

## 2.06 Advanced Trinomials

In the factoring of trinomials of the previous sections, it may have been assumed that the coefficient of  $x^2$  is 1 (or that the coefficient is a common factor of the entire trinomial). Notice that as you work through this section, because of combinations of numbers, the trial and error process becomes more and more challenging. However, let's begin with some examples that do not involve too many combinations of numbers.

**EXAMPLE 1.**  $5x^2 + 6x + 1$ .

**Solution:** Remember that this should be factored by **FL OI**.

The **F** term is  $5x^2$ ,

**L** is 1,

$$5x^2 + 6x + 1$$

$$5x^2 + 6x + 1$$

$$(5x \quad)(x \quad)$$

$$(\quad + 1)(\quad + 1)$$

and **OI** adds up to  $6x$ , as follows:

$$5x^2 + 6x + 1$$

**Final answer:**  $(5x + 1)(x + 1)$

**EXAMPLE 2.**  $3x^2 + 4x + 1$

**Solution:** The **F** term is  $3x^2$ , **L** is 1,

$$3x^2 + 4x + 1$$

$$3x^2 + 4x + 1$$

$$(3x \quad)(x \quad)$$

$$(\quad + 1)(\quad + 1)$$

and **OI** adds up to  $4x$ , as follows:

$$3x^2 + 4x + 1$$

**Final answer:**  $(3x + 1)(x + 1)$

**EXAMPLE 3.**  $8x^2 + 9x + 1$  (Trial and Error!)

**Solution:** The **F term** is  $8x^2$ , **L is 1,**

$$\begin{array}{l} 8x^2 + 9x + 1 \qquad \qquad \qquad 8x^2 + 9x + 1 \\ (8x \quad)(x \quad) \text{ or } (4x \quad)(2x \quad) \qquad \qquad \qquad (\quad + 1)(\quad + 1) \end{array}$$

Since the **OI term must add up to  $9x$** , use the  **$8x, 1x$**  combination.

$$8x^2 + 9x + 1$$

**Final answer:**  $(8x + 1)(x + 1)$

**EXAMPLE 4.**  $8x^2 - 6x + 1$  (Trial and Error!)

**Solution:** The **F term** is  $8x^2$ , **L is 1,**

$$\begin{array}{l} 8x^2 - 6x + 1 \qquad \qquad \qquad 8x^2 - 6x + 1 \\ (8x \quad)(x \quad) \text{ or } (4x \quad)(2x \quad) \qquad \qquad \qquad (\quad - 1)(\quad - 1) \end{array}$$

Since the **OI term must add up to  $-6x$** , use the  **$4x, 2x$**  combination.

$$8x^2 - 6x + 1$$

**Final answer:**  $(4x - 1)(2x - 1)$

**EXERCISES.** Factor each of the following trinomials.

1.  $3x^2 + 4x + 1$

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2.  $4x^2 + 5x + 1$

(      )(      )

3.  $7x^2 + 8x + 1$

(      )(      )

4.  $3x^2 - 4x + 1$

(      )(      )

5.  $4x^2 - 5x + 1$

(      )(      )

6.  $7x^2 - 8x + 1$

(      )(      )

- RULES:**
1. When the sign of the LAST is positive, the signs are the SAME.  
You find middle term by **ADDING** the O and I terms.
  2. When the sign of the LAST is negative, the signs are OPPOSITE.  
You find middle term by **SUBTRACTING** the O and I terms.

**EXAMPLE 5.**  $8x^2 - 2x - 1$  (Trial and Error!)

NOTE: Since the **L term is negative**, you must subtract **the O and I terms**.

**Solution:** The **F term is  $8x^2$** ,

**L is  $-1$** ,

$$8x^2 - 2x - 1$$

$$8x^2 - 2x - 1$$

$$(8x \quad)(x \quad) \text{ or } (4x \quad)(2x \quad) \quad ( \quad + 1)( \quad - 1) \text{ or } ( \quad - 1)( \quad + 1)$$

Since the **OI term** must subtract to give  $-2x$ , use the  $-4x, 2x$  combination.

$$8x^2 - 2x - 1$$

Final answer:  $(4x + 1)(2x - 1)$

**EXERCISES.** Factor each of the following trinomials.

7.  $3x^2 + 2x - 1$

8.  $4x^2 + 3x - 1$

9.  $7x^2 + 6x - 1$

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(      )(      )

(      )(      )

10.  $3x^2 - 2x - 1$

11.  $4x^2 - 3x - 1$

12.  $7x^2 - 6x - 1$

(      )(      )

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13.  $6x^2 + 5x + 1$

14.  $6x^2 + 7x + 1$

15.  $6x^2 - 7x + 1$

16.  $6x^2 - 5x + 1$

17.  $6x^2 + 5x - 1$

18.  $6x^2 - 5x - 1$

19.  $8x^2 - 9x + 1$

20.  $8x^2 + 7x - 1$

21.  $8x^2 + 2x - 1$

22.  $8x^2 + 6x + 1$

23.  $10x^2 + 7x + 1$

24.  $10x^2 - 11x + 1$

25.  $10x^2 - 3x - 1$

26.  $10x^2 + 9x - 1$

27.  $10x^2 + 11x + 1$

28.  $10x^2 - 7x + 1$

29.  $10x^2 - 9x - 1$

30.  $10x^2 + 3x - 1$

Of course, with larger numbers, with many more combinations of numbers this can become a very lengthy process of trial and error. There are some systematic methods of factoring these trinomials, which usually turn out to be somewhat complicated. In problems that are not too difficult, the trial and error method will be fairly simple and more than adequate for now.

