2.11 *Quadratic Applications*

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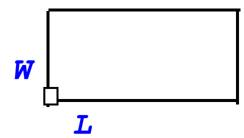
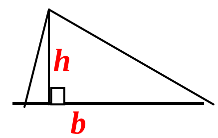
The question "What good is factoring?" was answered in part with the use of factoring to solve quadratic equations. This may have raised more questions than it answered: "What good are quadratic equations?" "What real life problems result in quadratic equations?" Questions like this usually end up in word problems. Some of the very best applications of quadratic equations involve some very important, yet simple concepts. A few basic formulas will be necessary.

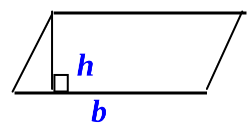
AREA of RECTANGLE $A = L \bullet W$



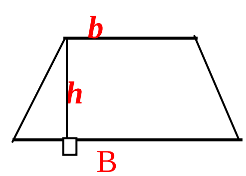
AREA of TRIANGLE $A = \frac{b\,h}{2}$ or $\frac{1}{2} b \bullet h$



AREA of PARALLELOGRAM $A = b \bullet h$

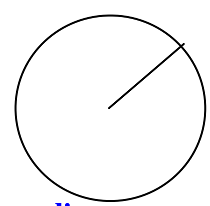


AREA of TRAPEZOID $A = \frac{1}{2}(B + b) \bullet h$



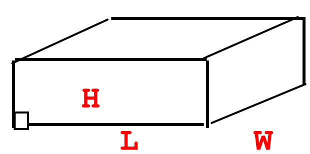
CIRCUMFERENCE of CIRCLE $C = \pi d$ or $C = 2\pi r$

AREA of CIRCLE $A = \pi\, r^2$



r = radius

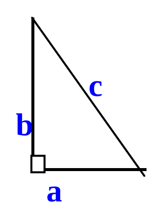
VOLUME of RECTANGULAR SOLID $V = L \bullet W \bullet H$



and of course,
THEOREM of PYTHAGORAS

In any right triangle, where "**a**" and "**b**" are legs and "**c**" the is hypotenuse:

$$a^2 + b^2 = c^2$$



PERIMETER/AREA/VOLUME

PERIMETER	**always 1 DIMENSIONAL**
Linear Units	**ft, in, m, cm, etc.**
AREA	**always 2 DIMENSIONAL**
Square Units	**sq ft, sq in, sq m, sq cm, etc.**
	(or ft^2, in^2, m^2, cm^2, etc.)
VOLUME	**always 3 DIMENSIONAL**
Cubic Units	**cu ft, cu in, cu m, cu cm, etc.**
	(or ft^3, in^3, m^3, cm^3, etc.)

Remember the 5 steps in solving word problems from **Section 1.10**:

STEP 1: ***IDENTIFY THE VARIABLE.***
STEP 2: ***WRITE THE EQUATION.***
STEP 3: ***SOLVE THE EQUATION.***
STEP 4: ***ANSWER THE QUESTION.***
STEP 5: ***CHECK.***

EXAMPLE 1. **The length of a rectangle is 3 times the width. If the area of the rectangle is 75 square centimeters, find the dimensions of the rectangle.**

STEP 1: **Let** x **=** width of rectangle
3 x **=** length of rectangle

STEP 2: **Eq: Width • Length = Area**
$x \cdot 3x = 75$

STEP 3: **Solve:** $3x^2 = 75$
$x^2 = 25$
$x = \pm 5$

STEP 4: **Answer:** Since -5 is meaningless for the dimensions of a rectangle, reject $x = -5$
However, x **= 5 cm** is an acceptable width; the length **3** x = **15 cm**.

The rectangle is **5 cm. by 15 cm.**

EXERCISES. **Complete the following exercises.**

1. **The length of a rectangle is 5 times the width. If the area of the rectangle is 245 square centimeters, find the dimensions of the rectangle.**

STEP 1: **Let** x **=** width of rectangle

______ = length of rectangle

STEP 2: **Eq: Width • Length = Area**

____

STEP 3: **Solve:**

STEP 4: **Answer:**

2. **The length of a rectangle is 3 more than the width. If the area of the rectangle is 10 square centimeters, find the dimensions of the rectangle.**

STEP 1: **Let** x **=** ________________

______ = ________________

STEP 2: **Eq: Width • Length = Area**

$x \bullet (\quad) =$ ______

STEP 3: **Solve:** $x^2 + 3x = 10$ (Quadratic: Set = 0)

(Factor!)

