3.04 *Adding and Subtracting Fractions*

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Before beginning the algebra of adding and subtracting fractions, it might be helpful to do a few "simple" fractions, especially since you have a calculator to help out. As always, when using the calculator, it is good to begin with simple exercises to check it out first.

Solve each of the following by calculator (especially if you have fractions capabilities) and check by regular LCD methods.

EXAMPLE 1. $\frac{1}{8} + \frac{3}{8}$

CALCULATOR:
If you have **a b/c** button, type **[1]**, **[a b/c]**, **[8]**, **[+]**, **[3]**, **[a b/c]**, **[8]**, **[=]**, _____ Your calculator should say [1 ⌋ 2], which means ½.

Without the **fractions key, you can enter [1], [÷], [8], [+], [3], [÷], [8], [=],** _____ The calculator gives the decimal 0.5 which is 1/2.

In your head: you add to get 4/8, which reduces to 1/2.

EXAMPLE 2. $\frac{3}{4} + \frac{1}{2}$

CALCULATOR:
If you have **a b/c** button, type **[3]**, **[a b/c]**, **[4]**, **[+]**, **[1]**, **[a b/c]**, **[2]**, **[=]**, _____ The calculator says **[1 1 ⌋ 4]**, which means a mixed fraction **[1 ¼]**, which is 5/4.

Without the fractions key, you can **enter [3], [÷], [4], [+], [1], [÷], [2], [=],** _____ The calculator gives the decimal 1.25 which is 1 1/4 or 5/4.

In your head: you must find **LCD** of 4. Then 3/4 + 2/4 gives you 5/4, or 1 1/4.

EXAMPLE 3. $\frac{5}{12} + \frac{3}{16}$

CALCULATOR:
If you have **a b/c** button, type **[5], [a b/c], [12], [+], [3], [a b/c], [16], [=],** _____
Did you get **[29 ⌋ 48]** (i.e. 29/48)?

Without fractions key, you get a decimal you may or may not be able to convert to a fraction.

In your head: It is much harder. Find **LCD** = 48. We'll save the rest of the check for later.

EXAMPLE 4. $\frac{1}{3} + \frac{1}{8} + \frac{3}{4} + \frac{7}{24}$

CALCULATOR:
If you have **a b/c** button, you can easily enter the numbers and get **[1 1 ⌋ 2]**, which means 1 ½ or 3/2.

Even without the fractions key, you should easily get the decimal value 1.5 which is 1 ½ or 3/2.

In your head: The **LCD** = 24, and with some work (later!) the answer reduces to 3/2.

EXERCISES. **Use a calculator to add or subtract the following fractions:**

1. $\frac{2}{3} + \frac{3}{5}$ 2. $\frac{3}{8} + \frac{5}{6}$ 3. $\frac{7}{6} - \frac{2}{3}$ 4. $\frac{5}{24} - \frac{1}{8}$

5. $\frac{2}{13} + \frac{5}{17}$ 6. $\frac{23}{36} - \frac{14}{23}$ 7. $\frac{5}{6} + \frac{17}{18}$ 8. $\frac{25}{28} - \frac{13}{42}$

The first question that must be answered when adding and subtracting fractions without a calculator is this: "Is there a common denominator?" If the fractions already have a common denominator, then just put down the common denominator as THE denominator of the answer. To get the numerator of the answer just add (or subtract) the numerators. Then, of course, try to reduce the fractions, if possible, as you did in an earlier section.

9. $\frac{2x}{3} + \frac{10y}{3}$

$= \frac{\_\_\_\_\_\_}{3}$

10. $\frac{3x}{5} - \frac{13y}{5}$

$=$ ________

11. $\frac{2x}{3} + \frac{10x}{3}$

$=$

$=$ ________ (Reduce!)

