3.03 *Least Common Denominators (LCD)*

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Addition and subtraction of fractions requires a completely different procedure from the multiplication and division of the last section. Fractions cannot be added (or subtracted) unless they have a **common denominator**. Fractions that have a common denominator are added in the same way that you combine like terms. Just like adding 3 apples plus 2 apples (which equals 5 apples), adding 3 sevenths plus 2 sevenths equals 5 sevenths. This is written in math symbols:

$$\frac{3}{7}+\frac{2}{7}=\frac{5}{7}.$$

When you add or subtract fractions **with a common denominator**, you add or subtract the numerators, and place that answer over the common denominator. And of course remember, you are still not allowed (and never will be allowed!) to divide by zero!

GENERALIZATION: $\frac{a}{d}+\frac{b}{d}-\frac{c}{d}=\frac{a+b-c}{d},\quad d\neq 0$

PRINCIPLE: **Before you can add or subtract fractions, you MUST have a common denominator.**

If the fractions do not have a common denominator, then the first order of business must be to find a common denominator. (It must really be lost! People have been looking for it for decades!) A common denominator is a number or an expression such that each denominator divides into the common denominator evenly. For example, to add $\frac{1}{2}+\frac{1}{3}$, the common denominator of **6** may be used, since both 2 and 3 divide evenly into 6. However, notice that any multiple of 6, like 12, 18, 24, 36, etc., are also divisible by 2 and 3, so these could also be considered "common denominators" for this addition problem. Finally, we can say that of all these common denominators, **6 is the least (lowest) common denominator (LCD),** and in MOST cases the **least common denominator is the best one to use.**

DEFINITIONS

A *COMMON DENOMINATOR* is a number or quantity such that each denominator divides evenly into it.

The *LEAST (lowest) COMMON DENOMINATOR (LCD)* is the smallest of all the common denominators.

Many of "least common denominators" are rather obvious. While there are many explanations and methods of finding **LCD**s, a good approach is to begin with "obvious" examples (intuitive method). From these examples, you will discover a strategy that will allow you to quickly and easily find most **LCD**s. Then, after further probing, you will discover a method that works to find all **LCD**s, even those you do not intuitively know. This method will not only guarantee your ability to find **LCD**s regardless of how complicated it may be--it will ensure your understanding of the entire concept of **LCD**s--the what, the how, and the why!

EXERCISES. **In the following exercises, by trial and error, find the least common denominators (LCD). Remember you are trying to find the smallest possible number that each of the denominators divides into evenly. For your convenience, the answers to this page are provided at the bottom of the page:**

1. $\frac{1}{2}$, $\frac{1}{4}$ **LCD** = _____
2. $\frac{1}{2}$, $\frac{1}{6}$ **LCD** = _____
3. $\frac{1}{2}$, $\frac{1}{8}$ **LCD** = _____
4. $\frac{1}{3}$, $\frac{1}{6}$ **LCD** = _____
5. $\frac{1}{3}$, $\frac{1}{12}$ **LCD** = _____
6. $\frac{1}{2}$, $\frac{1}{12}$ **LCD** = _____
7. $\frac{1}{4}$, $\frac{1}{12}$ **LCD** = _____
8. $\frac{1}{6}$, $\frac{1}{12}$ **LCD** = _____
9. $\frac{1}{5}$, $\frac{1}{10}$ **LCD** = _____
10. $\frac{1}{5}$, $\frac{1}{20}$ **LCD** = _____
11. $\frac{1}{20}$, $\frac{1}{60}$ **LCD** = _____
12. $\frac{1}{6}$, $\frac{1}{24}$ **LCD** = _____

13. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ LCD = _____

14. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{12}$ LCD = _____

15. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$ LCD = _____

16. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{24}$ LCD = _____

17. $\frac{1}{2}$, $\frac{1}{3}$ LCD = _____

18. $\frac{1}{3}$, $\frac{1}{4}$ LCD = _____

19. $\frac{1}{2}$, $\frac{1}{5}$ LCD = _____

20. $\frac{1}{3}$, $\frac{1}{5}$ LCD = _____

21. $\frac{1}{4}$, $\frac{1}{5}$ LCD = _____

22. $\frac{1}{5}$, $\frac{1}{7}$ LCD = _____

23. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ LCD = _____

24. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{7}$ LCD = _____

ANSWERS

1.4; **2.**6; **3.**8; **4.**6; **5.**12; **6.**12; **7.**12; **8.**12; **9.**10; **10.**20; **11.**60; **12.**24;
13.6; **14.**12; **15.**12; **16.**24; **17.**6; **18.**12; **19.**10; **20.**15; **21.**20; **22.**35; **23.**30; **24.**70.

What is the pattern in #1-16? ____

What is the pattern in #17-24? ____

25. $\frac{1}{4}$, $\frac{1}{6}$ LCD = _____

26. $\frac{1}{6}$, $\frac{1}{8}$ LCD = _____

27. $\frac{1}{10}$, $\frac{1}{15}$ LCD = _____

28. $\frac{1}{6}$, $\frac{1}{15}$ LCD = _____

29. $\frac{1}{10}$, $\frac{1}{25}$ LCD = _____

30. $\frac{1}{10}$, $\frac{1}{8}$ LCD = _____

#31 - 34 are harder to do by trial and error. This underscores the need for systematic strategies that will follow these exercises. (Before getting frustrated, you may want to sneak a peek ahead!)

31. $\frac{1}{9}$, $\frac{1}{60}$ LCD = _____

32. $\frac{1}{25}$, $\frac{1}{40}$ LCD = _____

33. $\frac{1}{27}$, $\frac{1}{36}$ LCD = _____

34. $\frac{1}{50}$, $\frac{1}{45}$ LCD = _____

From these exercises you probably noticed that sometimes the **LCD** is the larger (or largest) of the numbers (as in **#1-16**). Sometimes the **LCD** is the product of the numbers (as in **#17-24**). Sometimes you just have to use trial and error looking at multiples of the denominators (as in **#25-34**). Sometimes you could spend hours looking (as in **#35-40**)!

An easy way to find the LCD is to begin with the largest denominator--see if that is the LCD. If it is not, then take multiples of that largest denominator, trying each one in order, until you find one that "works"--that is, until you find one that each of the other denominators divides into evenly.

For example, if the denominators are **6, 12,** and **9**, begin with the largest number which is **12**. Since 12 is <u>not</u> divisible by 9, try multiples of 12: **24, 36, 48, 60, etc.** Notice that 24 does not work either, but **36 does work, since it is divisible by 6, 12, and 9.** Therefore, **the LCD is 36**. This is a quick and easy way to find most **LCD**s. Unfortunately, some problems could take a long time to do this way.

Perhaps you also correctly noticed that if the denominators have no common factors, then the **LCD** is the product of the denominators. Most importantly, perhaps you noticed that every **LCD** is built using the prime factors of the denominators involved. The following examples should add details to this idea, and enable you to develop a systematic way of finding the **LCD**, based upon the factors of the denominators. Begin by factoring each denominator into its prime factors.

EXAMPLE 1: $\frac{1}{2}, \frac{1}{10} = \frac{1}{2}, \frac{1}{2 \bullet 5}$

SOLUTIONS:
In the **LCD**, you need factors of **2 and 5**, but the extra 2 is not needed. **LCD = 2•5 = 10.**

EXAMPLE 2: $\frac{1}{5}, \frac{1}{7}, \frac{1}{2}$

The denominators consist of prime factors **2, 5, and 7**. Therefore the **LCD=2•5•7 = 70.**

EXAMPLE 3:

$$\frac{1}{6}, \frac{1}{9} = \frac{1}{2 \bullet 3}, \frac{1}{3^2}$$

SOLUTIONS:

