4.05 *More Slope*

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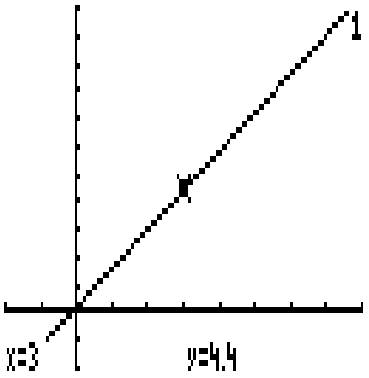
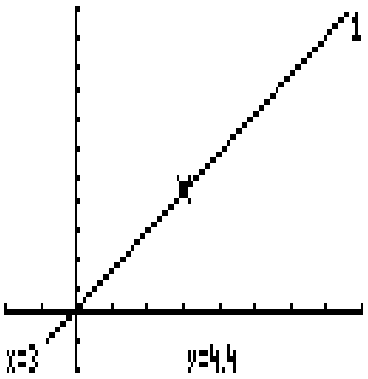
In a previous section, you learned that the slope of a line describes the "steepness" of a line. It is the **rise over the run**, the vertical divided by the horizontal distance between any two points on a given line. In this section, a formula will be developed for the **slope between two given points**. In addition, similar formulas can be developed for the **distance** and the **midpoint** between two points.



EXAMPLE 1. Find the distance between (2,3) and (7,10).

Solution. From the graph at the right you can see that the **rise (vertical distance) is 7** and the **run (horizontal distance) is 5**. What did you do to get the 7 and the 5? You can probably see that you subtracted the Y values to get 7, and you subtracted the X values to get 5. Then the slope between these points is **m=7/5**.



In the general case, consider the points with **subscripts (X_1,Y_1) and (X_2,Y_2).** Using the same procedure (subtraction), you can see that the **rise is Y_2-Y_1** and the **run is X_2-X_1.**

Therefore, the formula for slope is:

$$m = \frac{rise}{run} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

NOTE: Be sure the Y difference is in the numerator, and the X difference is in the denominator.

Use the slope formula to find the slope between the points:

1a) (X_1,Y_1) and (X_2,Y_2)
(2, 3) and (5, 15)

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \_\_\_\_\_$$

$$= \_\_\_\_\_$$

b) (X_1,Y_1) and (X_2,Y_2)
(5, 15) and (2, 3)

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \_\_\_\_\_$$

$$= \_\_\_\_\_$$

2a) (6, 1) and (4, -2)

b) (4, -2) and (6, 1)

Did you notice from these exercises that the order of the points does not matter? However, as mentioned before, be certain the Y difference is in the numerator and the X difference is in the denominator. Also, be sure to be consistent with the order of subtraction.

3. (-2, 4) and (6, 8)

4. (4, -2) and (6, 8)

5. (4, -2) and (6, -8)

6. (1, -3) and (-1, -5)

7. (-2, 2) and (4, -10)

8. (-2, 6) and (10, 4)

9. (-6, -12) and (4, -8)

10. (-4, -16) and (-16, -4)

11. (-8, 7) and (-2, 3)

12. (-2, 5) and (4, -4)

13. (0, 4) and (6, 0)

14. (4, 0) and (0, 6)

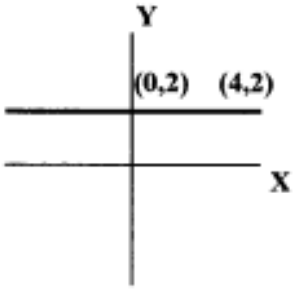
15. (-6, 0) and (0, 4)

16. (0, -6) and (4, 0)

17. (-8, 0) and (0, -4)

18. (-12, 0) and (0, -2)

19. Consider the points on the horizontal line Y = 2.

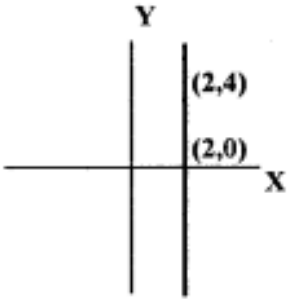


Find the slope between the indicated points.

rise = _____; run = _____

m = _____

20. Consider the points on the vertical line X = 2.



Find the slope between the indicated points.

rise = _____; run = _____

m = _____

21. The slope of any horizontal line is a)_____, since zero divided by any number is b) _____.

22. The slope of any vertical line is a)_____, since any number divided by zero is b) _____.

In 23 - 26, find the slope of each of the following lines.