12. $\frac{3y}{5} - \frac{13y}{5}$

$=$

$=$

13. $\frac{3}{4x} - \frac{9}{4x}$

14. $\frac{2}{7x^2} - \frac{9}{7x^2}$

15. $\frac{x^2}{x+4} + \frac{4x}{x+4} = \frac{\_\_\_\_\_\_}{x+4}$

Factor: $=$

Reduce: $=$

16. $\frac{y^2}{y-4} - \frac{4y}{y-4} =$

17. $\dfrac{x^2+4}{x+2} + \dfrac{4x}{x+2} = \dfrac{\qquad}{x+2}$

$= \dfrac{x^2+4x+4}{x+2}$

$=$

$=$

18. $\dfrac{y^2+9}{y-3} - \dfrac{6y}{y-3} =$

Notice that in the next few exercises, the primary concept in adding or subtracting fractions is putting down the **LCD** and then adding or subtracting numerators. Whether the numerators can be factored or not is usually irrelevant. Don't forget that if you were multiplying or dividing the fractions, the first step would be to factor everything, both numerators and denominators. However, when adding or subtracting fractions, ***there is usually no need to factor numerators*****!** (Even if they do factor, it is usually not a good idea!)

19. $\dfrac{x^2-12x}{x-6} + \dfrac{x^2-5x+30}{x-6}$

$= \dfrac{\qquad + \qquad}{x-6}$

$= \dfrac{2x^2 - 17x + 30}{x - 6}$

$= \dfrac{(2x - \quad)(x - \quad)}{x-6}$

$=$

20. $\dfrac{y^2+4y}{3y+5} + \dfrac{2y^2+4y+5}{3y+5}$

In the next exercises, be careful of the signs:

21. $\dfrac{3y^2+4y}{y+2} - \dfrac{2y^2-4}{y+2}$

$= \dfrac{-2y^2 + 4}{y+2}$

$=$

$=$

$=$

22. $\dfrac{5x^2}{x-2} - \dfrac{4x^2+3x-2}{x-2}$

23. $\dfrac{3x^2-4x}{x^2-4} - \dfrac{2x^2-3x+6}{x^2-4}$

$=$

$=$

$=$

$=$

$=$

24. $\dfrac{2x^3-6x^2-3}{x^3+x^2-6x} - \dfrac{x^3-8x-3}{x^3+x^2-6x}$

NOW, if the fractions do **not** have a common denominator, then finding the common denominator is your first priority! If the denominator is not in factored form, then you must factor each denominator completely and find the common denominator, as in the previous section. The next step will require a few examples to understand. You must compare each denominator of the problem to the LCD and in each case decide **"WHAT'S MISSING?"** You must multiply the numerator and denominator of each fraction by the "missing factors." Don't forget! Always multiply the **numerator** ***AND*** **denominator!** Then combine numerators adding or subtracting like terms, as illustrated in the next examples. On the next page is a summary in outline form. This will be a very helpful reference as you study the examples and do the exercises that follow.

ADDITION AND SUBTRACTION OF FRACTIONS
Summary

I. FIND THE LEAST COMMON DENOMINATOR (LCD).

A. Factor each denominator to determine what factors are needed for the common denominator.

B. For each of the denominator factors, determine the highest power of each factor. The LCD is the product of each factor raised to its highest power.

C. The LCD becomes the denominator of the fraction.

II. PLAY "WHAT'S MISSING?"

A. Compare each denominator to the LCD, and determine the missing factors for each denominator.

B. Multiply each numerator and denominator by "What's missing!"

III. ADD OR SUBTRACT NUMERATORS.

A. Add (or subtract) numerators, and place over the common denominator.

B. Combine like terms and reduce the resulting fraction, if possible.

EXAMPLE 5. $\frac{1}{4} + \frac{3}{5}$

Solution. **Step I (Find the LCD):** The LCD is 20.

Step II (What's Missing?): 1st denominator has 4, missing: 5

2nd denominator has 5, missing: 4

Multiply numerator and denominator of each fraction by "What's Missing":

$= \frac{1}{4} \bullet \frac{(\ )}{(\ )} + \frac{3}{5} \bullet \frac{(\ )}{(\ )}$

$= \frac{1}{4} \bullet \frac{(5)}{(5)} + \frac{3}{5} \bullet \frac{(4)}{(4)}$

Step III (Add or Subtract): Add the numerators and place over the LCD.

