3.02 *Multiplying and Dividing Fractions*

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When multiplying two fractions $\dfrac{a}{b}$ and $\dfrac{b}{c}$, simply multiply the numerator times numerator and denominator times denominator. Of course, the denominators cannot equal zero. That is:

PRINCIPLE: $\dfrac{a}{b} \bullet \dfrac{c}{d} = \dfrac{a \bullet c}{b \bullet d}, \quad b \neq 0, d \neq 0$

When dividing two fractions, you must remember to invert the second number (the divisor) and multiply. Again, remember that you denominators cannot equal zero, and you can't divide by zero. That is:

PRINCIPLE: $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \bullet \dfrac{d}{c}, \quad b \neq 0, c \neq 0, d \neq 0$

In both cases, you must remember to reduce all fractions completely.

EXAMPLE 1. Multiply $\dfrac{7}{8} \bullet \dfrac{5}{9}$ without using a calculator.

Solution: Multiply the numerator times numerator and the denominator times denominator. The answer is $\dfrac{35}{72}$, which does not reduce, so this is the final answer.

EXAMPLE 2. Multiply $\dfrac{7}{8} \bullet \dfrac{5}{9}$ using a calculator.

Solution #1: First, enter [7] [÷] [8] [x] [5] [÷] [9], and press [ENTER] or [=]. The calculator gives the answer in decimal form 0.486111111 ... Your calculator will probably convert this decimal to a fraction in lowest terms. As in the last section, for a TI 83/84, press [MATH] [ENTER] [ENTER]. For a TI 30, press [2nd] [F↔D]. The calculator should give you for a final answer, $\dfrac{35}{72}$.

Solution #2: If your calculator has an **[a b/c]** button (TI 30 and several other brands and models of calculators), then press [7] [a b/c] [8] [x] [5] [a b/c] [9] [=]. The calculator will give you **35 ⌋ 72**, which means $\dfrac{35}{72}$.

EXAMPLE 3.

Multiply $\dfrac{7}{8} \cdot \dfrac{16}{21}$ without using a calculator.

Solution #1: Multiply the numerator times numerator and the denominator times denominator. If you multiply it out, you get a rather large fraction $\dfrac{112}{168}$ that must be reduced. It takes some work, but the answer finally reduces to $\dfrac{2}{3}$.

Solution #2: It might be easier to write the fraction in the form $\dfrac{7 \cdot 16}{8 \cdot 21}$.

Then you can easily see that you can divide the numerator and denominator by the factors of 7 and 8 respectively. When you divide out the 7 and the 8, you get $\dfrac{\cancel{7} \cdot \cancel{16}\,2}{\cancel{8} \cdot \cancel{21}\,3}$. The final answer is $\dfrac{2}{3}$.

EXAMPLE 4.

Multiply $\dfrac{7}{8} \cdot \dfrac{16}{21}$ using a calculator.

Solution #1: First, enter [7] [÷] [8] [x] [16] [÷] [21], and press [ENTER] or [=]. The calculator gives the answer in decimal form 0.666666667 Your calculator will probably convert this decimal to a fraction in lowest terms. As in the last section, for a TI 83/84, press [MATH] [ENTER] [ENTER]. For a TI 30, press [2nd] [F↔D]. The calculator should give you for a final answer $\dfrac{2}{3}$.

Solution #2: If your calculator has an **[a b/c]** button (TI 30 and several other brands and models of calculators), then press [7] [a b/c] [8] [x] [16] [a b/c] [21] [=]. The calculator will give you **2 ⌋ 3**, which means $\dfrac{2}{3}$.

EXERCISES. In each of the following, perform the indicated operations, and reduce the fractions completely. If your calculator has "fractions" capabilities, check your answers with the calculator.

1. $\dfrac{3}{7} \cdot \dfrac{9}{5} =$ _____

2. $\dfrac{4}{5} \cdot \dfrac{3}{13} =$ _____

Note: In the next exercises and in future exercises, improper fractions are preferred, mixed fractions are accepted.