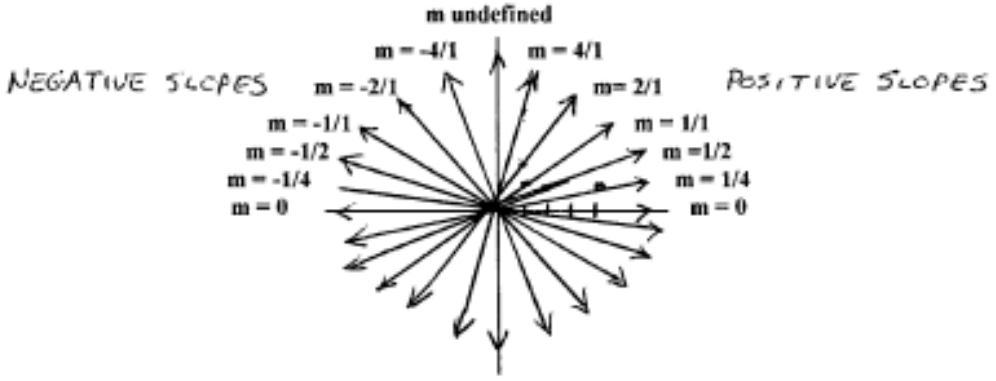
23. Y = 6 **m = _____**

24. Y = -6 **m = _____**

25. X = -6 **m = _____**

26. X = 6 **m = _____**

SLOPES of LINES



PARALLEL and PERPENDICULAR LINES

It should be pretty obvious that if two lines are parallel, then they are going in the same direction, so they have the same slope. What happens with slopes of perpendicular (⊥) lines is not so evident. You have probably noticed that any line that leans to the right (increasing from left to right) has a positive slope, while any line that leans to the left (decreasing from left to right) has a negative slope. If two lines are perpendicular, then one must lean to the left while the other leans to the right. Moreover, if one of the lines has a "steep" slope, then the other will have a "shallow" slope. This means that if one slope is positive, then the other slope must be negative, and if one slope is a "large" number, then the other must be a "small" (that is, a small fractional value) number. Putting this together, if two lines are perpendicular, then one slope must be the negative reciprocal of the other slope. This can be summarized as follows:

SLOPES OF PARALLEL AND PERPENDICULAR LINES

Parallel Lines--Same Slope

Perpendicular (⊥) Lines--Slope is the negative reciprocal*

* except for vertical and horizontal lines.

EXAMPLE 2. The slope of a line is 2/3. Find the slope of a line that is
a) parallel to the line **b)** perpendicular to the line.

Solution. m = 2/3 = slope of given line.
a) Line parallel: m=same slope, **m= 2/3.**
b) Line perpendicular: m= negative reciprocal, **m= -3/2.**

EXAMPLE 3. The slope of a line is -4. Find the slope of a line that is
a) parallel to the line **b)** perpendicular to the line.

Solution. m = -4 = slope of given line.
a) Line parallel: m=same slope, **m= -4.**
b) Line perpendicular: m= negative reciprocal, **m= 1/4.**

EXERCISES. Given the slope **m** of a line, find the slope of a line that is

a) parallel to the line **b)** perpendicular to the line.

27.	**m = 3**	a) _____	b) _____
28.	**m = 3/4**	a) _____	b) _____
29.	**m = - 3**	a) _____	b) _____
30.	**m = -3/2**	a) _____	b) _____
31.	**m = -1**	a) _____	b) _____
32.	**m = 1/7**	a) _____	b) _____
33.	**m = -2/5**	a) _____	b) _____
34.	**m = 5/2**	a) _____	b) _____
35.	**m = 2**	a) _____	b) _____
36.	**m = 0**	a) _____	b) _____

EXAMPLE 4. The equation of a line is $y = 5x + 4$. Find the slope of a line that is
a) parallel to the line **b)** perpendicular to the line.

Solution. m = 4 = slope of given line.
a) Line parallel: m=same slope, **m= 5.**
b) Line perpendicular: m= negative reciprocal, **m= -1/5.**

EXAMPLE 5. The equation of a line is $5x + 4y = 20$. Find the slope of a line that is
a) parallel to the line **b)** perpendicular to the line.