$= \frac{5}{20} + \frac{12}{20}$ or $\frac{17}{20}$

EXAMPLE 6. $\frac{1}{4x} + \frac{3}{5x}$

Solution. **Step I (Find the LCD):** **The LCD is 20*x*.**

Step II (What's Missing?): **1st denominator has 4 *x*, missing: 5**
2nd denominator has 5 *x*, missing: 4

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{1}{4x}\bullet\frac{(\quad)}{(\quad)} + \frac{3}{5x}\bullet\frac{(\quad)}{(\quad)}$$

$$= \frac{1}{4x}\bullet\frac{(5)}{(5)} + \frac{3}{5x}\bullet\frac{(4)}{(4)}$$

Step III (Add or Subtract): **Add the numerators and place over the LCD.**

$$= \frac{5}{20x} + \frac{12}{20x} \quad \text{or} \quad \frac{17}{20x}$$

EXAMPLE 7. $\frac{1}{8x} + \frac{3}{10y}$

Solution. **Step I (Find the LCD):** **The LCD is 40xy.**

Step II (What's Missing?): **1st denominator has 8*x*, missing: 5*y***
2nd denominator has 10*y*, missing: 4*x*

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{1}{8x}\bullet\frac{(\quad)}{(\quad)} + \frac{3}{10y}\bullet\frac{(\quad)}{(\quad)}$$

$$= \frac{1}{8x}\bullet\frac{(5y)}{(5y)} + \frac{3}{10y}\bullet\frac{(4x)}{(4x)}$$

Step III (Add or Subtract): **Add the numerators and place over the LCD.**

$$= \frac{5y}{40xy} + \frac{12x}{40xy}$$

$$= \frac{5y+12x}{40xy} \quad \text{or} \quad \frac{12x+5y}{40xy}$$

EXAMPLE 8. $\dfrac{5}{4x^2y} - \dfrac{2}{3xy^3}$

Solution. **Step I (Find the LCD):** **The LCD is $12x^2y^3$.**

Step II (What's Missing?): **1st denominator has $4x^2y$, missing: $3y^2$**
2nd denominator has $3xy^3$, missing: $4x$

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{5}{4x^2y} \cdot \frac{(\ \ )}{(\ \ )} - \frac{2}{3xy^3} \cdot \frac{(\ \ )}{(\ \ )}$$

$$= \frac{5}{4x^2y} \cdot \frac{(3y^2)}{(3y^2)} - \frac{2}{3xy^3} \cdot \frac{(4x)}{(4x)}$$

Step III (Add or Subtract): **Add the numerators and place over the LCD.**

$$= \frac{15y^2}{12x^2y^3} - \frac{8x}{12x^2y^3}$$

$$= \frac{15y^2 - 8x}{12x^2y^3}$$

EXAMPLE 9. $\dfrac{3}{8x^2y} + \dfrac{5}{6x^3}$

Solution. **Step I (Find the LCD):** **The LCD is $24x^3y$.**

Step II (What's Missing?): **1st denominator has $8x^2y$, missing: $3x$**
2nd denominator has $6x^3$, missing: $4y$

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{3}{8x^2y} \cdot \frac{(\ \ )}{(\ \ )} + \frac{5}{6x^3} \cdot \frac{(\ \ )}{(\ \ )}$$

$$= \frac{3}{8x^2y} \cdot \frac{(3x)}{(3x)} + \frac{5}{6x^3} \cdot \frac{(4y)}{(4y)}$$

Step III (Add or Subtract): **Add the numerators and place over the LCD.**

$$= \frac{9x + 20y}{24x^3y}$$

EXERCISES. **Add or subtract the fractions as indicated.**

25. $\frac{2}{3} + \frac{1}{4}$

Step I (Find the LCD): **The LCD is _____.**

Step II (What's Missing?): **1st denominator has 3,** **missing: _____**

2nd denominator has 4, **missing: _____**

Multiply numerator and denominator of each fraction by "What's Missing":

$= \frac{2}{3} \bullet \frac{(\ )}{(\ )} + \frac{1}{4} \bullet \frac{(\ )}{(\ )}$

Step III (Add or Subtract): **Add the numerators and place over the LCD.**

$= \frac{}{12} + \frac{}{12}$ **or** ______.

