquad)$

86a) $4xy + 3y$

= ______________

b) $4xa + 3a$

= ______________

c) 4x$ + 3$

= ______________

d) 4x (Junk) + 3 (Junk)

= ______________

e) $4x(y-7) + 3(y-7)$

= ______________

87. $a(x+4) - 5(x+4)$

88. $5a(b+7) + 3(b+7)$

89. $5a(3x+4) + 9b(3x+4)$

90. $10a(9y-7) - 3(9y-7)$

RULE

In order to factor a common factor, you must have an identical factor common to all terms.

Be sure to count terms first.

EXAMPLE 3. **Can you factor $5u(3x+4) + 9v(3x-4)$ in this manner?**

[See answer on next page!]

EXAMPLE 3 **Answer!** **NO! There is no factor common to both terms.]**

EXERCISES. **Factor the common factor in each of the following expression.**

91. $x(x-y) - y(x-y)$

92. $x(x-y) - y(x-y) + 4(x-y)$

93. $x(x-y) + y(x-y) - 4(x-y)$

94. $x(2x+3y) - y(2x+3y) + 4(2x+3y)$

EXTRA CHALLENGE:

95. $(x + y)^2 - z(x + y)$

$= (x + y)[(x + y) - \underline{\quad z \quad}]$

$= (\qquad)(\qquad\qquad)$

96. $(x - y)^2 - z(x - y)$

97. $(x - y)^2 - y(x - y)$

98. $(x + y)^2 - y(x + y)$

99. $(2x + 3y)^2 - 5(2x + 3y)$

100. $(2x - 3y)^2 - 5(2x - 3y)$

101. $(2x + 3y)^2 + 5(2x + 3y)$

102. $(x + y)^2 + (x - y)(x + y)$

LOOKING AHEAD

GUIDELINES TO FACTORING

1. **Common Factor (Factor Common Factor First FCFF!)**

2. **Trinomial (F OI L rearranged to spell F L OI)**

3. **Difference of Squares:** $x^2 - y^2 = (x - y)(x + y)$

   (Intermediate Algebra) Diff of Cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

   (Intermediate Algebra) Sum of Cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

4. **Factoring by Grouping**

ANSWERS 2.03

p. 140-145:

1. $3(x+5)$; **2.** $8(x+3)$; **3.** $7(x-4)$; **4.** $5(x-5)$; **5.** $17(2x+y)$; **6.** $14(x+2y)$; **7.** $12(x+3y)$; **8.** $15(x+4y)$; **9.** $7(x+1)$; **10.** $3(x^2+1)$; **11.** $21(2x^2+1)$; **12.** $15(2x^2+1)$; **13.** $5x(x+3)$; **14.** $7x(x+3)$; **15.** $7x(x+2)$; **16.** $7x(x-2)$; **17.** $3x(x+4)$; **18.** $3x(7x+10)$; **19.** $2x(8x-9)$; **20.** $6x(2x-5)$; **21.** $x^2(x+3)$; **22.** $x^2(x+4)$; **23.** $x(x^2+4)$; **24.** $x^2(x-4)$; **25.** $4x^2(x+2)$; **26.** $4x(x^2+2)$; **27.** $4x^2(3x-2)$; **28.** $8x^2(2x-3)$; **29.** $16x^2(x+2)$; **30.** $6x^2(2x+3)$; **31.** $6x^2(2x-3)$; **32.** $15x^2(3x+2)$; **33.** $12(x^2+1)$; **34.** $12(2x^2+1)$; **35.** $12x(2x+1)$; **36.** $12x(12x+1)$; **37.** $16x^2(1+3x)$; **38.** $16x^2(1-3x)$; **39.** $12x^3(2x+3)$; **40.** $24x^2(x+1)$; **41.** $24x(x^2+1)$; **42.** $8x^2(3x^2+2)$; **43.** $3(2x+3y-4)$; **44.** $3(x+2y+4z)$; **45.** $9(x+2y+1)$; **46.** $10(3x+2y+1)$; **47.** $5(6x+4y-1)$; **48.** $7(5x+4y-2z)$; **49.** $10(3x^2+2xy-y^2)$; **50.** $10x(3x^2+2y+x)$; **51.** $12x(x^2+2x+2)$; **52.** $3x(4x^2- 8x+1)$; **53.** $19x^2(x+y+2)$; **54.** $12x^2(3x+2y+1)$; **55.** $16x^2(1+2x)$; **56.** $16x^2(x+2)$; **57.** $4x^2(4-3x)$; **58.** $4x^2(4x-3)$; **59.** $y^3(y^2-14)$; **60.** $x^3(x^7+5)$; **61.** $4x^2y^2(4y-3x)$; **62.** $4x^2y^3(x+2y)$; **63.** $5x^2y(xy^2+2)$; **64.** $8x^4y^2(x-2y)$; **65.** $4x^3y^3(2x^2+3y)$;

66. $8x^2y^4(x+3y^2)$; **67.** $-(x^2+4x-6)$; **68.** $-(x^2-5x+1)$; **69.** $-(x^2-3x-7)$; **70.** $-6(x^2-2)$;
71. $-3(2x+3y-5)$; **72.** $-4(2x-5y+6)$; **73.** $-x(x+4)$; **74.** $-x(x+7)$; **75.** $-x(x-7)$; **76.** $-4x(1-3x)$;
77. $-4x(2+3x)$; **78.** $-4(x^2+y^2)$; **79.** $-x(x+3-8y)$; **80.** $-4(x^2+4x-4)$; **81.** $-(8-8x+x^2)$;
82. $-8(1+x+x^2)$; **83.** $-8(-1+2x+5x^2)$; **84.** $-5(-7-4x+x^2)$;
85a) $x(y+7)$; **b)** $a(y+7)$;**c)** $\$(y+7)$;**d)** $(\text{Junk})(y+7)$;**e)** $(x+4)(y+7)$;
86a) $y(4x+3)$; **b)** $a(4x+3)$; **c)** $\$(4x+3)$;**d)** $(\text{Junk})(4x+3)$;**e)** $(y-7)(4x+3)$;
87. $(x+4)(a-5)$; **88.** $(b+7)(5a+3)$; **89.** $(3x+4)(5a+9b)$; **90.** $(9y-7)(10a-3)$; **91.** $(x-y)^2$;
92. $(x-y)(x-y+4)$; **93.** $(x-y)(x+y-4)$; **94.** $(2x+3y)(x-y+4)$; **95.** $(x+y)(x+y-z)$; **96.** $(x-y)(x-y-z)$;
97. $(x-y)(x-2y)$; **98.** $x(x+y)$; **99.** $(2x+3y)(2x+3y-5)$; **100.** $(2x-3y)(2x-3y-5)$;
101. $(2x+3y)(2x+3y+5)$; **102.** $2x(x+y)$.