

3.01 Reducing Fractions

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Before beginning Chapter 3 on **FRACTIONS** be sure to have both of your calculators turned on--that is, the calculator in your hand, and the calculator between your ears! This chapter on fractions provides an excellent opportunity to use the hand-held calculator to solve problems and check your answers, especially if you have a calculator that works with fractions and converts decimals to fractions. It also gives you the opportunity to develop and sharpen your mental skills by using your "head-calculator." It is not a question of whether or not to use the hand-held calculator, or of which calculator to use. You really need to use **both!**

In this chapter there will be five types of fractions problems: reducing fractions, multiplying and dividing fractions, adding and subtracting fractions, solving fractional equations, and a few easy applications that (believe it or not!) you will really like.

A fraction can be “**reduced**” whenever a **factor** can be found that divides evenly into both the **numerator** and **denominator** of the fraction. For example, the fraction 8/12 can be reduced because **4** is a factor of the numerator and also the denominator. Dividing the numerator and denominator by 4 reduces the fraction to 2/3.

PRINCIPLE: If the numerator and denominator of a fraction can be multiplied or divided by the same (non-zero) number, then the resulting fraction is equivalent.

FORMAL STATEMENT: $\frac{a \bullet c}{b \bullet c} = \frac{a}{b}, b \neq 0, c \neq 0$

INTERPRETATION: In reducing fractions, you will be dividing the numerators and denominators by factors. It is often helpful to factor the numerator and denominator to see what factors are common to both.

EXAMPLE 1. Reduce the fraction $\frac{126}{216}$ completely without using a calculator.

Solution: Notice that both numerator and denominator are even numbers, and therefore divisible by 2. Dividing 126 and 216 each by 2, you get $\frac{63}{108}$. Next, notice that the numerator 63 and denominator 108 are each divisible by 9. Dividing 63 and 108 each by 9, you get $\frac{7}{12}$. Since there are no common factors that divide both 7 and 12, it is then reduced completely. The final answer is $\frac{7}{12}$.

EXAMPLE 2. Reduce the fraction $\frac{126}{216}$ completely using a calculator.

Solution #1: First, enter the numerator 126 divided by the denominator, and press [ENTER] or [=]. This changes the fraction to a decimal 0.583333333 . . . Most calculators can convert this decimal to a fraction, and the fraction will be reduced to lowest terms. For a TI 83/84, press [MATH] [ENTER] [ENTER]. For a TI 30, press [2^{nd}] [F \leftrightarrow D]. The calculator should give you for a final answer, $\frac{7}{12}$.

Solution #2: If your calculator has an [a b/c] button (TI 30 and several other brands and models of calculators), then press [126] [a b/c] [216] [=]. The calculator will give you $7 \downarrow 12$, which means $\frac{7}{12}$.

EXERCISES. In each of the following, reduce the fractions completely. As in the examples above, you reduce the fractions with or without a calculator, or use both methods. If you use a calculator, of course you will not need the hints nor the extra blanks that are provided.

1. $\frac{75}{100} = \underline{\hspace{2cm}}$ Divide numerator and denominator by 25.

2. $\frac{35}{75} = \underline{\hspace{2cm}}$ Divide numerator and denominator by 5.

3. $\frac{34}{51} = \underline{\hspace{2cm}}$ Divide numerator and denominator by 17.

4. $\frac{30}{135} = \underline{\hspace{2cm}}$ Divide numerator and denominator by 15.

5. $\frac{38}{95} = \underline{\hspace{2cm}}$ Divide numerator and denominator by $\underline{\hspace{1cm}}$.

6. $\frac{58}{87} = \underline{\hspace{2cm}}$ Divide numerator and denominator by $\underline{\hspace{1cm}}$.

7. $\frac{112}{140} = \underline{\hspace{2cm}}$ Divide numerator and denominator by $\underline{\hspace{1cm}}$.

8. $\frac{140}{112} = \underline{\hspace{2cm}}$ Divide numerator and denominator by $\underline{\hspace{1cm}}$.

9. $\frac{95}{38} = \underline{\hspace{2cm}}$ Divide numerator and denominator by $\underline{\hspace{1cm}}$.