26. $\frac{7}{8} - \frac{5}{6}$

Step I (Find the LCD): **The LCD is _____.**

Step II (What's Missing?): **1st denominator has 8,** **missing: _____**

2nd denominator has 6, **missing: _____**

Multiply numerator and denominator of each fraction by "What's Missing":

$= \frac{7}{8} \bullet \frac{(\ )}{(\ )} - \frac{5}{6} \bullet \frac{(\ )}{(\ )}$

Step III (Add or Subtract): **Add the numerators and place over the LCD.**

$= \frac{}{24} - \frac{}{24}$ **or** ______.

27. $\frac{5}{8} + \frac{3}{10}$

28. $\frac{5}{8} - \frac{3}{10}$

29. $\frac{7}{15} - \frac{5}{9}$

30. $\frac{7}{15} + \frac{5}{9}$

31. $\frac{3}{5x} + \frac{7}{15y}$

Step I (Find the LCD): The LCD is _______.

Step II (What's Missing?): 1st denominator has $5x$, missing: _____

2nd denominator has $15y$, missing: _____

Multiply numerator and denominator of each fraction by "What's Missing":

$= \frac{3}{5x} \bullet \frac{(\quad)}{(\quad)} + \frac{7}{15y} \bullet \frac{(\quad)}{(\quad)}$

Step III (Add or Subtract): Add the numerators and place over the LCD.

$= \frac{\quad}{15xy} + \frac{\quad}{15xy}$ or __________.

32. $\frac{5}{7x^2} - \frac{3}{28xy^2}$

Step I (Find the LCD): The LCD is _______.

Step II (What's Missing?): 1st denominator has $7x^2$, missing: _____

2nd denominator has $28xy^2$, missing: _____

Multiply numerator and denominator of each fraction by "What's Missing":

$= \frac{5}{7x^2} \bullet \frac{(\quad)}{(\quad)} - \frac{3}{28xy^2} \bullet \frac{(\quad)}{(\quad)}$

Step III (Add or Subtract): Add the numerators and place over the LCD.

$= \frac{\quad}{28x^2y^2} - \frac{\quad}{28x^2y^2}$ or __________.

33. $\frac{7}{8x} + \frac{3}{5x}$

34. $\frac{7}{8x} - \frac{5}{6y}$

35. $\frac{2}{9x^2} - \frac{5}{3xy}$

36. $\frac{3}{8xy^2} + \frac{5}{6x^2}$

37. $\dfrac{7}{6xy^2} + \dfrac{5}{4x^3y}$

38. $\dfrac{4}{3y^2} - \dfrac{8}{5x^2y}$

39. $\dfrac{6}{25x^3y^2} - \dfrac{7}{5xy}$

40. $\dfrac{5}{9y^2} - \dfrac{7}{18xy}$

41. $\dfrac{5}{24y} - \dfrac{8}{9xy^3}$

42. $\dfrac{7}{5xy^2} + \dfrac{8}{45x^4y^3}$

EXAMPLE 10. $\dfrac{2}{x^2+3x} + \dfrac{3}{x^2+4x+3}$

Solution. **Step I (Find the LCD):** **The first step is to factor the denominators.**

$$= \frac{2}{x(x+3)} + \frac{3}{(x+3)(x+1)}$$

The LCD is $x(x+3)(x+1)$.

