2.14 *Scientific Notation*

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When working with very large (astronomical!) numbers or very small (microscopic!) numbers, it is often convenient to write the numbers in **scientific notation.** **In scientific notation, the number must be expressed as a number between 1 and 9.99... times a power of 10**. Numbers with magnitude **10 or greater** will be expressed with a **positive power of 10**, while numbers with magnitude **smaller than 1** must be expressed with a **negative power of 10**.

Before using the calculator to compute with scientific notation, first it may be helpful to review the concept and make sure you understand how to convert from **standard decimal form** to **scientific notation** and back, using the calculator in your head! Your calculator may or may not convert from one to the other (this is something you can easily do in your head!), but it will certainly perform calculations and sometimes give you answers in scientific notation. First, the "non-calculator" explanation, for those times in which you do not have a calculator in hand and for those times, like certain exams, in which calculators are not allowed! By the way, converting from standard decimal form to scientific notation (and vice-versa) is so simple you hardly need a calculator to do it! Just count the number of places you must move the decimal to obtain a number between 1 and 10. Numbers larger than (astronomical!) 10 have a positive power of 10. Small decimal numbers (microbiology!) must have a negative power of 10. Study the following examples.

EXAMPLES:

STANDARD DECIMAL FORM		SCIENTIFIC NOTATION
1. 300	**= 3.0 × 100**	$= 3.0 \times 10^{2}$
2. 3000	**= 3.0 × 1000**	$= 3.0 \times 10^{3}$
3. 3250	**= 3.25 × 1000**	$= 3.25 \times 10^{3}$
4. 32,500,000	move decimal 7 places left	$= 3.25 \times 10^{7}$
5. 0.03	**= 3.0 × 1/100**	$= 3.0 \times 10^{-2}$
6. 0.0003	**= 3.0 × 1/10000**	$= 3.0 \times 10^{-4}$
7. 0.0000000325	move 8 places right	$= 3.25 \times 10^{-8}$
8. 0.00000000032	move 10 places right	$= 3.2 \times 10^{-10}$

EXERCISES: Express in scientific notation.

1. 450 = ______________________
2. 7500 = ______________________
3. 12,000,000 = ________________
4. 720,000,000,000 = _____________
5. 0.00325 = ___________________
6. 0.000000246 = ________________
7. 0.00000000325 = ______________
8. 0.00000000436 = _______________
9. 480,000,000 = ________________
10. 93,200,000,000 = ______________
11. 0.0000876 = _________________
12. 0.0000122 = __________________

Continuing with the non-calculator explanation, sometimes scientific notation can be a shortcut in calculation, especially if you do not have a calculator. However, calculators are really great later!

EXAMPLE 9. $2{,}000{,}000 \times 3{,}000{,}000{,}000$

Solution: First, change to scientific notation.

$2 \times 10^6 \times 3 \times 10^9$ Multiply 2 times 3 and add the exponents 6 plus 9.

$\mathbf{6 \times 10^{15}}$

EXAMPLE 10. $4{,}000{,}000 \times 3{,}000{,}000{,}000$

Solution: First, change to scientific notation.

$4 \times 10^6 \times 3 \times 10^9$ Multiply 4 times 3 and add the exponents 6 plus 9.

12×10^{15} Change 12 to scientific notation: 1.2×10^1

$1.2 \times 10^1 \times 10^{15}$

$\mathbf{1.2 \times 10^{16}}$

EXAMPLE 11. $0.0000032 \times 70{,}000{,}000{,}000$

Solution: First, change to scientific notation.

$3.2 \times 10^{-6} \times 7 \times 10^{10}$ Multiply 3.2 times 7 and add the exponents -6 plus 10.

22.4×10^4 Change 22.4 to scientific notation: 2.24×10^1

$2.24 \times 10^1 \times 10^4$

$\mathbf{2.24 \times 10^5}$ or **224,000**

EXAMPLE 12. $\dfrac{6{,}000{,}000}{0.00003}$

Solution: First, change to scientific notation.

