# 2.12 *Laws of Exponents: Positive Exponents*

***Dr. Robert J. Rapalje, Retired***
***Central Florida, USA***

The use of exponents (i.e., raising to powers) was introduced in the section on order of operations. As you already know, a number raised to a power is a shorthand notation for multiplication. For example, $\mathbf{2^5}$ really means **2·2·2·2·2**. In this notation, the **2 is called the base number**, and the **5 is called the exponent or the power.**

As you use the notation of exponents, you will discover certain patterns and rules. These are the **laws of exponents**, which will be summarized at the end of the section. In these exercises, try to see the pattern and predict the rule that is being developed.

## Multiplication with the Same Base Number

The first rule involves multiplication of expressions with powers. What does it mean if you multiply numbers that have the same base number? Look for the answer to this question as you study the examples and work the exercises that follow.

**EXAMPLE 1.** $2^2 \cdot 2^3$

**Solution:**

$(2{\cdot}2){\cdot}(2{\cdot}2{\cdot}2)$

$2{\cdot}2{\cdot}2{\cdot}2{\cdot}2$

$2^5$

Check: $2^2\ 2^3 = 2^5$

$4 \cdot 8 = 32$

**EXAMPLE 2.** $x^4 \cdot x^7$

**Solution:**

$(x{\cdot}x{\cdot}x{\cdot}x){\cdot}(x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x)$

$x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x{\cdot}x$

$x^{11}$

**EXERCISES.** Simplify the expressions. Do you see the pattern? Use your rule as a shortcut.

1. $2^3 \cdot 2^4$
   $(2{\cdot}2{\cdot}2){\cdot}(2{\cdot}2{\cdot}2{\cdot}2)$
   How many 2s?_____
   Answer:
2. $x^5 \cdot x^3$
3. $x^4 \cdot x^5$
4. $x^{11} \cdot x^{17}$

5. $5^2 \cdot 5^3$ 6. $3^1 \cdot 3^3$ 7. $2^5 \cdot 2^3$ 8. $\boldsymbol{x}^4 \cdot \boldsymbol{x}^7$

9. $a^4 \cdot a^6$ 10. $b^7 \cdot b^3$ 11. $c^8 \cdot c^6$ 12. $\boldsymbol{x}^{42} \cdot \boldsymbol{x}^{79}$

13. When you multiply with the same base number, you write down the a) ________________ and b)___________________ the exponents.

**GENERALIZATION:**

**When you multiply with the same base number, you ADD exponents!**

$$x^m \bullet x^n = x^{m+n}$$

## Division with the Same Base Number

The second rule involves division of expressions with powers. What does it mean if you divide numbers that have the same base number? Try to discover the answer to this question in the examples and exercises that follow.

**EXAMPLE 3.** $\frac{2^5}{2^3}$ (or $\frac{32}{8}$)

**Solution:** $\frac{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2 \bullet 2}$

**EXAMPLE 4.** $\frac{x^7}{x^4}$

**Solution:** $\frac{x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x \bullet x}$

When you divide out denominator factors with numerator factors, you are left with

2·2 (or 4). $\boldsymbol{x \cdot x \cdot x}$ (or $\boldsymbol{x}^3$).

Do you see a shortcut that has to do with the exponents?

14. If when you **multiply** you **add** exponents, then when you **divide** you ______________ the exponents.

**EXERCISES.** Express each of the following as a base number raised to a power and calculate the answer. You may want to check the numerical problems with a calculator.

15. $\frac{2^7}{2^4}$ 16. $\frac{3^5}{3^3}$ 17. $\frac{4^5}{4^3}$ 18. $\frac{2^{10}}{2^5}$

In 19 - 34, give answer as a power.

19. $\frac{x^7}{x^5}$ 20. $\frac{x^{10}}{x^5}$ 21. $\frac{x^6}{x^2}$ 22. $\frac{y^{16}}{y^4}$

23. $\frac{31^{64}}{31^8}$ 24. $\frac{a^{12}}{a^6}$ 25. $\frac{b^9}{b^3}$ 26. $\frac{c^{14}}{c^2}$

27. When you divide with the same base number, you write down the a)__________________ and b)_____________________ the exponents.