Step II (What's Missing?): **1st Denom missing: $(x+1)$**

2nd Denom missing: (x)

Multiply numerator and denominator of each fraction by "What's Missing".

$$= \frac{2}{x(x+3)} \bullet \frac{(\quad)}{(\quad)} + \frac{3}{(x+3)(x+1)} \bullet \frac{(\quad)}{(\quad)}$$

$$= \frac{2}{x(x+3)} \bullet \frac{(x+1)}{(x+1)} + \frac{3}{(x+3)(x+1)} \bullet \frac{(x)}{(x)}$$

Step III (Add or Subtract): **Add numerators and place over LCD.**

$$= \frac{2(x+1) + 3(x)}{x(x+3)(x+1)}$$

$$= \frac{2x+2+3x}{x(x+3)(x+1)}$$

$$= \frac{5x+2}{x(x+3)(x+1)}$$

43. $\dfrac{4x}{x^2-5x+6} + \dfrac{3}{x^2-3x+2}$

Step I (Find the LCD): Factor the denominators

$\dfrac{4x}{(\quad)(\quad)} + \dfrac{3}{(\quad)(\quad)}$ The LCD is______________.

Step II (What's Missing?):

$\dfrac{4x}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)} + \dfrac{3}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)}$

Step III (Add or Subtract): Add numerators and place over LCD.

$\dfrac{\quad + \quad}{(\quad)(\quad)(\quad)}$

$=$

44. $\dfrac{5}{x^2-5x+6} - \dfrac{3}{x^2-9}$

$= \dfrac{5}{(\quad)(\quad)} - \dfrac{3}{(\quad)(\quad)}$ The LCD is______________.

$= \dfrac{5}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)} - \dfrac{3}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)}$

$= \dfrac{}{(\quad)(\quad)(\quad)}$

$=$

45. $\dfrac{x}{x^2+4x+3} - \dfrac{4}{x^2-3x-4}$

46. $\dfrac{2x}{x^2-4} - \dfrac{3}{x^2+x-6}$

47. $\dfrac{x}{x^2-4x+3} - \dfrac{4}{x^2+3x-4}$

48. $\dfrac{2x}{x^2-4} - \dfrac{3}{x^2-x-6}$

EXAMPLE 11. $\dfrac{x}{x^2-6x+9} - \dfrac{4}{x^2+2x-15}$

Solution. **Step I (Find the LCD):** **The first step is to factor the denominators.**

$= \dfrac{x}{(x-3)^2} - \dfrac{4}{(x-3)(x+5)}$ **The LCD is $(x-3)^2(x+5)$.**

Step II (What's Missing?): **1st Denom missing: $(x+5)$**
2nd Denom missing: $(x-3)$

Multiply numerator and denominator of each fraction by "What's Missing".

$= \dfrac{x}{(x-3)^2} \cdot \dfrac{(\quad)}{(\quad)} - \dfrac{4}{(x-3)(x+5)} \cdot \dfrac{(\quad)}{(\quad)}$

$= \dfrac{x}{(x-3)^2} \cdot \dfrac{(x+5)}{(x+5)} - \dfrac{4}{(x-3)(x+5)} \cdot \dfrac{(x-3)}{(x-3)}$

Step III (Add or Subtract): **Add numerators and place over LCD.**

$= \dfrac{x^2+5x - 4x+12}{(x-3)^2(x+5)} = \dfrac{x^2+x+12}{(x-3)^2(x+5)}$

49. $\dfrac{4x}{x^2-6x+9} - \dfrac{3}{x^2-9}$

$\dfrac{4x}{(\quad)^2} - \dfrac{3}{(\quad)(\quad)}$ **The LCD is ________________.**

$\dfrac{4x}{(\quad)^2} \cdot \dfrac{(\quad)}{(\quad)} - \dfrac{3}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)}$

=

=

50. $\dfrac{4x}{x^2+6x+9} - \dfrac{3}{x^2-9}$

51. $\dfrac{5}{x^2-10x+25} - \dfrac{3}{x^2-5x}$

52. $\dfrac{5}{x^2-2x+1} + \dfrac{3}{x^2-x}$

53. $\dfrac{7x}{x^2-14x+49} + \dfrac{5}{x^2-2x-35}$

In #54 - 56, **factoring the numerators at the beginning of the problem, even if possible, is usually irrelevant! However, after you finish the problem, you may be able to reduce the fractions by factoring.**