3. $\dfrac{13}{5} \cdot \dfrac{7}{2} =$ _____

4. $\dfrac{7}{2} \cdot \dfrac{5}{13} =$ _____

Note: In the following exercises, remember, you may divide <u>any</u> factor of <u>any</u> numerator with <u>any</u> factor of <u>any</u> denominator. Be sure to reduce all fractions. (You may need one or more steps!)

5. $\dfrac{14}{5} \cdot \dfrac{15}{63} =$ _____

$=$ _____

6. $\dfrac{20}{13} \cdot \dfrac{26}{25} =$ _____

$=$ _____

7. $\dfrac{13}{20} \cdot \dfrac{16}{52} =$ _____

$=$ _____

8. $\dfrac{19}{57} \cdot \dfrac{7}{28} =$ _____

$=$ _____

9. $\dfrac{32}{108} \cdot \dfrac{120}{80} =$ _____

$=$ _____

10. $\dfrac{45}{85} \cdot \dfrac{102}{76} =$ _____

$=$ _____

Remember: **When dividing, your first step is to invert the second fraction, then multiply as in the problems above!**

EXAMPLE 5.   Divide $\dfrac{27}{8} \div \dfrac{33}{20}$ without using a calculator.

Solution:

$\dfrac{27}{8} \div \dfrac{33}{20}$ First, invert the second fraction, and multiply.

$\dfrac{27}{8} \cdot \dfrac{20}{33}$ Factor the numbers in the numerators and denominators.

$\dfrac{9 \cdot 3 \cdot 4 \cdot 5}{4 \cdot 2 \cdot 3 \cdot 11}$ Divide numerator and denominator by factors of 3 and 4.

$\dfrac{9 \cdot 5}{2 \cdot 11}$ or $\dfrac{45}{22}$ This can also be written as the mixed fraction $2\dfrac{1}{22}$.

EXAMPLE 6.   Divide $\dfrac{27}{8} \div \dfrac{33}{20}$ using a calculator.

Solution #1:   First, if you try to calculate this using only division [÷] signs such as [27] [÷] [8] [÷] [33] [÷] [20], there are just too many division signs, and the calculator, following the order of operations agreements, just begins with the first number and divides by all three of the succeeding numbers. Obviously, this is NOT what you had in mind. The easiest way to tell the calculator exactly what you want done is to insert parentheses around each of the two fractions to be divided. In other words, rewrite the problem like this: $\left(\dfrac{27}{8}\right) \div \left(\dfrac{33}{20}\right)$.

Enter it in the calculator as follows:

[( ] [27] [÷] [8] [ )] [÷] [( ] [33] [÷] [20] [ )] [ENTER]

The calculator gives the answer in decimal form 2.045454545 Your calculator will convert this decimal to either the improper fraction **45/22** or the mixed fraction **2 1/22**.

Solution #2:   If your calculator has an **[a b/c]** button, then you don't need parentheses. Press [27] [a b/c] [8] [÷] [33] [a b/c] [20] [=]. The calculator will give you **2   1 ⌋ 22**, which means $2\dfrac{1}{22}$ This can also be converted (by calculator or by hand) to the improper fraction **45/22**.

EXERCISES.   **Perform the indicated operations.**

11. $\dfrac{3}{10} \div \dfrac{5}{7} = \dfrac{3}{10} \cdot \dfrac{7}{5}$

$= \underline{\hspace{2cm}}$

12. $\dfrac{8}{3} \div \dfrac{7}{20} = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

13. $\dfrac{27}{25} \div \dfrac{9}{35} = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

14. $\dfrac{42}{13} \div \dfrac{7}{26} = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