**GENERALIZATION:**

**When you divide (same base number), you SUBTRACT exponents!**

$$\frac{x^m}{x^n} = x^{m-n}$$

## Raising a Power to a Power

Consider the following examples:

**EXAMPLE 5.** $(2^5)^2$ (or $32^2$ or 1024)

**Solution:** This means $(2^5)\bullet(2^5)$

$2^{10}$ (or 1024)

**EXAMPLE 6.** $(x^2)^3$

**Solution:** This means $(x^2)\bullet(x^2)\bullet(x^2)$

$x^6$

Do you see a shortcut that has to do with the exponents?

28. When you **raise a power to a power,** you ______________________ exponents.

**EXERCISES.** **Express each of the following as a number or variable raised to a power.**

29. $(3^3)^2$
30. $(2^4)^3$
31. $(x^2)^5$
32. $(y^3)^5$
33. $(x^7)^4$
34. $(x^6)^2$
35. $(y^4)^2$
36. $(z^9)^2$
37. $(10^3)^2$
38. $(a^7)^2$
39. $(b^2)^8$
40. $(c^5)^2$

**GENERALIZATION:**
**When you raise a power to a power, you MULTIPLY exponents!**

$$(x^m)^n = x^{m\,n}$$

## Combined Operations

**EXAMPLE 8.** $(2^3)^2 \bullet 2^4$

**Solution:** $2^6 \bullet 2^4$

$2^{10}$

**EXAMPLE 9.** $\dfrac{(x^5)^2}{x^7}$

**Solution:** $\dfrac{x^{10}}{x^7}$

$x^3$

**EXAMPLE 10.** $\dfrac{y^2 \bullet y^3 \bullet y^6}{y^8 \bullet y}$

**Solution:** $\dfrac{y^{11}}{y^9}$

$y^2$

**EXAMPLE 11.** $\dfrac{(b^5)^3 \bullet b^4}{b^{10} \bullet b^7}$

**Solution:** $\dfrac{b^{15} \bullet b^4}{b^{17}}$

$\dfrac{b^{19}}{b^{17}}$

$b^2$

**EXERCISES.** **Simplify each of the following.**

41. $z^2 \bullet z^4$

42. $a \bullet a^3$

43. $(p^3)^2$

44. $\dfrac{q^{10}}{q^2}$

45. $\dfrac{x^6}{x}$

46. $(y^2)^5$

47. $\dfrac{x^{12}}{x^6}$

48. $x \bullet x^5$

49. $\dfrac{3^{10} \bullet 3^2}{3^8}$

50. $\dfrac{4^4 \bullet 4^3}{4^2}$

51. $(x^5)^2 \bullet x^7$

52. $(2^4)^3 \bullet 2^2$

53. $\dfrac{(x^4)^3}{x^2}$

54. $(x^3)^2 \bullet (x^4)^3$

55. $\dfrac{(b^3)^3}{(b^4)^2}$

56. $\dfrac{(3^2)^4}{3^4 \bullet 3}$

57. $\dfrac{(x^5)^2}{x^7}$

58. $\dfrac{(y^4)^3}{y^2}$

59. $(x^4)^3 \bullet (x^2)^4$

60. $\dfrac{(x^6)^4}{(x^6)^3}$

Sometimes the expression inside parentheses is a product or quotient that cannot be simplified. There is a special rule for products and quotients that are raised to powers. What is the meaning, for example, of

$(3x)^4$ or $\left(\dfrac{3}{x}\right)^4$ ?

**EXAMPLE 11.** Remove the parentheses: $(3x)^4$

**Solution:** $(3x)^4$ actually means (3x)·(3x)·(3x)·(3x), which can be re-written as

$$3\cdot 3\cdot 3\cdot 3\cdot x\cdot x\cdot x\cdot x$$

or **$3^4x^4$ or $81x^4$.**

**EXAMPLE 12.** Remove the parentheses: $\left(\dfrac{3}{x}\right)^4$

**Solution:** $\left(\dfrac{3}{x}\right)^4$ actually means $\left(\dfrac{3}{x}\right)\bullet\left(\dfrac{3}{x}\right)\bullet\left(\dfrac{3}{x}\right)\bullet\left(\dfrac{3}{x}\right)$ which is the same as

$$\frac{3^4}{x^4} \text{ or } \frac{81}{x^4}.$$

**EXERCISES.** Remove the parentheses and simplify if possible.

61. $(3x)^3$
62. $(4y)^3$
63. $(5x)^2$
64. $(2z)^4$
65. $(2x^3)^4$
66. $(5y^4)^3$
67. $(x^2y^3)^6$
68. $(3x^4y^5)^2$