10. $\frac{234}{387} = \underline{\hspace{2cm}}$ Divide numerator and denominator by $\underline{\hspace{1cm}}$.

11. $\frac{9000}{40500} = \underline{\hspace{2cm}}$ Divide numerator and denominator by whatever factors you can
= $\underline{\hspace{2cm}}$ until it is reduced completely. [Hint: one to three steps!]
= $\underline{\hspace{2cm}}$

12. $\frac{3240}{13320} = \underline{\hspace{2cm}}$ Divide numerator and denominator by whatever factors you can
= $\underline{\hspace{2cm}}$ until it is reduced completely. [Hint: one to three steps!]
= $\underline{\hspace{2cm}}$

13. $\frac{306}{1071} = \underline{\hspace{2cm}}$ Divide numerator and denominator by whatever factors you can
= $\underline{\hspace{2cm}}$ until it is reduced completely. [Hint: one to three steps!]
= $\underline{\hspace{2cm}}$

14. $\frac{5400}{13500} = \underline{\hspace{2cm}}$ Divide numerator and denominator by whatever factors you can
= $\underline{\hspace{2cm}}$ until it is reduced completely. [Hint: one to three steps!]
= $\underline{\hspace{2cm}}$

15. $\frac{91x^3y^2}{49x^2y} =$ _____ Divide numerator and denominator by $7x^2y$.

[Remember: when you divide with variables, you subtract exponents!]

16. $\frac{34x^5y^8}{17x^3y^2} =$ _____ Divide numerator and denominator by _____.

17. $\frac{98x^2y^8}{14x^6y^6} =$ _____ .

Divide out 14, x^2 , and y^6 .

Avoid negative exponents!

18. $\frac{105x^3y^5}{10x^7y^3} =$ _____ .

19. $\frac{26x^8y^4}{39x^4y^{12}} =$ _____ .

20. $\frac{77x^9y^5}{121x^4y^4} =$ _____ .

21. $\frac{48x^3y^3}{32x^5y^6} =$ _____ .

22. $\frac{39x^4y^4}{117x^9y^{12}} =$ _____ .

23. $\frac{4(x-5)}{(x-5)(x+5)} =$ _____ Divide numerator and denominator by $(x-5)$.

24. $\frac{(x-y)(x+y)}{9(x+y)} =$ _____ Divide numerator and denominator by $(x+y)$.

25. $\frac{9(x-6)}{12(x-6)(x+3)} =$ _____ Divide numerator and denominator by $3(x-6)$.

26. $\frac{40(x-4)(x+10)}{75(x+10)(x-8)} =$ _____ Divide numerator and denominator by $5(x+10)$.

[Notice that in these exercises, only **FACTORS** (not **TERMS**!) can be divided out!]

$$27. \frac{9(x-6)(x+6)}{54(x+2)(x-6)} = \underline{\hspace{2cm}} \quad 28. \frac{5(x-5)(x+8)}{75(x+8)(x+5)} = \underline{\hspace{2cm}}$$

$$29. \frac{24(x+3)(x-8)}{16(x+3)(x+8)} = \underline{\hspace{2cm}} \quad 30. \frac{140(x-3y)(x+y)}{49(x+y)(x+3y)} = \underline{\hspace{2cm}}$$

In the next exercises, remember you must factor the numerator and denominator (if possible!) first. Divide out only factors that are common to the numerator and denominator.

$$31. \frac{x^2-4}{16x+32} = \frac{(\quad)(\quad)}{16(\quad)} \quad 32. \frac{x^2-9}{9x-27} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$33. \frac{x^2-4x}{x^2-16} = \frac{x(\quad)}{(\quad)(\quad)} \quad 34. \frac{x^2-9x}{x^2-81} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$35. \frac{x-8}{x^2-64} = \underline{\hspace{2cm}} \quad 36. \frac{x^2-25}{x-5} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$37. \frac{x^2-5x+4}{x^2-3x-4} = \frac{(\quad)(\quad)}{(\quad)(\quad)} \quad 38. \frac{x^2+7x+6}{x^2+12x+36} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$39. \frac{x^2 - 49}{x^2 - 14x + 49} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$40. \frac{x^2 + 8x + 16}{x^2 - 16} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$41. \frac{8x^3 - 8x^2}{x^2 - 1} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$42. \frac{8x^2 - 16x}{x^2 - 4x + 4} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$43. \frac{x^2 - 16}{x^2 - 8x + 16} = \underline{\hspace{2cm}}$$

$$44. \frac{x^2 - 25}{x^2 - 4x - 5} = \underline{\hspace{2cm}}$$

45a) The negative of x is _____.