15. $\dfrac{13x^3}{12y} \cdot \dfrac{48x^5}{39y^3}$

16. $\dfrac{24x^3}{36y^5} \cdot \dfrac{39x^4}{8y^4}$

17. $\dfrac{38x^6}{19y^4} \cdot \dfrac{36y^{10}}{16x^6}$

18. $\dfrac{52x^3}{39x^6} \cdot \dfrac{74x^4}{37y^2}$

19. $\dfrac{42x^7}{27y^3} \div \dfrac{14x^6}{9y^5}$

20. $\dfrac{24x^5}{15x^{10}} \div \dfrac{3y^3}{40y^6} =$

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21. $\dfrac{x^2-4x}{16} \bullet \dfrac{8x}{x^2-2x=8} = \dfrac{x(x-4)}{16} \bullet \dfrac{8x}{(x-4)(x+2)}$

$$=$$

22. $\dfrac{x^2-16}{x^2-3x-4} \bullet \dfrac{x^2-1}{x^2+5x+4} = \dfrac{(\quad)(\quad)}{(\quad)(\quad)} \bullet \dfrac{(\quad)(\quad)}{(\quad)(\quad)}$

$$=$$

23. $\dfrac{x^2-8x}{x^2-7x+12} \bullet \dfrac{x^2-4x+3}{x^2-9x+8} = \dfrac{x(\quad)}{(\quad)(\quad)} \bullet \dfrac{(\quad)(\quad)}{(\quad)(\quad)}$

$$=$$

24. $\dfrac{x^2-x}{x^2-x-12} \bullet \dfrac{x^2-3x-4}{x^2-1}$

25. $\dfrac{x^2-6x+8}{x^2-7x+10} \bullet \dfrac{x^2-3x-10}{x^2-4x}$

26. $\dfrac{x^2-12x+32}{x^2-36} \bullet \dfrac{x^2+6x}{x^2-4x}$

27. $\dfrac{x^2-2x}{x^2-5x+6} \div \dfrac{x^2-36}{x^2+3x-18}$

$\dfrac{x(\quad\quad)}{(\quad)(\quad\quad)} \bullet \dfrac{(x+6)(x-3)}{(\quad)(\quad\quad)}$

28. $\dfrac{x^2-9x+20}{x^2-16} \div \dfrac{x^2-10x+25}{x^2+8x+16}$

29. $\dfrac{3x^2-24x}{x^2-10x+24} \div \dfrac{6x^3+24x^2}{x^2-2x-24}$

30. $\dfrac{4x^2-24x}{6x^3-24x^2} \div \dfrac{x^2-12x+36}{x^2-9x+18}$

31. $\dfrac{x^2 - 49y^2}{x^2 + 12xy + 35y^2} \bullet \dfrac{x^2 - 3xy - 10y^2}{x^2 - 5xy - 14y^2}$

32. $\dfrac{x^2 - 8xy + 16y^2}{x^2 - 3xy - 10y^2} \bullet \dfrac{x^2 - 4y^2}{x^2 - 5xy + 4y^2}$

33. $\dfrac{4x - 16y}{x^2 y} \div \dfrac{x^2 - 3xy - 4y^2}{xy^2}$

$\dfrac{4(\quad)}{x^2 y} \bullet \dfrac{xy^2}{(\quad)(\quad)}$

34. $\dfrac{x^3 y^2}{6xy + 12x} \div \dfrac{y^3}{y^2 - 4}$

35. $\dfrac{6x^2 - x - 7}{12x^2 + 16x - 35} \div \dfrac{x^2 - x - 2}{2x^2 + x - 10}$

$\dfrac{(6x\quad)(x\quad)}{(6x\quad)(2x\quad)} \bullet \dfrac{(2x\quad)(x\quad)}{(\quad)(\quad)}$

36. $\dfrac{3x^2 - x - 2}{x^4 - x^3} \div \dfrac{3x^2 + 5x + 2}{x^3 - 2x^2}$

37. $\dfrac{9x^2 - 4y^2}{9x^2 - 12xy + 4y^2} \div \dfrac{x^2 - 2xy - 8y^2}{3x^2 - 14xy + 8y^2}$

38. $\dfrac{4x^2 - 9y^2}{4x^2 - 4xy - 3y^2} \div \dfrac{4x^2 + 8xy + 3y^2}{4x^2 - y^2}$

Frequently it is helpful (necessary) to factor a negative from one of the factors in order to help things "match-up".