69. $\left(\frac{x^2}{y^3}\right)^4$

70. $\left(\frac{2x}{y}\right)^3$

71. $\left(\frac{5}{x^5y^3}\right)^2$

72. $\left(\frac{2y^2}{4x^3}\right)^3$

73. $\left(\frac{2x^2}{y^4}\right)^3$

74. $\left(\frac{2x^3}{5y^2}\right)^3$

75. $\left(\frac{5x^2}{3y^3}\right)^2$

76. $\left(\frac{7x^5}{9y^4}\right)^2$

# SUMMARY

| LAWS OF (POSITIVE) EXPONENTS | GENERALIZATION |
|---|---|
| **1.** When you multiply (with the same base number), you add exponents. | $x^m \bullet x^n = x^{m+n}$ |
| **2.** When you divide (with the same base number), you subtract exponents. | $\frac{x^m}{x^n} = x^{m-n}$ |
| **3.** When you raise a power to a power, you multiply exponents. | $(x^m)^n = x^{mn}$ |
| **4.** When a product or a quotient is raised to a power, you raise each factor to the power. | $(xy)^m = x^m y^m$<br>$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ |

# ANSWERS 2.12

**p. 207-213:**

**1.** $2^7$; **2.** $x^8$; **3.** $x^9$; **4.** $x^{28}$; **5.** $5^5$; **6.** $3^4$ or 81; **7.** $2^8$; **8.** $x^{11}$; **9.** $a^{10}$; **10.** $b^{10}$; **11.** $c^{14}$; **12.** $x^{121}$; **13a)** base number, **b)** add; **14.** subtract; **15.** $2^3$ or 8; **16.** $3^2$ or 9; **17.** $4^2$ or 16; **18.** $2^5$ or 32; **19.** $x^2$; **20.** $x^5$; **21.** $x^4$; **22.** $y^{12}$; **23.** $31^{56}$; **24.** $a^6$; **25.** $b^6$; **26.** $c^{12}$; **27a)** base number, **b)** subtract; **28.** multiply; **29.** $3^6$; **30.** $2^{12}$; **31.** $x^{10}$; **32.** $y^{15}$; **33.** $x^{28}$; **34.** $x^{12}$; **35.** $y^8$; **36.** $z^{18}$; **37.** $10^6$; **38.** $a^{14}$; **39.** $b^{16}$; **40.** $c^{10}$; **41.** $z^6$; **42.** $a^4$; **43.** $p^6$; **44.** $q^8$; **45.** $x^5$; **46.** $y^{10}$; **47.** $x^6$; **48.** $x^6$; **49.** $3^4$ or 81; **50.** $4^5$; **51.** $x^{17}$; **52.** $2^{14}$; **53.** $x^{10}$; **54.** $x^{18}$; **55.** b; **56.** $3^3$ or 27; **57.** $x^3$; **58.** $y^{10}$; **59.** $x^{20}$; **60.** $x^6$; **61.** $27x^3$; **62.** $64y^3$; **63.** $25x^2$;

**64.** $16z^4$; **65.** $16x^{12}$; **66.** $125y^{12}$; **67.** $x^{12}y^{18}$; **68.** $9x^8y^{10}$; **69.** $\frac{x^8}{y^{12}}$; **70.** $\frac{8x^3}{y^3}$;

**71.** $\frac{25}{x^{10}y^6}$; **72.** $\frac{27y^6}{64x^9}$; **73.** $\frac{8x^6}{y^{12}}$; **74.** $\frac{8x^9}{125y^6}$; **75.** $\frac{25x^4}{9y^6}$; **76.** $\frac{49x^{10}}{81y^8}$.