2.04 *Factoring Trinomials*

Do you remember from **Section 2.01**, you learned to multiply the **product of two binomials** using the **F OI L** method? As an introduction to this most important section on factoring trinomials, it will be helpful to review the **F OI L** method of finding products.

REVIEW EXERCISES. Use the **F OI L** method to multiply each of the following:

1. $(x + 2)(x + 5)$ **2.** $(x + 3)(x + 2)$ **3.** $(x + 5)(x + 3)$

4. $(x - 2)(x - 5)$ **5.** $(x - 3)(x - 2)$ **6.** $(x - 5)(x - 3)$

7. $(x - 2)(x + 5)$ **8.** $(x + 3)(x - 2)$ **9.** $(x + 5)(x - 3)$

10. $(x + 2)(x - 5)$ **11.** $(x - 3)(x + 2)$ **12.** $(x - 5)(x + 3)$

In each of the previous exercises, you were given a **product** of two **binomials**, and with **F OI L**, in each case you obtained a **trinomial** with x^2 as the **first term**. Now the problem will be to work these problems in reverse. What if you were given a **trinomial**, such as $x^2+7x+10$ and asked **to factor** it—that is, **to express it as a product.** This product is the product of two **binomials.** When you factor the trinomial: $x^2 + 7x + 10$

you expect the product of binomials: **( )( ).**

Also, when factoring a trinomial, instead of thinking **F OI L**, you need to change the order and think **F L OI.** In other words, you need to find the correct **F (first times first)** combination, then skip to the **L (last times last)**. Finally, check to make sure the **OI (outer times outer, inner times inner) middle term** is correct. **F OI L**

EXAMPLE 1. **Factor** $x^2 + 7x + 10$.

Solution: $x^2 + 7x + 10$

( )( ) Product of two **binomials**;

(x)(x) **F term is x^2**, which is ***x* times *x***;

(x)(x) **L term is +10**.

You must find two numbers whose product is **+10**. Probably **2 times 5**, or it could be **1 times 10**. Try **2 times 5**. Since the **last sign** is **positive**, it will be positive times positive, or negative times negative. Also, the **middle terms (O and I)** must be **added** together.

(x + 2)(x + 5) **OI term is 7*x*.** This means the **outer times outer** and the **inner times inner** terms must **add up** to **7*x*.**

[NOTE: **The order does NOT matter!**
If you wrote **(x + 5)(x + 2)**, this is exactly equivalent to **(x + 2)(x + 5)**. **WHY??**]

EXERCISES: **Factor.** (Find two numbers whose **product** is "**L**," and whose **sum** is "**OI.**")

1. $x^2 + 5x + 6$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

2. $x^2 + 8x + 15$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

3. $x^2 + 5x + 4$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

4. $x^2 + 7x + 6$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

5. $x^2 + 6x + 8$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

6. $x^2 + 9x + 8$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

7. $x^2 + 3x + 2$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

8. $x^2 + 9x + 14$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

9. $x^2 + 7x + 10$

   $(x +\_\_\_\_)(x +\_\_\_\_)$

10. $x^2 + 12x + 35$

    $(x +\_\_\_\_)(x +\_\_\_\_)$

11. $x^2 + 13x + 22$

    $(x +\_\_\_\_)(x +\_\_\_\_)$

12. $x^2 + 9x + 18$

    $(x +\_\_\_\_)(x +\_\_\_\_)$

13. $x^2 + 13x + 40$

    $(x \quad)(x \quad)$

14. $x^2 + 14x + 40$

    $(x \quad)(x \quad)$

15. $x^2 + 41x + 40$

    $(x \quad)(x \quad)$

16. $x^2 + 17x + 16$

    $(x \quad)(x \quad)$

17. $x^2 + 10x + 16$

    $(x \quad)(x \quad)$

18. $x^2 + 17x + 72$

    $(x \quad)(x \quad)$

In the next exercises, notice that the sign of the middle term is negative while the sign of the last term is positive. This means that the signs will be like sign, both negative.

EXAMPLE 2. **Factor** $x^2 - 7x + 10$.

Solution: $x^2 - 7x + 10$ This begins exactly as Example 1.

( )( ) Product of two **binomials**;

(x)(x) **F term is x^2**, which is x times x;

(x)(x) **L term is +10**.

You must find two numbers whose product is **+10**. Probably **2 times 5**, or it could be **1 times 10**. Try **2 times 5**. Since the **last sign** is **positive**, it will be positive times positive, or negative times negative. Also, the **middle terms (O and I)** must be **added** together.

$(x - 2)(x - 5)$ **OI term is $-7x$.** This means the **outer times outer** and the **inner times inner** terms must **add up** to **$-7x$.**

19. $x^2 - 5x + 6$

(x)(x)

20. $x^2 - 8x + 15$

(x)(x)

21. $x^2 - 7x + 10$

(x)(x)

22. $x^2 - 9x + 14$

23. $x^2 - 6x + 8$

24. $x^2 - 7x + 12$

25. $x^2 - 15x + 14$

26. $x^2 - 9x + 8$

27. $x^2 - 13x + 42$

28. $x^2 - 15x + 54$

29. $x^2 - 13x + 36$

30. $x^2 - 20x + 36$

If the **"L" term** is negative, this means that the signs are opposite, and you must subtract the numbers to get the **middle term**:

F OI L **F OI L**

EXAMPLE 3. $x^2 + 2x - 8$ **EXAMPLE 4.** $x^2 - 2x - 8$

Solutions: Because the **L term** is negative in each of these examples, you must have opposite signs, probably **−2 times 4** or **−4 times 2**.