$\dfrac{6 \times 10^{6}}{3 \times 10^{-5}}$ Divide 6 by 3, and subtract the exponents 6 minus (-5).

$\mathbf{2 \times 10^{11}}$

EXAMPLE 13. $\dfrac{0.00006}{40{,}000{,}000}$

Solution: First, change to scientific notation.

$\dfrac{6 \times 10^{-5}}{4 \times 10^{7}}$ Divide 6 by 4, and subtract the exponents -5 minus 7.

$\mathbf{1.5 \times 10^{-12}}$

EXAMPLE 14. $\dfrac{3{,}000{,}000}{0.006}$

Solution: First, change to scientific notation.

$\dfrac{3 \times 10^{6}}{6 \times 10^{-3}}$ Divide 3 by 6, and subtract the exponents 6 minus (-3).

0.5×10^{9} Change 0.5 to scientific notation: 5×10^{-1}

$5 \times 10^{-1} \times 10^{9}$

$\mathbf{5 \times 10^{8}}$

Perhaps you noticed that all the numbers in the preceding examples were carefully "arranged" (mathematicians like the word "contrived"!) so everything came out even, and the arithmetic was easy even without a calculator. Of course, in the real world, life does NOT usually come out even, and neither does the math. Not to worry, however!! With most any calculator, you can easily handle such calculations, whether the numbers come out even or not. Calculators will really be nice, but first here are a few exercises to try without calculators.

EXERCISES. **Perform the calculations without using a calculator by converting to scientific notation.**

13. $20{,}000 \times 3{,}000{,}000$

14. $25{,}000{,}000 \times 300{,}000{,}000$

15. $400{,}000{,}000 \times 0.000008$

16. 0.0000006×0.0004

17. $\dfrac{0.00006}{20{,}000{,}000}$

18. $\dfrac{80{,}000}{0.00002}$

19. $\dfrac{0.000096}{0.000000008}$

20. $\dfrac{840{,}000}{0.000042}$

21. $\dfrac{150{,}000}{0.00006}$

22. $\dfrac{0.00006}{0.0000008}$

USING YOUR CALCULATOR

Your calculator may or may not be able to convert from standard decimal form to scientific notation and vice versa. For some calculators (certain Casios and the TI-85/86), you use the **[MODE]** button, and follow the instructions in your manual. For other calculators (including the TI-30 and other Texas Instrument models) there are separate functions called **[SCI]** and **[FLO]**. **FLO** means floating point, which is a decimal format. The details of how different calculators convert from scientific notation to standard decimal form vary greatly. However, when it comes to using them to calculate problems in this section, they are remarkably similar.

EXAMPLE 15. Use your calculator to compute 6,000 × 7,000. Express it in scientific notation.

Solution: Just enter [6000] [x] [7000] [= or ENTER]. The calculator gives you 42000000. Now you can either change it to scientific notation yourself or use the calculator. To do it yourself, just move the decimal to the left to write 4.2. Since you moved the decimal 7 places to the left, this means the answer is $\mathbf{4.2 \times 10^7}$.

TI-30 Note: If your calculator says 42000000, look for and press [2^{nd}] [Sci]. The calculator will convert automatically to scientific notation and give you 4.2 ^{7}. Of course this doesn't mean 4.2 raised to the 7^{th} power. You have to write it in its correct form: $\mathbf{4.2 \times 10^7}$. **NOTE: Some TI-30s behave as the TI83/84 described below!!**

TI83/84 Note: If your calculator says 42000000, find **[MODE]**. Press **[MODE]**, and a rather impressive MODE menu opens up on the calculator. In the top row of this menu, it says **[Normal]**, **[Sci]** (for Scientific Mode), and **[Eng]** (for Engineering Mode). Press the **[Right arrow]** key, and the cursor moves from [Normal] to [Sci]. Press **[ENTER]** to lock it in; press **[CLEAR]** to get out of this menu; press **[ENTER]**, and the calculator converts the last calculation to scientific notation, **4.2 E 7**. Of course, the final answer is $\mathbf{4.2 \times 10^7}$.