In this case the prime factors are 2 and 3. Since you probably already know the LCD is 18, you will need factors of **2** and of 3^2 **(the highest power of the 3 factors).** Therefore, **LCD = $2 \cdot 3^2 = 2 \cdot 9 = 18$.**

EXAMPLE 4:

$$\frac{1}{8}, \frac{1}{20} = \frac{1}{2^3}, \frac{1}{2^2 \bullet 5}$$

The prime factors are 2 and 5. If you take 2^3 **(the highest power of 2)** that will include the other 2^2. Therefore, **LCD = $2^3 \cdot 5 = 40$.**

EXAMPLE 5:

$$\frac{1}{24}, \frac{1}{18} = \frac{1}{2^3 \bullet 3}, \frac{1}{2 \bullet 3^2}$$

This time the highest power of 2 is 3, and the highest power of 3 is 2. Therefore, **LCD = $2^3 \cdot 3^2 = 8 \cdot 9 = 72$.**

PRINCIPLE:

To find the LCD, factor each denominator into prime factors. The LCD is the product of the factors of the denominators, *each factor being raised to the highest power* of the factor.

Find the LCDs by factoring each denominator into prime factors.

35. $\frac{1}{30}, \frac{1}{25}$ LCD = _____

36. $\frac{1}{27}, \frac{1}{60}$ LCD = _____

37. $\frac{1}{54}, \frac{1}{32}$ LCD = _____

38. $\frac{1}{24}, \frac{1}{52}$ LCD = _____

39. $\frac{1}{21}, \frac{1}{98}$ LCD = _____

40. $\frac{1}{72}, \frac{1}{40}$ LCD = _____

This technique is particularly useful where variables are involved.

SOLUTIONS:

EXAMPLE 6. $\frac{1}{3x^2}, \frac{1}{8x^3}$

The **LCD** for 3 and 8 is obviously 24. The highest power of x is x^3. **LCD** = $24x^3$.

EXAMPLE 7. $\frac{1}{3x^3y}, \frac{1}{3x^2y^4}$

LCD = $3x^3 \bullet y^4$.

EXAMPLE 8. $\frac{1}{3x}, \frac{1}{x+3}$

The factors are **3, x, and x +3**.
Notice that x+3 is a distinct factor.
LCD = **3 x (x +3).**

EXAMPLE 9.

$$\frac{1}{x^2+3x+2}, \frac{1}{x^2+5x+6}$$

$$\frac{1}{(x+2)(x+1)}, \frac{1}{(x+2)(x+3)}$$

First factor each denominator. Then the factors are **(x+2), (x+1), and (x+3).**
(Any order!)
LCD = **(x +1)(x +2)(x +3).**

EXAMPLE 10.

$$\frac{1}{x^2+5x+6}, \frac{1}{x^2+6x+9}$$

$$\frac{1}{(x+2)(x+3)}, \frac{1}{(x+3)^2}$$

The prime factors are **(x+2) and (x+3). (x+3)** to the highest power is **(x+3)^2**.
LCD = **(x+2)(x+3)^2**.