54. $\dfrac{3x^2-4}{2x^2-4x} - \dfrac{x}{x-2} - \dfrac{3}{2x}$

55. $\dfrac{2a^2-7ab-12b^2}{2a(3a-4b)} + \dfrac{2a+4b}{3a-4b}$

56. $\dfrac{y+x}{y-2x} + \dfrac{y^2-7xy}{(2y+x)(y-2x)} + \dfrac{-y+x}{2y+x}$

NOTE: Problems #57 - 59 are from *New School Algebra* (1898), by G.A. Wentworth.

57. $\dfrac{1}{a^2-7a+12} + \dfrac{2}{a^2-4a+3} - \dfrac{3}{a^2-5a+4}$

58. $\dfrac{1}{x-2} + \dfrac{1}{x^2-3x+2} - \dfrac{2}{x^2-4x+3}$

59. $\dfrac{3}{10a^2+a-3} - \dfrac{4}{2a^2+7a-4}$

FACTORS of "$x - y$" and "$y - x$" (Optional--if time permits!)

As in the first two sections of this chapter, it is frequently necessary to work with factors and their negatives. This may occur in reducing fractions, multiplying/ dividing fractions, or adding/subtracting fractions. Sometimes it is helpful to factor a "-1" from one of the factors, in order to make them "match up." Another helpful hint is to remember that any number (except zero, of course!) divided by its negative is "−1". Consider: $\frac{-6}{6} = -1$; $\frac{3}{-3} = -1$; $\frac{-125}{125} = -1$; $\frac{-x}{x} = -1$; $\frac{y}{-y} = -1$; and $\frac{-3z}{3z} = -1$.

Likewise, since "**4−x**" is the negative of "**x−4**", and "**y−x**" is the negative of "**x−y**": So, $\frac{x-4}{4-x} = -1$ and $\frac{x-y}{y-x} = -1$. We'll begin with a few review exercises.

EXERCISES.

60. $\frac{x-6}{6-x} =$ ____ **61.** $\frac{3x-8y}{8y-3x} =$ ____ **62.** $\frac{8y-3x}{3x-8y} =$ ____ **63.** $\frac{3x-8y}{-8y+3x} =$ ____

64. $\frac{x^2-16}{4-x} = \frac{(x-4)(x+4)}{4-x}$

$= -1(x+4)$

or $-x-4$

65. $\frac{x^2-36}{6-x} =$

When adding and subtracting fractions such as $\frac{x}{x-6} + \frac{4}{6-x}$, the first reaction may be to use the product as the least common denominator. True, the product is a common denominator, but it is not the least common denominator, and the answer you get will need to be reduced. It is much easier to multiply the numerator and denominator of one of the fractions (either one--your choice!) by "−1". Consider the following examples:

EXAMPLE 12: $\dfrac{x}{x-6} + \dfrac{4}{6-x}$

Solution.

$$= \frac{x}{x-6} + \frac{(-1)}{(-1)} \cdot \frac{4}{6-x}$$

$$= \frac{x}{x-6} + \frac{-4}{x-6}$$

$$= \frac{x-4}{x-6}$$

EXAMPLE 13: $\dfrac{x}{x^2-9} - \dfrac{3}{3-x}$

Solution.