Now, as a follow up, **calculate 3 x 4 [ENTER].** The calculator gives you **1.2 E1**, which means $\mathbf{1.2 \times 10^1}$. The answer you were looking for was **12**--obviously, the calculator is still in scientific notation mode. To change it back, press **[MODE] [ENTER] [CLEAR] [ENTER].** The calculator returns to Normal Mode, and it should give you the answer of **12**.

TI-83/84 Summary of Keystrokes

Given a number on the calculator in standard decimal form to convert to scientific notation, press **[MODE] [Right arrow] [ENTER] [CLEAR] [ENTER]**.

To convert back to standard decimal, press **[MODE] [ENTER] [CLEAR] [ENTER]**.

EXAMPLE 16. **Use your calculator to compute 6,000,000 × 7,000,000. Express it in scientific notation.**

Solution: Just enter [6000000] [×] [7000000] [= or ENTER]. The calculator automatically gives you scientific notation, since it can't doesn't have enough digits to display the standard decimal form. Depending upon the type of calculator, you will automatically get something like **4.2** ^{13}, **4.2 E 13,** or **4.2 13**. What these mean, and the way you must write the final answer is $\mathbf{4.2 \times 10^{13}}$.

EXERCISES. **Use your calculator to calculate. Express standard decimal or scientific notation.**

23. 20,000 × 3,000,000

24. 25,000,000 × 300,000,000

25. 400,000,000 × 0.000008

26. 0.0000006 × 0.0004

27. $\dfrac{0.00006}{20{,}000{,}000}$

28. $\dfrac{80{,}000}{0.00002}$

29. $\dfrac{0.000096}{0.000000008}$

30. $\dfrac{840{,}000}{0.000042}$

31. $\dfrac{150{,}000}{0.00006}$

32. $\dfrac{0.00006}{0.0000008}$

Sometimes the problem is given in scientific notation. If this is the case, then look for the **[EE]** or the **[EXP]** button on your calculator. **For the TI83/84, this is [2nd] [EE]**. This **EE (or EXP) function** makes **scientific notation** very easy to enter. If you have an **EE or EXP key** on your calculator, to enter $\mathbf{7.2 \times 10^{12}}$, simply enter **[7.2] TI83/84 [2nd] [EE (or EXP)] [12] [ENTER]**. It doesn't get any easier than that! ***CAUTION: DO <u>NOT</u> ENTER* [7.2] [×] [EE], etc.**

EXAMPLE 17. **Use your calculator to find the value of** $\dfrac{7.5 \text{ x } 10^{6}}{1.5 \text{ x } 10^{-2}}$.

Solution: Enter [7.5] **TI83/84 [2nd]** [EE or EXP] [6] [÷] [1.5] [EE or EXP] [+/- or (-)] [2] [ENTER]. **(Note: To enter -2 exponent, on some calculators you must enter [2], then [+/-].)** The calculator should give you $\mathbf{5 \times 10^{8}}$.

You can check this without using a calculator by dividing 7.5 by 1.5 which is 5. Then subtract exponents 6 - (-2) to get 8, which is the power of 10.

EXERCISES. **Use your calculator to find the value of each of the following.**

33. $7.2 \text{ x } 10^{12} \bullet 6.3 \text{ x } 10^{-8} =$
[7.2] **TI83/84 [2nd]** [EE] [12] **[x]**
[6.3] **TI83/84 [2nd]** [EE] [+/-] [8] [=]

34. $3.5 \text{ x } 10^{-4} \bullet 8.1 \text{ x } 10^{14} =$

35. $8.42 \text{ x } 10^{13} \bullet 5.8 \text{ x } 10^{8} =$

36. $8.42 \text{ x } 10^{-13} \bullet 5.8 \text{ x } 10^{-8} =$

37. $\dfrac{7.2 \text{ x } 10^{12}}{3.5 \text{ x } 10^{-4}} =$

38. $\dfrac{6.3 \text{ x } 10^{-8}}{8.1 \text{ x } 10^{14}} =$

39. $\dfrac{7.2 \text{ x } 10^{12}}{8.1 \text{ x } 10^{-14}} =$

40. $\dfrac{7.2 \text{ x } 10^{-12}}{8.1 \text{ x } 10^{-14}} =$

41. $\dfrac{10^{12}}{8 \times 10^{-4}}$ **[HINT: 10^{12} may be entered as either 1 EE 12 or 10 ^ 12, NOT 10 EE 12]**

