3.08 *Ratio and Proportion*

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Some of the best answers to the age-old question, "What good is math?" may be found in this section on ratio and proportion. The concept of ratio and proportion is a convenient way of organizing given information, setting up a simple equation, and using this to determine an unknown quantity. The applications to everyday life are innumerable.

First, what is a **ratio**, and what is a **proportion**? **A ratio is simply the quotient of two numbers.** This is where we get the word "**ratio**nal numbers." **A rational number is any number that can be expressed as the "ratio" or quotient of two integers (denominators ≠ 0).** Every time you write a fraction, you have written a ratio. **A proportion is simply the equating of two ratios. Whenever one ratio** (or fraction) **equals another ratio** (or fraction)**, this is a proportion.** In fact, the first concept and the first 14 exercises in the last section were **proportions**. Do you remember the **definition of equality of fractions?** Two fractions, $\frac{a}{b}$ and $\frac{c}{d}$, are equal if and only if $\mathbf{a \cdot d = b \cdot c}$.

$$\frac{a}{b} = \frac{c}{d} \quad \textbf{means that} \quad a \bullet d = b \bullet c$$

By the way, this section could be sub-titled "applications of fractional equations."

EXAMPLE 1: **If 5 packages of a product sell for \$6.00, how much should 8 packages cost?**

SOLUTION: One obvious solution is to find the cost of one package by dividing **\$6** by **5** (which is \$1.20), then multiply by the number of packages that you want (**8 pkg**) which is \$9.60. With a calculator, this is all very easy.

However, as an introduction to a larger and very useful method, let's set up a proportion. Remember in this that the expression **"how much"** always means **"x"**.

While there are many ways to set up a proportion, begin with the ratio: $\frac{\text{packages}}{\text{price}}$. The proportion will be $\frac{\text{packages}}{\text{price}} = \frac{\text{packages}}{\text{price}}$. However you set it up, it is very important that you be consistent. This is the ratio that you need:

$$\frac{\text{packages}}{\text{price}} : \frac{5\text{ packages}}{\$\,6} = \frac{8\text{ packages}}{\$\,x}$$

From this you can see:

$$\mathbf{5 \bullet x = \$6 \bullet 8}$$
$$\mathbf{5x = \$48}$$
$$\mathbf{x = \$48/5 \text{ or } \$9.60}$$

Alternate Solution:

$$\frac{\text{price}}{\text{packages}} : \frac{\$\,6}{5\text{ packages}} = \frac{\$\,x}{8\text{ packages}}$$

Notice—the equation is same as before:

$$\mathbf{5 \bullet x = \$6 \bullet 8}$$
$$\mathbf{5x = \$48}$$
$$\mathbf{x = \$48/5 \text{ or } \$9.60}$$

EXAMPLE 2: **If 5 packages of a product sell for \$6.00, how many packages can you buy for \$20?**

SOLUTION: Set up a ratio $\frac{\text{cost}}{\text{packages}} : \frac{\$6}{5\text{ packages}} = \frac{\$20}{x\text{ packages}}$

$$\mathbf{6x = 100}$$
$$\mathbf{x = 100/6 \text{ or } 16.7 \text{ pkg}}$$

(Technically, you could only buy **16 packages**, since they come in packages, and \$20 is not enough to buy the 17th package.)

EXERCISES:

1. If a 9-oz. bag of potato chips costs \$1.50, what would you expect to pay for a 16 oz. bag?

2. If 8 pounds of dog food costs \$6.50, what would you expect to pay for 25 pounds?

3. If a 20-oz. bottle of ketchup costs \$0.90, what would you expect to pay for a 44 oz. bottle?

4. If a 6 packages of a product cost \$2.50, what would you expect to pay for a 40 packages?

5. If on the interstate it takes 3 hours to travel 200 miles, how far can you travel at this rate in 16 hours?

6. If on the interstate it takes 3 hours to travel 200 miles, how long will it take to travel 750 miles at this rate?

7. If it takes 3 hours to drive 65 miles in the mountains, how long will it take to drive 100 miles at the same rate?

8. If it takes 35 minutes to drive 15 miles in the mountains, how far can you drive at this rate in 2 hours?

9. If a typist can type 5 pages in 13 hours, how long will it take him to type a 14 page report?

10. If a typist can type 13 pages in 5 hours, how many pages can she type in 14 hours?

11. If an author can complete 5 sections in 2 days, how long many sections can she complete in 17 days?

12. If an author can complete 2 sections in 5 days, how long will it take him to complete 17 sections?

13. A gardening chemical is to be applied at 5 teaspoons per 300 sq. ft. How many teaspoons should be applied for 10,000 sq.ft.?

14. A gardening chemical is to be applied at 7 teaspoons per 300 sq. ft. How many sq. ft. can be treated by 100 teaspoons of the chemical?

15. A salt water brine is to be made at the rate of 3 pounds of salt for each 8 gallons of water. How much salt should be used for 25 gallons of water?

16. A salt water brine is to be made at the rate of 3 pounds of salt for each 8 gallons of water. How much water should be used with 25 pounds of salt?

ANSWERS 3.08

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1. $2.67; **2.** $20.31; **3.** $1.98; **4.** $16.67; **5.** 1066.67 mi; **6.** 11.25 hr; **7.** 4.62 hr; **8.** 51.43 mi; **9.** 36.4 hr; **10.** 36.4 pg; **11.** 42.5 sec; **12.** 42.5 days; **13.** 166.67 tsp; **14.** 4285.71 sq ft; **15.** 9.375 lb; **16.** 66.67 gal.