EXERCISES. **In the following, find the least common denominator (LCD).**

41. $\dfrac{1}{16}, \dfrac{1}{18}, \dfrac{1}{24}$

42. $\dfrac{1}{16}, \dfrac{1}{24}, \dfrac{1}{30}$

43. $\dfrac{1}{36}, \dfrac{1}{20}, \dfrac{1}{25}$

44. $\dfrac{1}{72}, \dfrac{1}{30}, \dfrac{1}{20}$

45. $\dfrac{1}{5x^2}, \dfrac{1}{10x^3}, \dfrac{1}{2x^4}$

46. $\dfrac{1}{8x^3}, \dfrac{1}{10x}, \dfrac{1}{20x^2}$

47. $\dfrac{1}{25x^2y^3}, \dfrac{1}{10xy^2}, \dfrac{1}{2x^4y}$

48. $\dfrac{1}{18y^3}, \dfrac{1}{10xy^2}, \dfrac{1}{25x^2y^5}$

49. $\dfrac{1}{12x^3}, \dfrac{1}{20x^7(x+3)}$

50. $\dfrac{1}{12x^3(x+4)}, \dfrac{1}{15x(x+4)^2}$

51. $\dfrac{1}{15x^3(x-5)^3}, \dfrac{1}{20x^2(x-5)^2}$

52. $\dfrac{1}{9x^4(x-2)^2}, \dfrac{1}{15x^5(x-2)}$

53. $\dfrac{1}{x^2+\ 3x}, \quad \dfrac{1}{x^2+5x+6}$

$\dfrac{1}{x(\qquad)}, \quad \dfrac{1}{(\qquad)(\qquad)}$

LCD = ____________________

54. $\dfrac{1}{x^2-5x}, \quad \dfrac{1}{x^2-25}$

LCD = ____________________

55. $\dfrac{1}{x^2+5x-6}, \quad \dfrac{1}{x^2-36}$

$\dfrac{1}{(\qquad)(\qquad)}, \quad \dfrac{1}{(\qquad)(\qquad)}$

LCD = ____________________

56. $\dfrac{1}{x^2-4}, \quad \dfrac{1}{x^2-5x+6}$

$\dfrac{1}{(\qquad)(\qquad)}, \quad \dfrac{1}{(\qquad)(\qquad)}$

LCD = ____________________

57. $\dfrac{1}{x^2+9x+20}, \quad \dfrac{1}{x^2+6x+8}$

=

LCD = _______________

58. $\dfrac{1}{x^2-4x}, \quad \dfrac{1}{x^2+4x}$

=

LCD = _______________

59. $\dfrac{1}{x^2-2x+1}, \quad \dfrac{1}{x^2-x}$

=

LCD = _______________

60. $\dfrac{1}{x^2-4x+4}, \quad \dfrac{1}{x^2-4}$

=

LCD = _______________

61. $\dfrac{1}{x^2-8x+16}, \quad \dfrac{1}{x^2-5x+4}$

=

LCD = _______________

62. $\dfrac{1}{x^2+10x+25}, \quad \dfrac{1}{x^2+3x-10}$

=

LCD = _______________

ANSWERS 3.03

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1. 4; **2.** 6; **3.** 8; **4.** 6; **5.** 12; **6.** 12; **7.** 12; **8.** 12; **9.** 10; **10.** 20; **11.** 60; **12.** 24; **13.** 6; **14.** 12; **15.** 12; **16.** 24; **17.** 6; **18.** 12; **19.** 10; **20.** 15; **21.** 20; **22.** 35; **23.** 30; **24.** 70; #1-16, LCD is the largest denominator; #17-24, LCD is the product of the denominators; **25.** 12; **26.** 24; **27.** 30; **28.** 30; **29.** 50; **30.** 40; **31.** 180; **32.** 200; **33.** 108; **34.** 450; **35.** 150; **36.** 540; **37.** 864; **38.** 312; **39.** 294; **40.** 360; **41.** 144; **42.** 240; **43.** 900; **44.** 360; **45.** $10x^4$; **46.** $40x^3$; **47.** $50x^4y^3$; **48.** $450x^2y^5$; **49.** $60x^7(x+3)$; **50.** $60x^3(x+4)^2$; **51.** $60x^3(x-5)^3$; **52.** $45x^5(x-2)^6$; **53.** $x(x+2)(x+3)$; **54.** $x(x-5)(x+5)$; **55.** $(x-1)(x-6)(x+6)$; **56.** $(x-2)(x+2)(x-3)$; **57.** $(x+4)(x+5)(x+2)$; **58.** $x(x-4)(x+4)$; **59.** $x(x-1)^2$; **60.** $(x-2)^2(x+2)$; **61.** $(x-4)^2(x-1)$; **62.** $(x+5)^2(x-2)$.