$x^2 + 2x - 8$ $x^2 - 2x - 8$

$(x + \ )(x - \ )$ $(x + \ )(x - \ )$

Answers: $(x + 4)(x - 2)$ $(x + 2)(x - 4)$

[Also correct: $(x - 2)(x + 4)$ $(x - 4)(x + 2)$]

31. $x^2 + 2x - 3$ 32. $x^2 + 3x - 4$ 33. $x^2 + x - 6$

34. $x^2 + x - 2$ 35. $x^2 + 3x - 10$ 36. $x^2 + 5x - 14$

37. $x^2 - 4x - 5$ 38. $x^2 - 2x - 3$ 39. $x^2 - 5x - 14$

40. $x^2 - 7x - 18$ 41. $x^2 + 3x - 18$ 42. $x^2 - x - 20$

43. $x^2 + x - 20$

44. $x^2 + 2x - 35$

45. $x^2 - 9x - 22$

46. $x^2 - 6x - 27$

47. $x^2 + 2x - 24$

48. $x^2 - 5x - 24$

49. $x^2 - 2x - 24$

50. $x^2 + 2x - 24$

51. $x^2 + 10x - 24$

52. $x^2 - 7x - 30$

53. $x^2 - x - 30$

54. $x^2 - 13x - 30$

Perhaps you have noticed that the key to factoring trinomials is breaking down the LAST term. It may be helpful to list all possible combinations of factors. To do this, begin with 1 times the number, then try 2, 3, 4, 5, etc. Notice that as the first number gets larger, the second number gets smaller, and the numbers "meet" in the middle. When the numbers meet, you have all the combinations. Here are some examples:

10	**12**	**36**	**60**
1 **X** 10	1 **X** 12	1 **X** 36	1 **X** 60
2 **X** 5	2 **X** 6	2 **X** 18	2 **X** 30
	3 **X** 4	3 **X** 12	3 **X** 20
		4 **X** 9	4 **X** 15
		6 **X** 6	5 **X** 12
			6 **X** 10

It is also worth noting that not all trinomials can be factored. The trinomials $x^2 + x + 2$, $x^2 + 2x + 6$, and $x^2 - 4x - 6$ are examples of trinomials that cannot be factored. These are called prime trinomials. The following is a summary of trinomial (**F L OI**) factoring.

SUMMARY of RULES

1. **When the sign of the LAST is positive, the signs are the SAME. You find middle term by ADDING the O and I terms.**

2. **When the sign of the LAST is negative, the signs are OPPOSITE. You find middle term by SUBTRACTING the O and I terms.**

EXERCISES. **Factor each of the following trinomials.**

55. $x^2 - 7x + 12$ 56. $x^2 - x - 12$ 57. $x^2 + 13x + 12$

58. $x^2 - 8x + 12$ 59. $x^2 - 4x - 12$ 60. $x^2 - 13x + 12$

61. $x^2 - 13x + 36$ 62. $x^2 - 20x + 36$ 63. $x^2 + 16x - 36$

64. $x^2 - 5x - 36$ 65. $x^2 - 35x - 36$ 66. $x^2 - 12x + 36$

67. $x^2 + 17x + 60$

68. $x^2 + 16x + 60$

69. $x^2 + 19x + 60$

70. $x^2 - 4x - 60$

71. $x^2 + 7x - 60$

72. $x^2 - 28x - 60$

73. $x^2 - 32x + 60$

74. $x^2 + 11x - 60$

75. $x^2 + 61x + 60$

76. $x^2 - 23x + 60$

77. $x^2 - 17x - 60$

78. $x^2 - 59x - 60$

79. $x^2 - x - 56$

80. $x^2 - x - 72$

81. $x^2 + 17x + 72$

82. $x^2 - 11x + 28$

83. $x^2 - 3x - 28$

84. $x^2 + 16x + 28$

85. $x^2 + 13x + 42$

86. $x^2 - x - 42$

87. $x^2 - 23x + 42$

88. $x^2 - 5x - 50$

89. $x^2 + 23x - 50$

90. $x^2 - 21x + 38$

Frequently, it is necessary to **FACTOR THE COMMON FACTOR FIRST (FCFF)**. When there is a common factor in the problem, always remember to **FCFF**!

NOTE: These exercises require TWO steps––the "**Factoring Two Step!**"

EXERCISES. **Factor the trinomials completely. Be sure to FCFF!!**

91. $3x^2 + 6x - 9$

$3(x^2 + 2x - 3)$

$3(\quad)(\quad)$

92. $5x^2 + 15x - 20$

$5(\quad)$

$5(\quad)(\quad)$

93. $8x^2 + 8x - 48$

$\_\_\_\_(\quad)$

$\_\_\_\_(\quad)(\quad)$

94. $6x^2 + 6x - 12$

$\_\_\_\_(\quad)$

$\_\_\_\_(\quad)(\quad)$

95. $12x^2 + 36x - 120$

96. $10x^2 + 30x - 100$

97. $x^3 - 4x^2 - 5x$

98. $x^3 + 5x^2 + 6x$

99. $2x^3 - 14x^2 + 20x$

100. $5x^3 + 5x^2 - 10x$

101. $7x^3 + 49x^2 + 42x$

102. $8x^3 - 40x^2 + 32x$

103. $x^4 + x^3 - 20x^2$

104. $x^4 + 2x^3 - 35x^2$

105. $15x^4 - 45x^3 - 60x^2$

106. $20x^4 - 20x^3 - 120x^2$

107. $30x^4 + 90x^3 - 300x^2$

108. $18x^4 + 54x^3 + 36x^2$