$$= \frac{x}{(x-3)(x+3)} - \frac{(-1)}{(-1)} \cdot \frac{3}{3-x}$$

$$= \frac{x}{(x-3)(x+3)} + \frac{3}{x-3}$$

$$= \frac{x}{(x-3)(x+3)} + \frac{3}{x-3} \cdot \frac{(x+3)}{(x+3)}$$

$$= \frac{x \, + \, 3x+9}{(x-3)(x+3)}$$

$$= \frac{4x+9}{(x-3)(x+3)}$$

66. $\dfrac{5}{s-5} + \dfrac{5}{5-s}$

67. $\dfrac{x}{x-2} - \dfrac{x}{2-x}$

68. $\dfrac{16}{x-8} + \dfrac{2x}{8-x}$

69. $\dfrac{2x}{x-10} + \dfrac{20}{10-x}$

70. $\dfrac{x^2}{x-2} + \dfrac{4}{2-x}$

71. $\dfrac{x^2}{x-2} + \dfrac{2x}{2-x}$

72. $\dfrac{x^2}{3x-9} + \dfrac{3}{3-x}$

73. $\dfrac{x^2}{5x-25} + \dfrac{5}{5-x}$

74. $\dfrac{x}{x^2-25} - \dfrac{5}{5-x}$

ANSWERS 3.04

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1. 19/15; **2.** 29/24; **3.** 1/2; **4.** 1/12; **5.** 99/221; **6.** 25/828; **7.** 16/9; **8.** 7/12; **9.** $\frac{2x+10y}{3}$;

10. $\frac{3x-13y}{5}$; **11.** $4x$; **12.** $-2y$; **13.** $-\frac{3}{2x}$; **14.** $-\frac{1}{x^2}$; **15.** x; **16.** y; **17.** $x+2$; **18.** y−3; **19.** $2x-5$;

20. y+1; **21.** y+2; **22.** $x-1$; **23.** $\frac{x-3}{x-2}$; **24.** $\frac{x-4}{x+3}$; **25.** $\frac{11}{12}$; **26.** $\frac{1}{24}$; **27.** $\frac{37}{40}$; **28.** $\frac{13}{40}$;

29. $\frac{-4}{45}$; **30.** $\frac{46}{45}$; **31.** $\frac{9y+7x}{15xy}$; **32.** $\frac{20y^2-3x}{28x^2y^2}$; **33.** $\frac{59}{40x}$; **34.** $\frac{21y-20x}{24xy}$; **35.** $\frac{2y-15x}{9x^2y}$;

36. $\frac{9x+20y^2}{24x^2y^2}$; **37.** $\frac{14x^2+15y}{12x^3y^2}$; **38.** $\frac{20x^2-24y}{15x^2y^2}$; **39.** $\frac{6-35x^2y}{25x^3y^2}$; **40.** $\frac{10x-7y}{18xy^2}$; **41.** $\frac{15xy^2-64}{72xy^3}$;

42. $\frac{63x^3y+8}{45x^4y^3}$; **43.** $\frac{4x^2-x-9}{(x-3)(x-2)(x-1)}$; **44.** $\frac{2x+21}{(x-2)(x-3)(x+3)}$; **45.** $\frac{x^2-8x-12}{(x+1)(x+3)(x-4)}$;

46. $\frac{2x^2+3x-6}{(x-2)(x+2)(x+3)}$; **47.** $\frac{x^2+12}{(x-3)(x-1)(x+4)}$; **48.** $\frac{2x^2-9x+6}{(x-2)(x+2)(x-3)}$; **49.** $\frac{4x^2+9x+9}{(x-3)^2(x+3)}$;

50. $\frac{4x^2-15x-9}{(x+3)^2(x-3)}$; **51.** $\frac{2x+15}{x(x-5)^2}$; **52.** $\frac{8x-3}{x(x-1)^2}$; **53.** $\frac{7x^2+40x-35}{(x-7)^2(x+5)}$; **54.** $\frac{x-1}{2x}$;

55. $\frac{2a+3b}{2a}$; **56.** $\frac{y-x}{y-2x}$; **57.** 0; **58.** $\frac{x-4}{(x-2)(x-3)}$; **59.** $\frac{-17a}{(5a+3)(2a-1)(a+4)}$; **60.** −1;

61. −1; **62.** −1; **63.** 1; **64.** $-(x+4)$ or $-x-4$; **65.** $-(x+6)$ or −x−6; **66.** 0; **67.** $\frac{2x}{x-2}$;

68. −2; **69.** 2; **70.** $x+2$; **71.** x; **72.** $\frac{x+3}{3}$; **73.** $\frac{x+5}{5}$; **74.** $\frac{6x+25}{(x-5)(x+5)}$.