39. $\dfrac{5 - x}{x^2 - 25}$

40. $\dfrac{3 - x}{3x^2 - 9x}$

41. $\dfrac{4 - x}{x^2 - 2x - 8}$

42. $\dfrac{25 - x^2}{x^2 - 3x - 10}$

43. $\dfrac{4 - 2x}{x^2 - 4}$

44. $\dfrac{x^2 + x}{1 - x^2}$

45. $\dfrac{5-x}{6xy} \bullet \dfrac{2xy^2}{x^2-25}$

$\dfrac{-1(x-5)}{6xy} \bullet \dfrac{2xy^2}{(x-5)(x+5)}$

_____

46. $\dfrac{6-x}{6xy^5} \bullet \dfrac{9x^3y^2}{x^2-7x+6}$

47. $\dfrac{25-x^2}{4x^3y} \div \dfrac{x^2-10x+25}{12xy^3}$

48. $\dfrac{4-x^2}{12x^3y^4} \div \dfrac{x^2+2x-8}{4xy^2}$

49. $\dfrac{6x-3x^2}{x^2+5x-14} \bullet \dfrac{x^2-49}{x^2-14x+49}$

50. $\dfrac{25-x^2}{x^2+x-20} \div \dfrac{x^2+3x-40}{x^2+4x-32}$

ANSWERS 3.02

1. 27/35; **2.** 12/65; **3.** 91/10; **4.** 35/26; **5.** 2/3; **6.** 8/5; **7.** 1/5; **8.** 1/12; **9.** 4/9; **10.** 27/38;

11. 21/50; **12.** 160/21; **13.** 21/5; **14.** 12; **15.** $\dfrac{4X^8}{3Y^5}$; **16.** $\dfrac{13X^7}{4Y^9}$; **17.** $\dfrac{9Y^6}{2}$; **18.** $\dfrac{8X}{3Y^2}$; **19.** XY^2 ;

20. $\dfrac{64Y^3}{3X^5}$; **21.** $\dfrac{X^2}{2(X+2)}$; **22.** $\dfrac{X-1}{X+1}$; **23.** $\dfrac{X}{X-4}$; **24.** $\dfrac{X}{X+3}$; **25.** $\dfrac{X+2}{X}$;

26. $\dfrac{X-8}{X-6}$; **27.** $\dfrac{X}{X-6}$; **28.** $\dfrac{X+4}{X-5}$; **29.** $\dfrac{X-8}{2X(X-4)}$; **30.** $\dfrac{2(X-3)}{3X(X-4)}$; **31.** $\dfrac{X-5Y}{X+5Y}$;

32. $\dfrac{(X-4Y)(X-2Y)}{(X-5Y)(X-Y)}$; **33.** $\dfrac{4Y}{X(X+Y)}$; **34.** $\dfrac{X^2(Y-2)}{6Y}$; **35.** 1; **36.** $\dfrac{X-2}{X(X+1)}$;

37. $\dfrac{3X+2Y}{X+2Y}$; **38.** $\dfrac{2X-Y}{2X+Y}$; **39.** $\dfrac{-1}{X+5}$; **40.** $\dfrac{-1}{3X}$; **41.** $\dfrac{-1}{X+2}$; **42.** $\dfrac{-(X+5)}{X+2}$ or $\dfrac{-X-5}{X+2}$;

43. $\dfrac{-2}{X+2}$; **44.** $\dfrac{X}{1-X}$ or $\dfrac{-X}{X-1}$; **45.** $\dfrac{-Y}{3(X+5)}$; **46.** $\dfrac{-3X^2}{2Y^3(X-1)}$; **47.** $\dfrac{-3Y^2(X+5)}{X^2(X-5)}$;

48. $\dfrac{-(X+2)}{3X^2Y^2(X+4)}$; **49.** $\dfrac{-3X}{(X-7)}$; **50.** -1.