42. $\dfrac{10^{-12}}{4 \times 10^{-4}}$

43. $\dfrac{10^{-6}}{6 \times 10^{-10}}$

COMBINED OPERATIONS

Remember, when numerators and denominators have more than one term or factor, you must use parentheses around the entire numerator and the entire denominator. (See Section 1.04, p. 26.)

EXAMPLE 18. Calculate: $\dfrac{7.2 \text{ X } 10^{12} \bullet 6.3 \text{ X } 10^{-8}}{3.5 \text{ X } 10^{-4} \bullet 8.1 \text{ X } 10^{14}}$

Solution: [( ] [7.2] **TI83/84 [2nd]** [EE] [12] **[x]** [6.3] [EE] **TI83/84 [2nd]** [+/- or (-)] [8] **[ )]** **[÷]**
[( ] [3.5] **TI83/84 [2nd]** [EE] [+/- or (-)] [4] **[x]** [8.1] **TI83/84 [2nd]** [EE] [14] **[ )]** [=]
The answer should be **1.6×10^{-6}**.

44. $\dfrac{6.3 \text{ X } 10^{-23} \bullet 9.5 \text{ X } 10^{4}}{7.5 \text{ X } 10^{12} \bullet 4.3 \text{ X } 10^{-3}} =$

45. $\dfrac{6.3 \text{ X } 10^{23} \bullet 9.5 \text{ X } 10^{-4}}{7.5 \text{ X } 10^{-12} \bullet 4.3 \text{ X } 10^{3}} =$

46. $\dfrac{6.3 \text{ X } 10^{-23} \bullet 9.5 \text{ X } 10^{-4}}{7.5 \text{ X } 10^{-12} \bullet 4.3 \text{ X } 10^{-3}} =$

ANSWERS 2.14

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1. 4.5×10^{2}; **2.** 7.5×10^{3}; **3.** 1.2×10^{7}; **4.** 7.2×10^{11}; **5.** 3.25×10^{-3}; **6.** 2.46×10^{-7}; **7.** 3.25×10^{-9}; **8.** 4.36×10^{-9}; **9.** 4.8×10^{8}; **10.** 9.32×10^{10}; **11.** 8.76×10^{-5}; **12.** 1.22×10^{-5}; **13.** 6×10^{10}; **14.** 7.5×10^{15}; **15.** 3.2×10^{3} or 3200 **16.** 2.4×10^{-10}; **17.** 3×10^{-12}; **18.** 4×10^{9}; **19.** 1.2×10^{4}; **20.** 2×10^{10} **21.** 2.5×10^{9}; **22.** 7.5×10^{1} or 75; **23.** 6×10^{10}; **24.** 7.5×10^{15}; **25.** 3.2×10^{3} or 3200 **26.** 2.4×10^{-10}; **27.** 3×10^{-12}; **28.** 4×10^{9}; **29.** 1.2×10^{4}; **30.** 2×10^{10} **31.** 2.5×10^{9}; **32.** 7.5×10^{1} or 75; **33.** 4.54×10^{5}; **34.** 2.84×10^{11}; **35.** 4.88×10^{22}; **36.** 4.88×10^{-20}; **37.** 2.06×10^{16}; **38.** 7.78×10^{-23}; **39.** 8.89×10^{25}; **40.** 8.89×10^{1} or 88.89; **41.** 1.25×10^{15}; **42.** 2.5×10^{-9}; **43.** 1.67×10^{3} or 1666.67; **44.** 1.86×10^{-28}; **45.** 1.86×10^{28}; **46.** 1.86×10^{-12}.