**EXAMPLE 6.**  $5x^2 + 8x + 3$  (Trial and Error!!!)

**Solution:** The **F** term is  $5x^2$ , **L** is 3,

$$\begin{array}{cc} 5x^2 + 8x + 3 & 5x^2 + 8x + 3 \\ (5x \quad)(x \quad) & (\quad + 3)(\quad + 1) \text{ or } (\quad + 1)(\quad + 3) \end{array}$$

The **OI** term must add up to  $8x$ .

$$\begin{array}{c} 5x^2 + 8x + 3 \\ (5x + 3)(x + 1) \text{ or } (5x + 1)(x + 3) \end{array}$$

**Final answer:**  $(5x + 3)(x + 1)$

**EXAMPLE 7.**  $5x^2 + 16x + 3$  (Trial and Error!)

**Solution:** The **F** term is  $5x^2$ , **L** is 3,

$$\begin{array}{cc} 5x^2 + 16x + 3 & 5x^2 + 16x + 3 \\ (5x \quad)(x \quad) & (\quad + 3)(\quad + 1) \text{ or } (\quad + 1)(\quad + 3) \end{array}$$

The **OI** term must add up to  $16x$ .

$$\begin{array}{c} 5x^2 + 16x + 3 \\ (5x + 3)(x + 1) \text{ or } (5x + 1)(x + 3) \end{array}$$

**Final answer:**  $(5x + 1)(x + 3)$

**EXAMPLE 8.**

$$3x^2 + 10x + 7$$

$$(\quad)(\quad)$$

$$(3x \quad)(x \quad)$$

**EXAMPLE 9.**

$$3x^2 + 22x + 7$$

$$(\quad)(\quad)$$

$$(3x \quad)(x \quad)$$

In Examples 8 and 9, the **F term** must obviously be  $3x \cdot x$ . The **L term** is 7, which must be either  $7 \cdot 1$  or  $1 \cdot 7$ . The possibilities are

$(3x + 7)(x + 1)$  whose **middle term** is  $3x + 7x = 10x$ ,

or  $(3x + 1)(x + 7)$  whose **middle term** is  $21x + 1x = 22x$ .

**Final answers:** Example 8.  $3x^2 + 10x + 7 = (3x + 7)(x + 1)$

Example 9.  $3x^2 + 22x + 7 = (3x + 1)(x + 7)$

In Examples 10 and 11, the **L term** is negative, so you must subtract the **O and I terms**.

**EXAMPLE 10.**

$$3x^2 + 4x - 7$$

$$(\quad)(\quad)$$

$$(3x \quad)(x \quad)$$

**EXAMPLE 11.**

$$3x^2 - 20x - 7$$

$$(\quad)(\quad)$$

$$(3x \quad)(x \quad)$$

In Examples 9 and 10, the **F term** again is obviously  $3x \cdot x$ , and the **L term** is  $-7$ , which means  $-7 \cdot 1$  or  $7 \cdot -1$  (opposite signs!). The possibilities are:

$(3x-7)(x+1)$  whose **middle term** is  $3x-7x = -4x$ ,

$(3x+7)(x-1)$  whose **middle term** is  $-3x+7x = 4x$ ,

$(3x-1)(x+7)$  whose **middle term** is  $21x-1x = 20x$ ,

or  $(3x+1)(x-7)$  whose **middle term** is  $-21x+1x = -20x$

**Final answers:** Example 10.  $3x^2 + 4x - 7 = (3x + 7)(x - 1)$

Example 11.  $3x^2 - 20x - 7 = (3x + 1)(x - 7)$

**EXERCISES.****Factor each of the following trinomials.**

31.  $3x^2 + 8x + 5$

32.  $3x^2 + 16x + 5$

33.  $3x^2 + 2x - 5$

34.  $3x^2 - 14x - 5$

35.  $3x^2 - 14x + 11$

36.  $3x^2 + 8x - 11$

37.  $3x^2 - 8x - 11$

38.  $3x^2 + 34x + 11$

39.  $3x^2 - 34x + 11$

40.  $3x^2 + 32x - 11$

41.  $5x^2 + 6x - 11$

42.  $5x^2 + 54x - 11$

43.  $5x^2 + 41x + 8$

44.  $5x^2 + 14x + 8$

45.  $5x^2 + 22x + 8$

46.  $5x^2 + 13x + 8$

47.  $5x^2 - 14x + 8$

48.  $5x^2 - 41x + 8$

49.  $5x^2 + 3x - 8$

50.  $5x^2 + 39x - 8$

51.  $5x^2 + 6x - 8$

52.  $5x^2 - 6x - 8$

53.  $5x^2 + 18x - 8$

54.  $5x^2 - 13x + 8$

55.  $5x^2 - 31x + 6$

56.  $5x^2 - 11x + 6$

57.  $5x^2 + 13x + 6$

58.  $5x^2 + 17x + 6$

59.  $5x^2 - 13x - 6$

60.  $5x^2 - 7x - 6$

61.  $5x^2 - x - 6$

62.  $5x^2 + 29x - 6$

63.  $6x^2 + 49x + 8$

64.  $6x^2 - 49x + 8$

65.  $6x^2 + 47x - 8$

66.  $6x^2 + 19x + 8$

67.  $6x^2 - 19x + 8$

68.  $6x^2 - 13x - 8$

69.  $6x^2 + 13x - 8$

70.  $6x^2 + 19x + 10$

71.  $6x^2 + 11x - 10$

72.  $6x^2 + 17x + 10$