STEP 4: **Answer:**

3. **The length of a rectangle is 5 more than the width. If the area of the rectangle is 50 square meters, find the dimensions of the rectangle.**

EXAMPLE 2: **The base of a triangle is 3 more than twice the height. If the area of the triangle is 10 square centimeters, find the dimensions of the triangle.**

SOLUTION: **Let x = height of triangle**

$2x + 3$ = base of triangle

Equation: **$\frac{1}{2} \cdot bh$ = Area**

$\frac{1}{2} \cdot x(2x+3) = 10$ **Mult both sides of equation by 2**

$x(2x+3) = 20$ **Add -20 to both sides of equation**

$2x^2 + 3x - 20 = 0$

$(2x - 5)(x + 4) = 0$

$x = 5/2$; $x = -4$ Reject -4

$x = 5/2$ cm. height of triangle

$2x + 3 = 8$ cm. base of triangle

4. **The base of a triangle is 5 times the height. If the area of the triangle is 90 square feet, find the dimensions of the triangle.**

SOLUTION: **Let x =** height of triangle

_____ = base of triangle

Equation: **$\frac{1}{2} \cdot bh$ = Area**

5. **The base of a triangle is 3 less than twice the height. If the area of the triangle is 10 square centimeters, find the dimensions of the triangle.**

6. **The height of a parallelogram is 3 less than the base. If the area of the parallelogram is 40 square meters, find the dimensions of the parallelogram.**

SOLUTION: **Let x = __________**

_____ = __________

Equation: **b • h = Area**

7. **The base of a parallelogram is 3 more than twice the height. If the area is 35 square centimeters, find the dimensions of the parallelogram.**

8. **The base of a parallelogram is 3 less than twice the height. If the area is 35 square centimeters, find the dimensions of the parallelogram.**

EXAMPLE 4: **A rectangular box with height 3 centimeters has a volume of 24 cubic centimeters. The length of the box is two more than the width. Find the dimensions of the box.**

SOLUTION: **Let x = width of box**

$x + 2$ = length of box

3 = height of box

Equation: **L W H = Volume of box**

$$3x(x)(x+2) = 24$$

$$3x^2 + 6x - 24 = 0$$

$$3(x^2 + 2x - 8) = 0$$

$$3(x+4)(x-2) = 0$$

$x = -4$; $x = 2$ cm. Width of box; Reject $x=-4$

$x + 2 = 4$ cm. Length of box

3 cm. Height of box

Check: **2cm•4cm•3cm= 24 cu.cm. Volume of box**

9. **A rectangular box with height 4 meters has a volume of 576 cubic meters and a square base. Find the dimensions of the box.**

SOLUTION: **Let** x **= width of box**
x **= length of box**
4 = height of box

Equation: **L W H = Volume of box**

10. **A rectangular box with height 2 centimeters has a volume of 50 cubic centimeters. The length of the box is 5 more than twice the width. Find the dimensions of the box.**

SOLUTION: **Let** x **= _____ of box**
_____ = _____ of box
_____ = _____ of box

Equation:

11. **A rectangular box with height 2 centimeters has a volume of 50 cubic centimeters. The length of the box is 5 less than twice the width. Find the dimensions of the box.**

ANSWERS 2.11

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1. 7 cm, 35 cm; **2.** 2 cm, 5 cm; **3.** 5 cm, 10 cm; **4.** h=6 ft, b=30 ft; **5.** h=4 cm, b=5 cm;

6. b=8 m, h=5 m; **7.** h=7/2 cm, b=10 cm; **8.** h=5 cm, b=7 cm; **9.** 12 m, 12 m, 4 m;

10. 5/2 cm, 10 cm, 2 cm; **11.** 5 cm, 5 cm, 2 cm.