b) The negative of y is _____.

c) The negative of $x - y$ is $-(x - y)$ or _____ or _____.

d) The negative of $y - x$ is $-(y - x)$ or _____ or _____.

e) The negative of $a - b$ is $-(a - b)$ or _____ or _____.

f) The negative of $b - a$ is $-(b - a)$ or _____ or _____.

g) If you take the negative of $6 - 3$ (which is $+3$), you could say $3 - 6$ or _____.

46a) When any number is divided by its negative, what is the result? _____

Examples: $\frac{3}{-3}$, $\frac{-3}{3}$, $\frac{6}{-6}$, $\frac{-6}{6}$, $\frac{12}{-12}$, etc. Answer always = _____.

b) $\frac{x}{-x} = \underline{\hspace{2cm}}$. c) $\frac{y}{-y} = \underline{\hspace{2cm}}$. d) $\frac{-a}{a} = \underline{\hspace{2cm}}$. e) $\frac{-b}{b} = \underline{\hspace{2cm}}$.

$$47. \frac{10 - 3}{3 - 10} = \underline{\hspace{2cm}}$$

$$48. \frac{x - 3}{3 - x} = \underline{\hspace{2cm}}$$

$$49. \frac{x + 3}{3 + x} = \underline{\hspace{2cm}}$$

$$50. \frac{5 - y}{y - 5} = \underline{\hspace{2cm}}$$

$$51. \frac{x - y}{y - x} = \underline{\hspace{2cm}}$$

$$52. \frac{-x + y}{y - x} = \underline{\hspace{2cm}}$$

ANSWERS 3.01

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1. $3/4$; 2. $7/15$; 3. $2/3$; 4. $2/9$; 5. $2/5$; 6. $2/3$; 7. $4/5$; 8. $5/4$ or $1\ 1/4$; 9. $5/2$ or $2\ 1/2$; 10. $26/43$;

11. $2/9$; 12. $9/37$; 13. $2/7$; 14. $2/5$; 15. $\frac{13XY}{7}$; 16. $2X^2Y^6$; 17. $\frac{7Y^2}{X^4}$; 18. $\frac{21Y^2}{2X^4}$;

19. $\frac{2X^4}{3Y^8}$; 20. $\frac{7X^5Y}{11}$; 21. $\frac{3}{2X^2Y^3}$; 22. $\frac{1}{3X^5Y^8}$; 23. $\frac{4}{X+5}$; 24. $\frac{X-Y}{9}$;

25. $\frac{3}{4(X+3)}$; 26. $\frac{8(X-4)}{15(X-8)}$; 27. $\frac{X+6}{6(X+2)}$; 28. $\frac{X-5}{15(X+5)}$; 29. $\frac{3(X-8)}{2(X+8)}$; 30. $\frac{20(X-3Y)}{7(X+3Y)}$;

31. $\frac{X-2}{16}$; 32. $\frac{X+3}{9}$; 33. $\frac{X}{X+4}$; 34. $\frac{X}{X+9}$; 35. $\frac{1}{X+8}$; 36. $X+5$; 37. $\frac{X-1}{X+1}$; 38. $\frac{X+1}{X+6}$;

39. $\frac{X+7}{X-7}$; 40. $\frac{X+4}{X-4}$; 41. $\frac{8X^2}{X+1}$; 42. $\frac{8X}{X-2}$; 43. $\frac{X+4}{X-4}$; 44. $\frac{X+5}{X+1}$;

45a) $-x$; b) $-y$; c) $-x+y$ or $y-x$; d) $-y+x$ or $x-y$; e) $-a+b$ or $b-a$; f) $-b+a$ or $a-b$; g) -3 ;

46a) -1 ; b) -1 ; c) -1 ; d) -1 ; e) -1 ; 47. -1 ; 48. -1 ; 49. 1 ; 50. -1 ; 51. -1 ; 52. 1 .