Solution. You must first find the slope of the given line by solving for y.

$5x + 4y = 20$ Add -5x to each side of the equation.
$4y = -5x + 20$ Divide both sides by 4.
$y = -5/4\ x + 5$ The slope of the given line is m = -5/4.

a) Line parallel: m=same slope, **m= -5/4.**
b) Line perpendicular: m= negative reciprocal, **m= 4/5.**

[NOTE: In the examples above, the y-intercepts of the given lines are irrelevant and not even used!]

EXERCISES. Find the slope of a line that is
a) parallel to the given line **b)** perpendicular to the given line.

37. $y = 2x + 3$

38. $y = -3x - 5$

39. $y = -\frac{2}{3}x - 2$

40. $y = \frac{5}{2}x + 3$

41. $2x + y = 6$

42. $4x - y = 8$

43. $3x - 2y = -6$

44. $3x + 5y = 15$

45. $-8x - y = 12$

46. $-12x + 9y = 16$

SLOPE SUMMARY

1. The equations of straight lines are usually given in **slope-intercept form: y = mx + b** or in **standard form Ax + By = C**.

2. If the equation is in the form y = mx + b, then the **slope** is **m**, the coefficient of x. The **y-intercept** is **b**, the number term. The equation can most easily be graphed by the **slope-intercept method** of **Section 4.02**.

3. If the equation is given in the form Ax + By = C, then you must first solve for y in order to find the slope of the line. The equation can most easily be graphed by the **two-intercept method** (i.e., let x = 0 and find y; let y = 0 and find x) of **Section 4.03**.

4. The slope between two points (X_1,Y_1) and (X_2,Y_2) is $m = \frac{rise}{run} = \frac{Y_2 - Y_1}{X_2 - X_1}$.

5. A line with a positive slope always leans to the right (uphill).
   A line with a negative slope always leans to the left (downhill).
   The slope of a horizontal line is zero. The slope of a vertical line is undefined.

6. **Parallel lines** have the **same slope**.
   For **perpendicular lines**, the slope of one line is the **negative reciprocal** of the other.

47. Find the slope of the line between (4, -3) and (-6, -2).

48. Find the slope of the line 2x - 7y = 14.

49. Find the slope of the line 7x + 3y = 9.

50. Find the slope of the line between (-3, 5) and (-5, -3).

51. Given the line y = -2/3 x + 6, find the slope of a line that is a) parallel to it; b) perpendicular to it.

52. Given the line 2x - 5y = 10, find the slope of a line that is a) parallel to it; b) perpendicular to it.

ANSWERS 4.05

p. 334 - 340:

1. 4; **2.** 3/2; **3.** 1/2; **4.** 5; **5.** -3; **6.** 1; **7.** -2; **8.** -1/6; **9.** 2/5; **10.** -1; **11.** -2/3; **12.** -3/2; **13.** -2/3; **14.** -3/2; **15.** 2/3; **16.** 3/2; **17.** -1/2; **18.** -1/6; **19.** 0; **20.** Undefined; **21a)** 0, **b)** 0; **22a)** Undefined, **b)** Undefined; **23.** 0; **24.** 0; **25.** Undefined; **26.** Undefined; **27a)** 3, **b)**-1/3; **28a)** 3/4, **b)** -4/3; **29a)** -3, **b)** 1/3; **30a)** -3/2, **b)** 2/3; **31a)** -1, **b)** 1; **32a)** 1/7, **b)** -7; **33a)** -2/5, **b)** 5/2; **34a)** 5/2, **b)** -2/5; **35a)** 2, **b)** -1/2; **36a)** 0, **b)** Undef; **37a)** 2, **b)** -1/2; **38a)** -3, **b)** 1/3; **39a)** -2/3, **b)** 3/2; **40a)** 5/2, **b)** -2/5; **41a)** -2, **b)** 1/2; **42a)** 4, **b)** -1/4; **43a)** 3/2, **b)** -2/3; **44a)** -3/5, **b)** 5/3; **45a)** -8, **b)** 1/8; **46a)** 4/3, **b)** -3/4;**47.** -1/10; **48.** 2/7; **49.** -7/3; **50.** 4; **51a)** -2/3; **b)** 3/2; **52a)** 2/5; **b)** -5/2.